9. Wind Data Analysis

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WIND POWER MEASUREMENT AND DATA

- The collection of sufficient wind data for the estimation of the generation is an essential task in the assessment of a wind project at a specified site.

- Various measurement devices – cup, sonic detection and ranging (SODAR), and light detection and ranging (LIDAR) anemometers – provide the ability to measure wind speed, its direction and other relevant metrics of interest.
Wind is a highly uncertain phenomenon with high variability and wide changes over a brief period of time; thus, wind speed exhibits much volatility and randomness.

While wind speed is a continuous variable, wind speed data are collected on a sampled basis: values are measured on a periodic basis, such as hourly, every 10 minutes, or every minute.
Wind data for wind analysis requires the collection around-the-clock of wind speed measurements at the altitude of interest and with a frequency commensurate with the nature and scope of the analysis.

The measurement scheme requires the specification of the smallest indecomposable unit of time:
WIND POWER MEASUREMENT AND DATA

- for planning evaluation and assessment, the collection of data on an hourly or half-hourly basis is, typically, adequate.

- for the analysis of dynamic phenomena such as stability, the collection has to be at a much finer resolution than hourly to capture the short time constants of such phenomena.
The wind data collected may be used to approximate the probability distribution of wind at a specified site.

We make use of such approximations under the assumption that natural phenomena, such as wind, continue to behave in the future in a way similar to their past behavior.
Suppose we wish to *probabilistically* characterize the wind speed at a given site and at a specified altitude: for that purpose, we collect hourly measurements over a long period of time and construct a *histogram* of the measured values.

We discretize the speed axis – we use the integer values of wind speed from 0 to 25 m/s – and we create 26 “buckets” of wind speed values.
We place each hourly measured value in the appropriate "bucket" and we construct a histogram of the historical data such as shown below.
We interpret the height of each bar at wind speed value $v$ in the histogram as the number of hours with wind speed value $v$.

We normalize the vertical axis values by dividing the number of hours of each bar by the total number of hours to obtain the fraction of the total hours at a particular wind speed $v$.

Clearly, each bar has a value $< 1$ and the sum of all the bars must be exactly 1.
In effect, we obtain a probability mass function of the wind speed.

To understand the probability interpretation, we view that wind speed is a random variable $(r.v.) \, Y$ and that the *normalized histogram* provides the probability associated with each of its possible discrete–valued outcomes or realizations.
The bar of the mass density function at the wind speed \( v \) provides
\[
P \{ \tilde{V} = v \} = \text{probability of wind speed at } v \text{ m/s}
\]
- We discretized the values of \( \tilde{V} \) by creating the 26 discrete buckets 0, 1, 2, ..., 25 but in reality, wind speed does not take discrete values since it is a continuously-valued variable.
- Alternatively, we may consider to make use of an increasingly finer resolution grid so as to capture the fact that \( \tilde{V} \) is a continuous r.v.
We associate with the continuous r.v. $\mathcal{V}$ a probability density function (p.d.f.) $f_{\mathcal{V}}(v)$ with the following properties

- $f_{\mathcal{V}}(v) \geq 0 \quad \forall v \geq 0$
- $\int_{0}^{\infty} f_{\mathcal{V}}(v) \, dv = 1$
for an infinitesimally small $\delta > 0$

$$\mathbb{P}\left\{ v < V \leq v + \delta \right\} \approx f_{V}(v) \delta$$

$$\mathbb{P}\left\{ v_1 < V \leq v_2 \right\} = \int_{v_1}^{v_2} f_{V}(v) \, dv$$

The p.d.f. $f_{V}(\cdot)$ provides a complete analytic characterization of the continuous r.v. $V$
PROBABILITY DENSITY

average wind speed

area under entire curve = 1

shaded area is the probability that wind is between $v_1$ and $v_2$

$P\{v_1 < V \leq v_2\}$

wind speed $v$
We may readily compute any function of $\tilde{V}$,

- **average wind speed:**
  
  $$\bar{v} = \int_{0}^{\infty} v \ f_{\tilde{V}}(v) \ dv$$

- **wind speed cubed:**
  
  $$E\left\{ V^3 \right\} = \int_{0}^{\infty} v^3 \ f_{\tilde{V}}(v) \ dv$$
PROBABILITY DENSITY

- number of annual hours $v_1 < V \leq v_2$: we define

an indicator function $i(x)$ with the property

$$i(x) = \begin{cases} 
1 & v_1 < x \leq v_2 \\
0 & \text{otherwise}
\end{cases}$$

and compute

$$8,760 \int_0^\infty i(v) f_V(v) \, dv = 8,760 \int_{v_1}^{v_2} (1) f_V(v) \, dv$$
The general Weibull distribution given by

\[ f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} e^{-\left( \frac{v}{c} \right)^k} \]

- \( k = \) shape parameter
- \( c = \) scale parameter

is often used to approximate the p.d.f. of \( \sim \).
WEIBULL DISTRIBUTION

\[ k = 1 \]
\[ k = 2 \]
\[ k = 3 \]

\[ c = 7 \]

probability density

wind speed \( v \) (m/s)
For $k = 2$, the *Weibull distribution* is called the *Rayleigh p.d.f.*

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2} \text{ Rayleigh p.d.f.}$$

The Rayleigh distribution is widely used in the analytic characterization of wind.
Note that for $V \sim Rayleigh$, the mean is given by

$$\bar{V} = \int_0^\infty v f_V \, dv = 2\int_0^\infty \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} \, dv = \frac{\sqrt{\pi}}{2} c$$

and so we may restate the expression for $f_V(\cdot)$ as

$$f_V(v) = \frac{v \pi}{2 (\bar{V})^2} e^{-\frac{\pi}{4} \left(\frac{v}{\bar{V}}\right)^2}$$
As $\bar{v}$ increases, $f_{\bar{v}}(\cdot)$ becomes flatter and shifts to the right, as shown below.
The wide use of Rayleigh distribution is in light of the good approximations it provides for the average wind power $\bar{v}$.

We have that

$$\bar{v} = \frac{\sqrt{\pi}}{2} c$$

and so we evaluate

$$E(V^3) = \int_0^\infty v^3 \frac{\pi v}{2(\bar{v})^2} e^{-\left[\frac{\pi}{4(\bar{v})^2}\right]} dv = \frac{6}{\pi} \left(\bar{v}\right)^3 \approx 1.91 \left(\bar{v}\right)^3$$
This closed-form solution for Rayleigh-based wind distribution allows us to calculate the average power in wind

\[ \bar{p} = \frac{1}{2} \rho a \left( \bar{v} \right)^3 (1.91) \]

and therefore, it becomes very clear that we cannot simply use \( \left( \bar{v} \right)^3 \) directly to evaluate \( \bar{p} \) but need to also explicitly include the \( \frac{6}{\pi} \approx 1.91 \) factor.
Wind power output is a function of the r.v. $V$ and therefore wind power output is itself a r.v., i.e.,

$$P = g(V) = \frac{1}{2} \rho a(V)^3$$

For wind r.v. $V \sim \text{Weibull p.d.f.}$ with

$$f_V(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} e^{-\left( \frac{v}{c} \right)^k}$$

the cumulative distribution function is given by

$$F_V(v) = \mathbb{P} \{ V \leq v \} = \int_0^v \frac{k}{c} \left( \frac{\xi}{c} \right)^{k-1} e^{-\left( \frac{\xi}{c} \right)^k} d\xi$$
Since, we can introduce a change of variables, we set

\[ u = \left( \frac{\xi}{c} \right)^k \] and \( du = \frac{k}{c} \left( \frac{\xi}{c} \right)^{k-1} d\xi \)

so that

\[ F_Y(v) = \int_0^v e^{-u} \, du = 1 - e^{-\left( \frac{v}{c} \right)^k} \]
For the special case of Rayleigh p.d.f.

\[ F_V(v) \bigg|_{Rayleigh} = 1 - e^{-\frac{\pi}{4}v} \left(\frac{v}{\bar{v}}\right)^2 \]

Note that the probability that Rayleigh wind

exceeds the value \( v \) is

\[ P\{V > v\} = 1 - F_V(v) \bigg|_{Rayleigh} = e^{-\frac{\pi}{4}v} \left(\frac{v}{\bar{v}}\right)^2 \]
ALTAMONT PASS, CA: HISTORICAL DATA vs. RAYLEIGH p.d.f.s

Rayleigh p.d.f. with $v = 6.4 \text{ m/s}$

historical-data-based p.d.f. at Altamont Pass, CA

windspeed $v (\text{m/s})$
EXAMPLE: AVERAGE POWER IN THE WIND

- Based on data from a standard anemometer at a height of 10 m, \( \bar{v}(10) = 6 \text{ m/s} \)

- The plan is to erect a 50 m tower to place the nacelle and we need to estimate the average power under the assumptions
  - Hellman exponent \( \alpha = \frac{1}{7} \)
  - \( \rho = 1.225 \frac{\text{kg}}{\text{m}^3} \)
  - Rayleigh distribution may be used
EXAMPLE: AVERAGE POWER IN THE WIND

The first step is to compute \( \bar{v}(50) \)

\[
\bar{v}(50) = \bar{v}(10) \left( \frac{50}{10} \right)^{\frac{1}{7}} = 7.55 \, \frac{m}{s}
\]

Since Rayleigh distribution holds

\[
\frac{\bar{p}(50)}{\alpha} = \frac{6}{\pi} \cdot \frac{1}{2} \rho \left[ \bar{v}(50) \right]^3 = 1.91 \cdot \frac{1}{2} \cdot 1.225 \cdot (7.55)^3 = 504 \, \frac{W}{m^2}
\]

Sensitivity case for an 80–m tower:

\[
\frac{\bar{p}(80)}{\alpha} = \left( \frac{80}{50} \right)^{\frac{3}{7}} \quad \frac{\bar{p}(80)}{\alpha} = \frac{504 \left( \frac{80}{50} \right)^{\frac{3}{7}}}{\alpha} = 616 \, \frac{W}{m^2}
\]
Each turbine manufacturer provides a plot of the electrical power output of the entire system – the blades, the gearbox, the generator, and the other components – as a function of wind speed.

Such a plot is called an *idealized wind turbine power curve*.

The typical shape of an idealized wind turbine power curve is given below.
THE IDEALIZED WIND TURBINE POWER CURVE

Generator is shut down for \( v > v_F \) generated power below cut-in wind speed.

- \( v_C \) is the cut-in wind speed.
- \( v_R \) is the rated wind speed.
- \( v_F \) is the cut-out wind speed.

Output power increases with wind speed until it reaches the rated power \( P_R \) at \( v_R \), after which the generator is shut down.

Wind speed \( v \) is in meters per second (m/s).
At low speeds, wind has insufficient energy to overcome friction in the turbine drive train, even if the generator rotor is spinning: below the cut-in wind speed $v_C$, the power output is 0.

Above $v_C$, the power output is a cubic function of $v$; at the rated wind speed $v_R$, the generator delivers its rated power $p_R$. 

THE IDEALIZED WIND TURBINE POWER CURVE
At $v > v_R$, controls are deployed to shed some of the wind so as not to exceed $p_R$.

When wind speed reaches the cut–out value $v_F$ – sometimes called by the sailing term furling wind speed – the machine is shut down and the mechanical brakes lock down the rotor shaft above $v_F$ wind speeds and the output power is 0.
We can assess the impact of two key design parameters:

- the diameter $d$ of the blade rotor
- the rated generator capacity

on the power output determined via the idealized power curve

The power output $p \propto d^2$ since $d^2$ determines the area swept by the blades.
For a generator with rated power $p_R$, an increase in $d$ produces a shift in the power curve to the left and the output $p_R$ is reached at a lower speed.
For a fixed rotor diameter $d$, an increase in the generator rated capacity may be accommodated by the continuation of the power curve up to the higher corresponding to the higher $p_R$. 

![Graph showing impacts of design parameters with placeholders for smaller and larger generator capacities and corresponding power levels.]

- For a fixed rotor diameter $d$, an increase in the generator rated capacity may be accommodated by the continuation of the power curve up to the higher corresponding to the higher $p_R$.

- The graph illustrates smaller and larger generator capacities with respective power levels $v_R$ and $v_F$.
Actual power curves do not veer too far from the idealized ones with much of the variance due to the inability of wind shedding techniques to control the power outputs at speeds $\nu > \nu_R$; in certain cases, the value of $\nu_R$ is difficult to determine.
<table>
<thead>
<tr>
<th>wind power class</th>
<th>wind power density (W/m²)</th>
<th>10 m (33 ft)</th>
<th>speed m/s (mph)</th>
<th>wind power density (W/m²)</th>
<th>50 m (164 ft)</th>
<th>speed m/s (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt; 100</td>
<td>&lt; 4.4 (9.8)</td>
<td></td>
<td>&lt; 200</td>
<td>&lt; 5.6 (12.5)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>100 - 150</td>
<td>4.4 (9.8)/5.1 (11.5)</td>
<td>200 - 300</td>
<td>5.6 (12.5)/6.4 (14.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>150 - 200</td>
<td>5.1 (11.5)/5.6 (12.5)</td>
<td>300 - 400</td>
<td>6.4 (14.3)/7.0 (15.7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>200 - 250</td>
<td>5.6 (12.5)/6.0 (13.4)</td>
<td>400 - 500</td>
<td>7.0 (15.7)/7.5 (16.8)</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>250 - 300</td>
<td>6.0 (13.4)/6.4 (14.3)</td>
<td>500 - 600</td>
<td>7.5 (16.8)/8.0 (17.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>300 - 400</td>
<td>6.4 (14.3)/7.0 (15.7)</td>
<td>600 - 800</td>
<td>8.0 (17.9)/8.8 (19.7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>&gt; 400</td>
<td>&gt; 7.0 (15.7)</td>
<td></td>
<td>&gt; 800</td>
<td>&gt; 8.8 (19.7)</td>
<td></td>
</tr>
</tbody>
</table>

Source: http://www.awea.org/faq/basicwr.html
WIND POWER EQUI - DENSITY
CONTOURS AT 50 m

Table 6.5

It is not possible to extract 100 % of the power in the wind as the rotor spills high-speed winds and the little energy at low-speed winds is lost.

The energy generated depends on rotor, gearbox, generator, tower, controls, terrain, and the wind.

Overall conversion efficiency $\eta_r \eta_g$ is around 30 %.
MANUFACTURER POWER CURVES

Gamesa G90-2.0 MW

Vestas V52-850 kW

GE 1.5sle/xle-1.5 MW