SINUSOIDAL VARIABLES

- In AC circuits, the basic current and voltage variables are considered to be sinusoidal functions.

- A general expression for a sinusoidal function is

\[ x(t) = X_m \cos(\omega t + \theta) \]

- Here, \( x(t) \) is the sinusoidal variable, \( X_m \) is the magnitude or amplitude, \( \omega \) is the angular frequency, and \( \theta \) is the phase angle.
SINUSOIDAL VARIABLES

- In our work, the argument of the sinusoidal function is expressed in radians and so:
  - $\theta$ is expressed in radians
  - $\omega$ is expressed in radians/s

\[ \omega T = 2\pi \]

---

ANGULAR FREQUENCY $\omega$

- We can also express the argument of the sinusoidal function in terms of the frequency $f$ stated in Hz or cycles per second with

\[ \omega = 2\pi f \]

- $\omega$ is in radians/s, $\omega$ is in radians/cycle, $f$ is in Hz.
ANGULAR FREQUENCY $\omega$

- The sinusoidal function is periodic with a period of $T$ s, where
  
  \[ s/\text{cycle} \quad \rightarrow \quad T = \frac{1}{f} \quad \text{cycles/s} \]

  that is each cycle (or period) lasts $T$ s

- We may express $x(t)$ therefore as
  
  \[ x(t) = X_m \cos(\omega t + \theta) = X_m \cos\left(2\pi f t + \theta\right) \]
  
  \[ = X_m \cos\left(\frac{2\pi}{T} t + \theta\right) \]

AC SYSTEM

- The current in the AC system is specified by
  
  \[ i(t) = I_m \cos(\omega t + \theta_i) \]

- The use of the cosine function is totally arbitrary since for any arbitrary angle $\phi$
  
  \[ \sin(\phi) = \cos\left(\frac{\pi}{2} - \phi\right) \]

  or equivalently
  
  \[ \cos(\phi) = \sin\left(\frac{\pi}{2} - \phi\right) \]
AC SYSTEM

- The voltage is also sinusoidal
  \[ v(t) = V_m \cos(\omega t + \theta_v) \]

- The power is
  \[ p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \]

- Recall that
  \[ \cos \phi \cos \xi = \frac{1}{2} [\cos(\phi + \xi) + \cos(\phi - \xi)] \]

POWER EXPRESSION: AC NETWORK

- We are interested to evaluate the average value of the power \( p(t) \)

- As we consider two periodic functions, each with the identical period \( T \), the average value is given by the average value over any single period
AC SYSTEM

Therefore

\[ p_{av} = \frac{1}{T} \int_0^T p(t) \, dt \]

\[ = \frac{V_m I_m}{2T} \int_0^T \left[ \cos(2\omega t + \theta_i + \theta_v) + \cos(\theta_i - \theta_v) \right] \, dt \]

\[ = \frac{V_m I_m}{2T} \int_0^T \left[ \cos(2\omega t + \theta_i + \theta_v) + \cos(\theta_v - \theta_i) \right] \, dt \]

\[ = \frac{V_m I_m}{2T} \cos \left( \frac{\theta_v - \theta_i}{\theta} \right) \cdot T , \]

where, we use the fact that the average value of any sinusoid is \( \theta \), as the positive and negative areas under the curve cancel each other out.

We use the standard definition for the angle \( \theta \)

\[ \theta = \theta_v - \theta_i \]

Therefore,

\[ p_{av} = \frac{1}{2} V_m I_m \cos \theta \]
EFFECTIVE VALUE

- The effective value of a periodic variable is the square root of the average of the squared value of the variable.

- For current

\[ I = \left( \frac{1}{T} \int_0^T i^2(t) \, dt \right)^{\frac{1}{2}} \]

We next evaluate \( I \)

\[ I = \left[ \frac{1}{T} \int_0^T I_m^2 \cos^2(\omega t + \theta_i) \, dt \right]^{\frac{1}{2}} \]

\[ = \frac{1}{2} \left[ \cos 2(\omega t + \theta_i) + \cos(\theta) \right]^{\frac{1}{2}} \]

\[ = \frac{1}{2} \left[ 1 + \cos 2(\omega t + \theta_i) \right] \]

\[ I = \frac{I_m}{\sqrt{2}} \]
EFFECTIVE VALUE

- The value $I$ is referred to as the r.m.s. value
- The r.m.s. value of a sinusoid is equal to its amplitude divided by $\sqrt{2}$
- The 240–V 60–Hz voltage at which electricity is supplied to a dryer is understood to mean that $V = 240 \text{ V}$
  and so we compute $V_m$ from the r.m.s. value, to get the value of the amplitude $V_m$ as

AC SYSTEM

$$V_m = 240 \sqrt{2} = 339.41 \text{ V}$$
with the angular frequency
$$\omega = 2\pi \cdot 60 = 377 \text{ radians/s}$$
and so we have the sinusoid
$$v(t) = 339.41 \cos(377t + \theta_v)$$
- We, henceforth, adopt the convention that the input voltage has $\theta_v = 0$ and measure all other variables’ phase angles with respect to the phase angle of the input voltage
**r.m.s. VALUE OF A SQUARE WAVE**

We consider the square wave

\[ x(t) = \begin{cases} 
  a & t_0 + (n-1)T \leq t \leq t_0 + (2n-1)\frac{T}{2} \\
  0 & t_0 + (2n-1)\frac{T}{2} < t < t_0 + nT 
\end{cases} \quad n = 1, 2, ... 

**r.m.s. VALUE OF A SQUARE WAVE**

We compute the r.m.s. value of \( x(t) \) by evaluating the average value over a cycle

\[
X = \left\{ \frac{1}{T} \int_{t_0}^{t_0+T} [x(t)]^2 \, dt \right\}^{\frac{1}{2}} = \left\{ \frac{1}{T} \int_{t_0}^{t_0+\frac{T}{2}} a^2 \, dt \right\}^{\frac{1}{2}} = \left[ \frac{a^2 \cdot T}{2} \right]^{\frac{1}{2}} = \frac{a}{\sqrt{2}}
\]
IDEAL RESISTOR IN AC NETWORKS

We analyze the behavior of an ideal resistor in a circuit with a sinusoidal voltage source

\[ v(t) = V_m \cos \omega t \]

or in the terms of the r.m.s. voltage \( V \)

\[ v(t) = \sqrt{2} V \cos \omega t \]

Now,

\[ i(t) = \frac{v(t)}{R} = \sqrt{2} \frac{V}{R} \cos \omega t \]
IDEAL RESISTOR IN AC NETWORKS

is the current through the resistor whose \textit{r.m.s.} value is

\[ I = \frac{V}{R} \]

- Since there is a \( \theta \) angle phase difference between the \( v(t) \) and \( i(t) \) sinusoids, we say that the two sinusoids are \textit{in phase} with each other.

The evaluation of the average power is

\[ p_{\text{avg}} = V I \cos(\theta_v - \theta_i) = V I = \frac{V^2}{R} = I^2R \]

- In AC networks, power is always interpreted as \textit{average power} and so we drop the \textit{avg} subscript and write

\[ P = V I = I^2R = \frac{V^2}{R} \]

and \( P \) represents the average power.
EXAMPLE: CUISINART TOASTER

- The two–slot Cuisinart toaster is a 1,500–W load when plugged into a 120–V socket at 60 Hz; we can model the appliance as a simple resistor.
- We compute from

\[ P = \frac{V^2}{R} \]

the value of the resistance

\[ R = \frac{V^2}{P} = \frac{120 \cdot 120}{1,500} = \frac{14,400}{1,500} = 9.6 \, \Omega \]

EXAMPLE: CUISINART TOASTER

- The current is

\[ I = \frac{V}{R} = \frac{120}{9.6} = 12.5 \, A \]

- Now, consider a voltage spike of 125 V and so the dissipated power becomes

\[ P = \frac{V^2}{R} = \frac{125 \cdot 125}{9.6} = 1,627.6 \, W \]

representing an increase of 127.6 W in the toaster consumption – a rather serious 8.5% increase.
Recall the \textit{equation of motion} for a capacitor

\[ i(t) = C \frac{dv}{dt} \]

For a sinusoidal voltage in an AC network

\[ i(t) = C \frac{d}{dt} \left[ \sqrt{2} V \cos \omega t \right] = -\omega C \sqrt{2} V \sin \omega t \]

We use the identity

\[ \sin \phi = \cos \left( \frac{\pi}{2} - \phi \right) = -\cos \left( \frac{\pi}{2} - \phi - \pi \right) = -\cos \left( \phi + \frac{\pi}{2} \right) \]

Thus

\[ i(t) = \omega C \sqrt{2} V \cos \left( \omega t + \frac{\pi}{2} \right) \]
Thus, the voltage across the capacitor and the current through it are

- identical frequency sinusoids
- there is a $\frac{\pi}{2}$ radians difference between the two waveforms
- the current leads the voltage by $\frac{\pi}{2}$ radians

Let

$$I = \omega CV$$

and so

$$i(t) = \sqrt{2}I \cos \left( \omega t + \frac{\pi}{2} \right)$$
IDEALIZED CAPACITOR IN AC NETWORKS

- We summarize
  
  \[ V = \left( \frac{1}{\omega C} \right) I \quad \text{"AC version" of Ohm's Law for capacitors} \]

- The power dissipated by the capacitor is
  
  \[ p(t) = v(t) i(t) = \sqrt{2} V \cos \omega t \sqrt{2} I \cos \left( \omega t + \frac{\pi}{2} \right) \]

  and this simplifies to

\[ p(t) = 2V I \cdot \frac{1}{2} \left[ \cos \left( 2\omega t + \frac{\pi}{2} \right) + \cos \left( -\frac{\pi}{2} \right) \right] \]

\[ = V I \cos \left( 2\omega t + \frac{\pi}{2} \right) \]

- Since the average power value of a sinusoid is 0,
  
  \[ p_{\text{avg}} = 0 \]

  and so for a capacitor

  \[ P = 0 \]
**CAPACITOR EXAMPLE**

- We consider the current through a 200 μF capacitor supplied by a 120–V, 60–Hz source.
- The voltage is given by
  \[ v(t) = \sqrt{2} \, 120 \, \cos \omega t \]
  and the current is therefore
  \[ i(t) = \sqrt{2} \, I \, \cos \left( \omega t + \frac{\pi}{2} \right) \]
  with
  \[ I = (2\pi 60)(120)(200 \cdot 10^{-6}) = 9.048 \, A \]

**IDEALIZED INDUCTOR IN AC NETWORKS**

- Recall the equation of motion for an inductor
  \[ v(t) = L \frac{di}{dt} \]

\[ v(t) = \sqrt{2} \, V \, \cos \omega t \]
and so
\[ i(t) = \frac{1}{L} \int_0^t v(\xi) \, d\xi \]

- For the sinusoidal voltage
\[ v(t) = \sqrt{2} V \cos \omega t \]
we have
\[ i(t) = \frac{1}{L} \int_0^t \sqrt{2} V \cos \omega \xi \, d\xi = \frac{\sqrt{2} V}{\omega L} \sin \omega t \]

- We use the identity
\[ \sin \phi = \cos \left( \phi - \frac{\pi}{2} \right) \]

Thus
\[ i(t) = \sqrt{2} \frac{V}{\omega L} \cos \left( \omega t - \frac{\pi}{2} \right) \]

- Therefore, the voltage across the inductor and the current through it are
  - identical frequency sinusoids
  - there is a \( \frac{\pi}{2} \) radians difference between the two waveforms
IDEALIZED INDUCTOR IN AC NETWORKS

- The current *lags* behind the voltage by \( \frac{\pi}{2} \)

Let

\[
I = \frac{1}{\omega L} V
\]

and so

\[
i(t) = \sqrt{2} I \cos \left( \omega t - \frac{\pi}{2} \right)
\]

We summarize:

\[
V = \omega LI
\]

"AC version" of Ohm’s Law for inductors

The power dissipated by the inductor is

\[
p(t) = v(t) i(t) = \sqrt{2} V \cos \omega t \sqrt{2} I \cos \left( \omega t - \frac{\pi}{2} \right)
\]

and this simplifies to

\[
p(t) = 2VI \cdot \frac{1}{2} \left[ \cos \left( 2\omega t - \frac{\pi}{2} \right) + \cos \left( \frac{\pi}{2} \right) \right]
\]

\[
= VI \cos \left( 2\omega t - \frac{\pi}{2} \right)
\]
IDEALIZED INDUCTOR IN AC NETWORKS

- Clearly

\[ \text{\( p_{avg} = 0 \)} \]

and so

\[ \text{\( P = 0 \)} \]

- Neither capacitors nor inductors consume real power

POWER FACTOR

- We generalize the expressions for resistors, capacitors and inductors for a sinusoidal

\[ \text{\( v(t) = \sqrt{2} V \cos(\omega t + \theta_v) \)} \]

and a current

\[ \text{\( i(t) = \sqrt{2} I \cos(\omega t + \theta_i) \)} \]
### POWER FACTOR

Now, we have shown that the angle $\theta$ takes on the specific values

\[
\theta = \begin{cases} 
0 & \text{for a resistor} \\
\frac{\pi}{2} & \text{for an inductor} \\
-\frac{\pi}{2} & \text{for a capacitor}
\end{cases}
\]

but for a network with an arbitrary combination of $R$, $L$ and $C$ components, $\theta$ is unknown.

- We also showed earlier that the average value of power is

\[
p_{\text{avg}} = V I \cos(\theta) \quad (*)
\]

for

\[
\theta = \theta_v - \theta_i
\]

- Power engineers call the quantity $\cos \theta$ the **power factor**

\[
p.f. \triangleq \cos \theta
\]
The expression in (*) is general and may apply to any circuit or circuit element, any combination of $R$, $L$ and $C$ elements and, more importantly, any component with sinusoidal voltage and current.

We interpret $p.f.$ to be the fraction that the real power represents of the total apparent power used by a particular component or system.

A $p.f.$ EXAMPLE

A small industrial customer is supplied by a $24-kV$, $60-Hz$ source to run a $1.5-MW$ real power load through a line with resistance $R$.

We compute the ratio of the real power line losses on the feeder line under two distinct $p.f.$ values:

$$P_{p.f.1} = 0.5 \quad \text{and} \quad P_{p.f.2} = \frac{\sqrt{3}}{2}$$
EXAMPLE ON p.f.

Basic assumption: the voltage drop through $R$ is negligibly small

$P = 1.5 \text{ MW}$

$v(t) = \sqrt{2} V \cos \omega t$

EXAMPLE ON p.f.

Since

$P = VI \cos \theta = 1.5 \text{ MW}$,

the r.m.s. value of the feeder current we compute under $p.f._1$

$I_1 = \frac{1.5 \text{ MW}}{\frac{1}{2}\left(24 \text{ kV}\right)}$

and also under $p.f._2$

$I_2 = \frac{1.5 \text{ MW}}{\frac{\sqrt{3}}{2}\left(24 \text{ kV}\right)}$

$\frac{\text{MW}}{\text{kV}} = \text{kA}$
EXAMPLE ON p.f.

The ratio of the losses is therefore

\[
\frac{I_1^2 R}{I_2^2 R} = \left( \frac{I_1}{I_2} \right)^2 = \left( \frac{\sqrt{3}}{2} \right)^2 = 3
\]

The losses are 3 times higher under the poor value p.f.₁ than under the higher value p.f.₂

APPARENT, REAL AND REACTIVE POWER

\[
\tilde{S} = \tilde{V} \tilde{I}^* \\
\tilde{S} = Ve^{j\theta_v} (I e^{j\theta_i})^* = Ve^{j\theta_v} (I e^{-j\theta_i}) \\
\tilde{S} = VI \cos (\theta_v - \theta_i) + j VI \sin (\theta_v - \theta_i) \\
\tilde{S} = \frac{VI \cos (\theta_v - \theta_i)}{\theta} + \frac{j VI \sin (\theta_v - \theta_i)}{\theta} \\
\tilde{S} = \frac{P}{\theta} + j \frac{Q}{\theta} \\
\tilde{S} = P + j Q
\]
There is an important relationship between the apparent power $S$, the real power $P$ and the reactive power $Q$; we represent this relationship by the so-called power triangle in the complex plane.

The power triangle is drawn as follows: 

$S = VI \cos \theta$

$Q = VI \sin \theta$

$P = VI \cos \theta$

$W = VI \sin \theta$
THE POWER TRIANGLE

\[
\theta > 0 \quad \text{current } lags \text{ voltage}
\]

\[
\theta < 0 \quad \text{current } leads \text{ voltage}
\]

\[
S = VI
\]

\[
P = S \cos \theta \quad \text{real power}
\]

\[
Q = S \sin \theta \quad \text{reactive power}
\]

\[
S^2 = P^2 + Q^2 \quad \text{apparent power}
\]

---

THE POWER TRIANGLE

- For any arbitrary load
  
  \( P > 0 \)

  but,

  \( Q > 0 \) for an inductive load

  \( Q < 0 \) for a capacitive load

- The real power consumed by a load is the rate at which work is done and is measured in \( W \)

- The reactive power is incapable to do work and its average is 0 for a capacitive/inductive element
THE POWER TRIANGLE

- Power suppliers, typically, charge for the $P$ consumption but are also impacted by the $Q$ since the larger the $Q$, the higher the line losses; in certain cases, charges are imposed on the basis of $S$ or explicitly take into account the $p.f.$
- The presence of electric motors, which are highly inductive loads, leads to increased losses on transmission lines.

EXAMPLE: POWER TRIANGLE

- We consider a 250–$V$ induction motor that draws 20 $A$ of current to generate 4.33 $kW$ of real power delivered to its shaft.
- We draw the power triangle using:

  
  $S = V I = 250 \times 20 = 5,000 \text{ } VA = 5 \text{ } kVA$
  
  $P = 4.33 \text{ } kW$

  $\cos \theta = \frac{P}{S} = \frac{4.33}{5} = 0.866$

  $\theta = \cos^{-1}(0.866) = \frac{\pi}{6}$

  $Q = S \sin \theta = 2.5 \text{ } kVAR$
EXAMPLE: POWER TRIANGLE

- Reactive power $Q = 2.5\, kVAR$
- Active power $P = 4.33\, kW$
- $5.0\, kVA$

POWER FACTOR CORRECTION

- The smaller the $p.f.$, the worse the utilization of power is; the ideal is to get as near as possible to the perfect $p.f.$ of 1.0
- Sometimes, it is desirable or necessary to use capacitors to correct the $p.f.$ to offset the $VARs$ of the inductive elements
- A $p.f.$ corrective action can result in increased real power delivery to the loads
EXAMPLE: POWER FACTOR CORRECTION

- A transformer is operating close to its $kVA$ rating and is used to deliver 600 $kVA$ at a 0.75 $p.f.$
- There is a 20% forecasted growth in the real power demand for next year
- This growth needs to be accommodated without any investment in a new transformer by installing capacitors for $p.f.$ correction
EXAMPLE: POWER FACTOR CORRECTION

- The current situation is characterized by

\[ p \cdot f. = 0.75 = \cos \theta \]

\[ \theta = \cos^{-1}(0.75) = 0.72 \text{ radians} \]

\[ P = 600 \cdot 0.75 = 450 \text{ kW} \]

\[ Q = 600 \cdot 0.66 = 397 \text{ kVAR} \]

- The forecasted situation is

\[ P_{\text{new}} = 450(1.2) = 540 \text{ kW} \]

\[ p \cdot f._{\text{new}} = \frac{540}{600} = 0.9 \]

- The difference between \( Q = 476 \text{ kVAR} \) and \( Q_{\text{new}} = 261 \text{ kVAR} \) is compensated by the installation of capacitors with

\[ Q_c = 476 - 261 = 215 \text{ kVAR} \]
EXAMPLE: POWER FACTOR CORRECTION

We can determine the capacitance of the p.f. correction capacitors

\[ Q_c = V_c I_c = V_c \left( \omega CV_c \right) \]

\[ C = \frac{Q_c}{\omega V_c^2} \]

We assume that the input voltage to the capacitors is at 12 kV, and so

\[ C = \frac{215 \text{kVAR}}{\left(377\right)\left(12\right)^2 \left(kV\right)^2} = \left(3.96\right) \times 10^{-3} \text{ F} \]
In the US, residential service is typically provided from a 4.16–kV feeder line through a step-down transformer to the 120/240 V household voltage.

- All outlets provide 120 V
- Some outlets provide 240 V electricity (air conditioning, heavier duty appliances)
The provision of 240–V service is done by:

- grounding the center tap of the secondary side of the transformer
- using the other two ends of the windings at the ±120 V supply to obtain the 240–V potential
THE RESIDENTIAL ELECTRICITY SUPPLY

\[ v_1 = 120\sqrt{2} \cos(377t) \]

\[ v_2 = -120\sqrt{2} \cos(377t) \]

\[ v_1 - v_2 = 240\sqrt{2} \cos(377t) \]

- Analytically

\[ v_1(t) = 120\sqrt{2} \cos 377t \]

\[ v_2(t) = 120\sqrt{2} \cos(377t + \pi) \]

\[ = -120\sqrt{2} \cos 377t \]

and therefore

\[ v_1(t) - v_2(t) = 240\sqrt{2} \cos 377t \]
We consider the three loads served by a three–wire 120 / 240–V system with

1,200 W at 120 V on phase A, p.f. = 1.0
2,400 W at 120 V on phase B, p.f. = 1.0
4,800 W at 240 V, p.f. = 1.0

We wish to compute the currents in the wires

We start with the relationship

\[ P = V I \cos \theta = VI \]
RESIDENTIAL LOAD EXAMPLE

- For the 4,800 W load
  \[ I_{4,800} = \frac{4,800}{240} = 20 \, A \]

- For the 2,400 W load
  \[ I_{2,400} = \frac{2,400}{120} = 20 \, A \]

- For the 1,200 W load
  \[ I_{1,200} = \frac{1,200}{120} = 10 \, A \]

Note that KCL induces a current of 10 A in the neutral leg and therefore the unbalanced load creates a nonzero current in the neutral.

This case differs from the typical, balanced conditions we encounter in which each hot leg has the same magnitude current and the neutral current vanishes.
THREE – PHASE AC NETWORKS

- Today’s power systems use the three–phase $(3\phi)$ generators to produce electricity and $3\phi$ transmission lines to “transport” it to various parts of the network.
- The interconnection of network elements into a $3\phi$ network is done typically using either the delta $(\Delta)$ or the wye $(Y)$ configuration.
- We examine a $Y$–connected $3\phi$ generator to a $3\phi$ load.
The phase voltages are measured with respect to the neutral.

\[
v_a(t) = V \sqrt{2} \cos \omega t \quad \leftrightarrow \quad \vec{V}_a = V e^{j0}
\]

\[
v_b(t) = V \sqrt{2} \cos \left( \omega t + \frac{2\pi}{3} \right) \quad \leftrightarrow \quad \vec{V}_b = V e^{j \frac{2\pi}{3}}
\]

\[
v_c(t) = V \sqrt{2} \cos \left( \omega t - \frac{2\pi}{3} \right) \quad \leftrightarrow \quad \vec{V}_c = V e^{-j \frac{2\pi}{3}}
\]

Note that the voltages are equal in magnitude and are exactly \( \pm \frac{2\pi}{3} \) radians from one another (balanced voltages).
THREE – PHASE AC NETWORKS

- Consequently,
\[ \vec{V_a} + \vec{V_b} + \vec{V_c} = 0 \]
- The voltage between two-phases are typically called line voltages; for example the line \( a \) to the line \( b \) voltage is
\[ v_{ab}(t) = v_{a0}(t) + v_{0b}(t) = v_{a0}(t) - v_{b0}(t) \]
and so
\[ v_{ab}(t) = V \sqrt{2} \cos \omega t - V \sqrt{2} \cos \left( \omega t + \frac{2\pi}{3} \right) \]

- Now, for a balanced network, the phase voltage \( r.m.s. \) values are equal
\[ V_a = V_b = V_c = V_p \leftarrow r.m.s. \text{ phase voltage} \]
- Therefore
\[ v_{ab}(t) = V_p \sqrt{2} \cos \omega t - V_p \sqrt{2} \cos \left( \omega t + \frac{2\pi}{3} \right) \]
- We make use of the identity
\[ \cos \phi - \cos \xi = -2 \sin \left[ \frac{1}{2} (\phi + \xi) \right] \sin \left[ \frac{1}{2} (\phi - \xi) \right] \]
THREE – PHASE AC NETWORKS

So we obtain

\[ v_{ab}(t) = V_p \sqrt{2} \cdot (-2) \sin \left( \omega t + \frac{\pi}{3} \right) \cdot \sin \left( -\frac{\pi}{3} \right) \]

\[ = V_p \sqrt{2} \cdot 2 \sin \frac{\pi}{3} \cdot \sin \left( \omega t + \frac{\pi}{3} \right) \]

\[ = \sqrt{3} \frac{V_p}{V_l} \sqrt{2} \sin \left( \omega t + \frac{\pi}{3} \right) \]

\[ = V_l \sqrt{2} \sin \left( \omega t + \frac{\pi}{3} \right) \]

The relationship of the r.m.s. value of line-to-line voltage \( V_l \) relative to that of the phase voltage \( V_p \) is given by

\[ V_l = \sqrt{3} V_p \]

Examples of typical values

<table>
<thead>
<tr>
<th>service type</th>
<th>( V_l )</th>
<th>( V_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>buildings</td>
<td>202 V</td>
<td>120 V</td>
</tr>
<tr>
<td>commercial</td>
<td>480 V</td>
<td>277 V</td>
</tr>
<tr>
<td>residential</td>
<td>416 V</td>
<td>240 V</td>
</tr>
</tbody>
</table>
THREE – PHASE AC NETWORKS

Each phase has apparent power

\[ S_\phi = I_p V_p \]

and so the 3\(\phi\) system has apparent power

\[ S_{3\phi} = 3I_p V_p \]
\[ = \sqrt{3} I_p \sqrt{3} V_p \]
\[ = \sqrt{3} I_p V_t \]

Therefore,

\[ P_{3\phi} = S_{3\phi} \cos \theta \]
\[ Q_{3\phi} = S_{3\phi} \sin \theta \]

where \(\theta\) is the phase angle between the phase current and the voltage and is identical for each phase under balanced conditions.

In fact, we can show that

\[ p_a(t) + p_b(t) + p_c(t) = 3P_\phi \]

and is constant and such a smooth constant level of power constitutes a key advantage of 3\(\phi\)-systems in contrast to 1\(\phi\), where \(p(t)\) is sinusoidal.
THREE – PHASE AC NETWORKS

\[ \text{total power } p_a + p_b + p_c \text{ is constant} \]

\[ \text{average power in } p_a, p_b \text{ or } p_c \]

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EXAMPLE: 3φ NETWORK \( p. f. \) CORRECTION

- The 1φ-motors in a small enterprise are supplied by a 3φ, 208-V transformer.
- The real power demand is 80 kW with a \( p. f. = 0.5 \) and incurs losses of 4 kW.
- We compute \( S_{3\phi} \) using

\[ P_{3\phi} = \sqrt{3}V_l I_p \cos \theta = S_{3\phi} \cdot 0.5 = 80 \text{ kW} \]

so that

\[ S_{3\phi} = 160 \text{ kVA} \]

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EXAMPLE: 3φ NETWORK p. f. CORRECTION

- We also evaluate

\[ I_p = \frac{S_{3φ}}{\sqrt{3} V_t} = \frac{160}{\sqrt{3} 208} = .444 \text{ kA} \]

- Next consider a p.f. correction to 0.9 and so

\[ S'_{3φ} = \frac{80}{0.9} = 88.9 \text{ kVA} \ll 160 \text{ kVA} \]

EXAMPLE: 3φ NETWORK p. f. CORRECTION

- Also, the corresponding phase current is

\[ I'_p = \frac{88.9}{\sqrt{3} 208} = .247 \text{ kA} \]

- We also evaluate the losses under corrected p.f.

\[ R \left( I'_p \right)^2 = \frac{4}{\left( .444 \right)^2} \left( .247 \right)^2 = 1.24 \text{ kW} \]
The other way to connect 3φ elements is the Δ connection without a neutral line.

The comparison of the key characteristics of the two connection schemes is summarized by the table:

<table>
<thead>
<tr>
<th>variable</th>
<th>Y–connection</th>
<th>Δ–connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>r.m.s. current</td>
<td>$I_t = I_p$</td>
<td>$I_t = \sqrt{3} I_p$</td>
</tr>
<tr>
<td>r.m.s. voltage</td>
<td>$V_t = \sqrt{3} V_p$</td>
<td>$V_t = V_p$</td>
</tr>
<tr>
<td>3φ power</td>
<td>$P_{3φ} = 3 V_p I_p \cos \theta$</td>
<td>$P_{3φ} = \sqrt{3} V_t I_t \cos \theta$</td>
</tr>
</tbody>
</table>