Problem 1: An anemometer mounted at a height of 10 m above a surface (Hellman exponent $\alpha = 0.2$) shows a wind speed of 10 m/s at the temperature of 0°C, when the air density 1.291 kg/m³.

(i) Evaluate the wind power density in W/m² where the anemometer is mounted (20 points)

(ii) Determine the wind speed in m/s at the height of 40 m (20 points)

(iii) Estimate the wind power density in terms of W/m² at the height of 40 m if the temperature is 30°C at the height of 40 m. (20 points)

\[
\text{i) Power density } = \frac{P}{A} = \frac{1}{2} \rho v^3 = \frac{1}{2} (1.291)(10)^3 = 645 \text{ W/m}^2
\]

\[
\text{ii) } \left( \frac{v_{40}}{v_{10}} \right)^{\alpha} \Rightarrow v_{40} = v_{10} \left( \frac{H_{40}}{H_{10}} \right)^{\alpha} = 10 \left( \frac{40}{10} \right)^{0.2} = 13.2 \text{ m/s}
\]

\[
\text{iii) Correct the density for 30°C } \\
\rho_{30°C,40m} = \frac{353.1 \times e^{-0.0342 \times 40}}{273 + 30} = 1.16 \text{ kg/m}^3
\]

\[
\text{Power density } = \frac{P}{A} = \frac{1}{2} \left( \rho_{30°C,40m} \right) v_{40m}^3 = \frac{1}{2} \left( 1.16 \right)(13.2)^3 = 1331 \text{ W/m}^2
\]

If you used the approximation,

\[
\frac{P_{40}}{P_{10}} = \left( \frac{H_{40}}{H_{10}} \right)^{3\alpha} \Rightarrow P_{40} = P_{10} \left( \frac{H_{40}}{H_{10}} \right)^{3\alpha} \Rightarrow P_{40} = 645 \left( \frac{40}{10} \right)^{3 \times 0.2} = 1482 \text{ W/m}^2
\]

This is roughly off by 10%, because it does not account for density correction at the new temperature. This is not the best equation to use in this case.
Problem 2: [40 points] Determine whether each of the following statements is True or False.

(i) Doubling the velocity of the wind increases the power in the wind by eightfold.

[✓ TRUE] [ ] FALSE

(ii) Doubling the blade diameter increases the power in the wind by a factor of four.

[✓ TRUE] [ ] FALSE

(iii) Horizontal axis Assume the following conditions hold: 15 °C temperature, 1-atm air pressure and a cross-sectional area $A = 1 \text{ m}^2$. The amount of wind energy produced — under the specified conditions — by a wind regime of 100 hours at a speed of 6 m/s is equal to the amount of energy produced by a wind regime of 50 hours at 3 m/s plus 50 hours at 9 m/s (i.e., an average wind speed of 6 m/s).

[ ] TRUE [✓ FALSE]

(iv) According to Betz' limit, the maximum theoretical efficiency of a wind turbine cannot be higher than 65%.

[ ] TRUE [✓ FALSE]