ECE 333 Green Electric Energy

Midterm Exam 1
October 16, 2018
7:00 – 9:00 p.m.
Room ECEB 2017
Closed Book/Closed Notes
Calculators and two cheat sheets allowed
NO CELL PHONES

Name: ____________ Student ID number: ____________

The exam consists of 4 problems and 1 set of true or false and multiple choice questions. Read each problem carefully. In the solution, state clearly each assumption and its justification in every problem and show all your work. Provide all relevant units, as appropriate. Follow the instructions for each problem.

Please write your name on the upper left hand corner and make sure that your name appears on any additional pages you submit.

GOOD LUCK!

<table>
<thead>
<tr>
<th>problem</th>
<th>type</th>
<th>points</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average &amp; RMS value</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>AC analysis</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Wind power</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Rayleigh distribution</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Wind turbine and wind farm design</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>True or False</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Multiple choice</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>
Problem 1: [10 points] Average and RMS value

Consider the sawtooth voltage waveform shown below. Determine in terms of \( V \) and \( T \):

(a) [5 points] Average value of the waveform

\( x(t) = \frac{V}{T} \) for \( 0 \leq t \leq T \)

(b) [5 points] RMS value of the waveform

\[
\text{a) Average value} \quad \text{Avg}(x(t)) = \frac{1}{T} \int_{0}^{T} x(t) \, dt
\]

\[
= \frac{1}{T} \int_{0}^{T} \frac{V}{T} t \, dt = \frac{1}{T} \frac{V}{T} \left[ \frac{t^2}{2} \right]_{0}^{T} = \frac{1}{T} \frac{V}{T} \left[ \frac{T^2}{2} \right] = \frac{V}{2}
\]

\[
\text{b) RMS value} \quad \text{RMS}(x(t)) = \sqrt{\frac{1}{T} \int_{0}^{T} x(t)^2 \, dt}
\]

\[
= \sqrt{\frac{1}{T} \int_{0}^{T} \left( \frac{V}{T} t \right)^2 \, dt}
\]

\[
= \sqrt{\frac{1}{T} \int_{0}^{T} \frac{V^2}{T^2} t^2 \, dt}
\]

\[
= \sqrt{\frac{1}{T} \frac{V^2}{T^2} \left[ \frac{t^3}{3} \right]_{0}^{T}}
\]

\[
= \sqrt{\frac{1}{T} \frac{V^2}{T^2} \left[ \frac{T^3}{3} \right]}
\]

\[
= \frac{V}{\sqrt{3}}
\]
Problem 2: [25 points] AC Analysis

A 10-MVA AC source supplies an inductive load $L$ characterized by an active power demand of 4 MW at 0.8 power factor.

(a) [15 points] Draw the circuit diagram. Determine the value of each of the following variables: magnitude of complex power (apparent power), active power and reactive power drawn by the load. Draw the power triangle and indicate the three power values previously computed.

(b) [5 points] Determine how much additional load $\Delta L$ with the same power factor as $L$ may be added without the violation of the limit of the 10-MVA source. Provide your answer in MVA.

(c) [5 points] Suppose we add a load $L'$ in parallel with load $L$ in the original network in order to obtain a resultant load $(L + L')$ that has the same active power as $L$ and a unity power factor. Draw this new network. Determine the active, the reactive and the apparent powers for $L'$. What type of load is $L'$?

\[ P = \boxed{4 \text{ MW}} \]

\[ \begin{align*}
\text{Apparent Power } & = \frac{P}{\text{pf}} = \frac{4}{0.8} = \boxed{5 \text{ MVA}} \\
\text{Reactive Power } & = \sqrt{s^2 - p^2} = \sqrt{5^2 - 4^2} = \boxed{3 \text{ Mvar}}
\end{align*} \]

b) The old load $|S_{old}| = 5 \text{ MVA}$

The new max load (without violating the limit of the supply) $|S_{new}| = 10 \text{ MVA}$

\[ \begin{align*}
\Delta L & = |S_{new}| - |S_{old}| \\
& = 10 - 5 = \boxed{5 \text{ MVA}}
\end{align*} \]
The combined new load has a power factor of 1.

All of the real power being consumed is by the load $L'$, which is the same as before.

$$P_{L'} = 10 \text{ MW}$$

Since the $pf = 1$,

$$Q_{L+L'} = Q_L + Q_{L'} = 0$$

$$\therefore Q_{L'} = -Q_L = -3 \text{ MVAR}$$

$$S_{L'} = \sqrt{P_{L'}^2 + Q_{L'}^2} = \sqrt{0^2 + (-3)^2} = 3 \text{ MVA}$$
Problem 3: [25 points] Wind power analysis

An anemometer is mounted at the hub part of the nacelle at a height of 50 m above the ground surface. A wind speed of 7 m/s is indicated in the last observed measurement at the turbine site.

The pressure at 50 m is 1 atm, the temperature is 30°C and the Hellman exponent is 0.1 at the turbine site.

(a) [5 points] Evaluate the air density at 50 m.

(b) [5 points] Use the value in (a) to evaluate the wind power density at 50 m.

(c) [5 points] Evaluate the wind power density at a height of 210 m at 30°C, using the Hellman exponent and your result from part (b).

(d) [5 points] Evaluate the wind power density at a height of 210 m at 30°C again, but this time, first compute the air density, and use this new density value to compute the wind power density.

(e) [5 points] Are the results you obtained in (c) and (d) different? If so, why do you think they are different?

\[\rho_{30 \text{ m}, 30^\circ C} = \frac{353.1 \exp (-0.0342 \times 2 \times T)}{T} = \frac{1.1587 \text{ kg/m}^3}{\text{kg/m}^3}\]

\[b) \quad \frac{P}{A} = \frac{1}{2} \left(\frac{\rho_{30 \text{ m}, 30^\circ C}}{\rho_0}\right) v^3 = \frac{1}{2} \times 1.1587 \times 7^3 = 198.7 \text{ W/m}^2\]

\[c) \quad \frac{\rho}{\rho_0} = \left(\frac{H}{H_0}\right)^{3 \alpha} \Rightarrow \rho_{210 \text{ m}} = \rho_0 \left(\frac{210 \text{ m}}{50 \text{ m}}\right)^{3 \times 0.1} = 305.6 \text{ W/m}^2\]

\[d) \quad \rho_{210 \text{ m}, 30^\circ C} = \frac{4}{8} \frac{353.1 \exp (-0.0342 \times 2 \times T)}{T} = \frac{1.1380 \text{ kg/m}^3}{\text{kg/m}^3}\]

\[\frac{v}{v_0} = \left(\frac{H}{H_0}\right)^\alpha \Rightarrow v = v_0 \left(\frac{210 \text{ m}}{50 \text{ m}}\right)^{0.1} = 7 \times \left(\frac{210 \text{ m}}{50 \text{ m}}\right)^{0.1} = 8.080 \text{ m/s}\]
\[
\frac{P}{A} = \frac{1}{2} \left( P_{20\text{m}, 30\text{c}} \right) v_{20}^3 = \frac{1}{2} \left( 1.1380 \right) \left( 8.080 \right)^3
\]

= 300.1 \text{ W/m}^2

c) There is a small difference between the results (approx 1.6\%) which could primarily be attributed to the change in air density with height.

Recall that to derive the expression

\[
\frac{P}{P_0} = \left( \frac{H}{H_0} \right)^{3\kappa}
\]

we assume the air density at \( H \) to be the same as \( H_0 \).
Problem 4: [20 points] Rayleigh Distribution

We consider a wind turbine whose cut-in speed is 5 m/s, a rated wind speed of 14 m/s and cut-out speed is 22 m/s. At the wind turbine site, the air density is 1.225 kg/m$^3$.

(a) [5 points] Based on the collected wind speed data at the turbine location, a Rayleigh distribution with average wind speed of 12 m/s provides a good approximation of the wind speed at the site. Evaluate the average wind power density.

(b) [15 points] Evaluate the expected number of hours in a year that the turbine will not generate because winds are outside the range from the cut-in to the cut-out wind speeds. Show all your work.

\[
\begin{align*}
a) \quad E\left(\frac{P}{A}\right) &= \frac{1}{2} \rho \overline{E}(v^3) = \frac{6}{\pi} \cdot \frac{1}{2} \rho \overline{v}^3 = \frac{6 \cdot 1.225}{\pi} (12)^3 \\
&= 2021.4 \text{ W/m}^2
\end{align*}
\]

\[
\begin{align*}
b) \quad F(\overline{v} \geq V_{co}) + F(\overline{v} \leq V_{ci}) &= 1 - F(\overline{v} \leq V_{co}) + F(\overline{v} \leq V_{ci}) \\
&= 1 - \left(1 - \exp\left[-\frac{\pi}{4} \left(\frac{V_{co}}{\overline{v}}\right)^2\right]\right) + \left(1 - \exp\left[-\frac{\pi}{4} \left(\frac{V_{ci}}{\overline{v}}\right)^2\right]\right) \\
&= \exp\left[-\frac{\pi}{4} \left(\frac{V_{co}}{\overline{v}}\right)^2\right] - \exp\left[-\frac{\pi}{4} \left(\frac{V_{ci}}{\overline{v}}\right)^2\right] + 1 \\
&= 0.1988
\end{align*}
\]

Expected hours the turbine will not generate

\[
= 0.1988 \times 8760 = 1741 \text{ hours}
\]
Problem 5: [20 points] Wind Turbine and Wind Farm Design

We would like to construct a wind farm. The tentative site chosen requires a buffer zone of 10 d by local regulation. The chosen layout is 5 d x 10 d. The selected turbines are 5 MW units with a rotor diameter of 120 m.

\( \text{a (c) [5 points]} \) Evaluate, (in terms of d), the size of the land required if 30 turbines are installed in 5 rows of 6 turbines each.

\( \text{b (d) [15 points]} \) We have a choice of two turbines: 5 MW turbines by Hitachi, which have a rotor diameter of 120 m, and 3 MW turbines by GE, which have a rotor diameter of 90 m. Calculate the power density for each case. Deduce which case will provide a higher power density.

\( \text{c (e) [5 points]} \) We know that the energy in the wind is proportional to the cube of the wind speed. Now imagine a wind turbine is installed at such a site. Assuming an ideal power curve, state under what conditions the electrical power extracted by the turbine does not change with the wind speed? [Hint: Draw an ideal power curve of a wind turbine and indicate all values of wind speed that are of interest]

\[ \text{Area} = (60d) \times (45d) = 2700 \, \text{d}^2 \]

6) \text{Hitachi turbines:}

\[ \text{Power density} = \frac{\text{Total Installed Capacity}}{\text{Area of farm}} = \frac{30 \times 5 \times 10^6}{2700 \times (120)^2} = 3.85 \, \text{W/m}^2 \]
GE turbines:

Power density = \frac{\text{Total installed capacity}}{\text{Total farm area}} = \frac{30 \times 3 \times 10^6}{2400 \times (90)^2} = 4.11 \text{ W/m}^2

The GE turbines will provide higher power density.

c

\[ P \]

\[ P_r \]

\[ v_{\text{cut-in}} \]

\[ v_{\text{rated}} \]

\[ v_{\text{cut-out}} \]

The power extracted by the turbine does not change with the wind speed in the following regions:

When \( 0 \leq v \leq v_{\text{cut-in}} \)

\( v_{\text{rated}} \leq v \leq v_{\text{cut-out}} \)

\( v_{\text{cut-out}} \leq v \)
Problem 6 [20 points] True or False

Each one of the following problems provides statements that require you to state its veracity. Each True or False statement is worth 4 points. Do not guess since incorrect answers without any explanation are penalized. Each explanatory statement in making your selection is carefully considered in the grading. You need to do so for each selected and non-selected item.

a) Under ideal conditions, a highly efficient wind turbine is able to extract 70% of the power in wind for electricity production.

True ______ False __X__

The theoretical maximum efficiency is bounded by the Betz limit, which is approximately 59%.

b) Consider a $4d \times 4d$ layout of a wind farm with 3 rows of 10 turbines in each row. If the buffer area is 10 d, the total area that the wind farm requires is 900 d$^2$.

True ______ False __X__

Area = \((36d + 20d)(8d + 20d)\)

\[= 1568d^2\]
e) The United States leads the world in cumulative installed capacity in wind and solar power.

True ____  False ___

China leads the world in cumulative installed capacity in wind and solar.

d) The wind in the Midwest tends to blow strongest when the electric load is the highest, so there is a virtually perfect match between wind power generation and load.

True ____  False _____

The wind in the Midwest blows strongest when the electric load is the lowest, so there is a virtually perfect mismatch between wind power generation and load.

e) Given a certain wind speed and temperature, the power density in the wind in $W/m^2$ increases exponentially with altitude.

True ____  False ___

$$P = \frac{1}{2} \rho v^3$$

If $v$ is constant, then $P$ depends only on density $\rho$.

$$\rho = \frac{353.1 \exp(-0.0342\frac{T}{T})}{T}$$

If $T$ does not change, then $\rho$ can be written as

$$\rho(z) = K_1 \exp(-K_2 z)$$

where $K_1 = \frac{353.1}{T}$, $K_2 = -0.0342 \frac{T}{T}$

$\rho$ decreases exponentially with $z$. 
Problem 7 [30 points] Multiple Choice Questions

Each multiple-choice question is worth 5 points. The questions may have more than one correct option. Circle all the options that apply.

a) The following loads have a power factor of 0:
   i. Resistive load
   ii. Inductive load
   iii. Capacitive load
   iv. None of the above

   *Ideal inductive and capacitive loads do not consume any real power, so their power factor is 0.*

b) The US state that leads in the amount of cumulative wind capacity installed is:
   i. North Carolina
   ii. California
   iii. Texas
   iv. Iowa

c) The cumulative installed wind capacity in the US is of the order of:
   i. 10 GW
   ii. 1 GW
   iii. 100 GW
   iv. 1000 MW

\[ \sim 89.1 \text{ GW at end of 2017} \]
d) Given a certain wind speed and temperature, how does the power density in the wind in \( W/m^2 \) change with altitude?

i. By a factor of 8

ii. By a factor of 4

iii. Linearly decreases

iv. Decreases exponentially

e) Doubling the diameter of the blades of a wind turbine increases the extracted energy by a factor of:

i. Four

ii. Eight

iii. Three

iv. Two

\[
P = \frac{1}{2} \rho A v^3
\]

\[
A = \frac{\pi d^2}{4} \Rightarrow P = \left( \frac{1}{2} \rho v^3 \right) \left( \frac{\pi}{4} \right) d^2
\]

If \( d' = 2d \)

\[
P' = \left( \frac{1}{2} \rho v^3 \right) \left( \frac{\pi}{4} \right) (2d)^2 = 4 \left( \frac{1}{2} \rho v^3 \right) \left( \frac{\pi}{4} \right) d^2
\]

\[
\therefore P' = 4P
\]

f) The total installed wind capacity in the world is of the order of:

i. 500 GW

\(~ 539.6 \text{ GW} \text{ at the end of 2017}\)

ii. 100 GW

iii. 50 GW

iv. 2000 GW