Problem 1 (20 pts.)

a) A single-phase 460 V, 60 Hz load uses 12 kW at a lagging PF of 0.75. Taking voltage as reference the current $\bar{I}$ is ________ and the complex power is ________.

$$\bar{V} = 460\angle 0^\circ V$$

$$\bar{I} = \frac{P}{V \cdot PF} \angle -\cos^{-1} 0.75 = \frac{12 \times 10^3}{460 \times 0.75} \angle -41.4^\circ A = \boxed{34.78 \angle -41.4^\circ A}$$

$$\bar{S} = P + jQ = P + jP\tan(41.4^\circ) = 12(1 + j0.88) = \boxed{12 + j10.6 \text{ kVA}}$$

b) Two loads in parallel have the complex powers as $141.4\angle 45^0$ kVA and 50+j50 kVA. The total complex power is ________. The value of Q required to make PF unity is ________.

$$\bar{S}_1 = 141.5\angle 45^\circ = 100 + j100$$

$$S_2 = 50 + j50$$

$$\bar{S}_T = \bar{S}_1 + \bar{S}_2 = \boxed{150 + j150 \text{ kVA}}$$

$$Q_{required} = \boxed{-150 \text{ kVAr}}$$

c) The voltage across a two terminal network is 100 cos(t-30^0)V and the current into the network is 500 cos (t+30^0)A. The power factor is 0.5 and is leading or lagging (circle one).

$$PF = \cos(-60^\circ) = 0.5$$

d) A 2300 V three-phase power system supplies 2 kW to a delta connected balanced load at a PF of 0.8 lagging. The magnitude of line current is 0.628 A, the magnitude of phase current is 0.362 A and the complex impedance of each leg (phase) is $6348\angle 37^\circ$ Ω.

$$P_{3\phi} = \sqrt{3}\, V_{\ell-\ell} I_\ell\, PF$$

$$I_\ell = \frac{P_{3\phi}}{\sqrt{3} V_{\ell-\ell} PF} = \frac{2 \times 10^3}{\sqrt{3} \times 2300 \times 0.8} = 0.628 A$$

$$I_\phi = \frac{I_\ell}{\sqrt{3}} = 0.362 A$$

$$|Z| = \frac{2300}{0.362} = 6348 \Omega \Rightarrow Z = 6348 \angle \cos^{-1} 0.8 = 6348 \angle 36.9^\circ \Omega$$

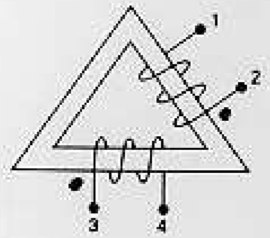
e) A coil of 1000 turns is wound on an iron core whose reluctance $\mathfrak{R} = 4.6 \times 10^6$ At/W. The inductance of the coil is __0.217__ H.

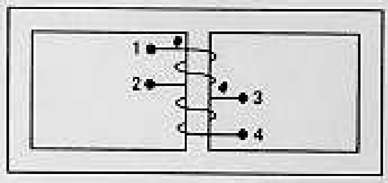
$$L = \frac{N^2}{\mathfrak{R}} = \frac{1000^2}{4.6 \times 10^6} = 0.217\,H$$

f) Two coils which are coupled have self-inductances of 10 and 20 mH respectively and a coupling coefficient of 0.9. The mutual inductance is __12.73__ mH.

$$M = k\sqrt{L_1 L_2} = 0.9\sqrt{10 \times 20} = 12.73\,mH$$

g) Put the dot markings on the coils in the structures below.





h) If $\lambda = \frac{i}{x^2}$, $W_m(\lambda,x) =$ __$\frac{x^2\lambda^2}{2}$__, $W'_m(i,x) =$ __$\frac{i^2}{2x^2}$__.

$$W_m = \int_0^\lambda i\,d\lambda' = \int_0^\lambda \lambda' x^2\,d\lambda' = \frac{x^2\lambda^2}{2}$$

$$W'_m = \int_0^i \lambda\,di' = \int_0^i \frac{i'}{x^2}\,di' = \frac{i^2}{2x^2}$$

i) An 8 pole induction motor with 5% slip (s = 0.05) is supplied by a 60 Hz generator. The speed of the motor is __855__ RPM and the frequency of rotor current is __3__ Hz.

$$N_s = \frac{120f}{P} = \frac{120 \times 60}{8} = 900\text{ RPM}$$

$$N_{act} = N_s(1-s) = 900(1-0.05) = 855\text{ RPM}$$

$$f_r = s f_s = 0.05 \times 60 = 3\text{ Hz}$$

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j) The rotor copper losses of a three phase, 6 pole, 60 Hz induction motor running at 1100 RPM is 5 kW. The input power to the motor is 60 kW and the torque of electric origin is 477.5 N-m Neglect stator copper losses.

$$N_s = \frac{120 \times 60}{6} = 1200 \text{ RPM}$$

$$s = \frac{N_s - N_{act}}{N_s} = \frac{1200 - 1100}{1200} = \frac{1}{12}$$

$$P_r = s P_{ag} \Rightarrow P_{ag} = \frac{P_r}{s} = \frac{5 \text{ kW}}{1/12} = \boxed{60 \text{ kW}}$$

$$P_m = P_{ag}(1-s) = 60\left(1 - \frac{1}{12}\right) = 55 \text{ kW}$$

$$T^e = \frac{P_m}{\omega_m} = \frac{55 \times 10^3}{2\pi\left(\frac{60}{6/2}\right)(1-s)} = \boxed{477.5 \text{ N-m}}$$

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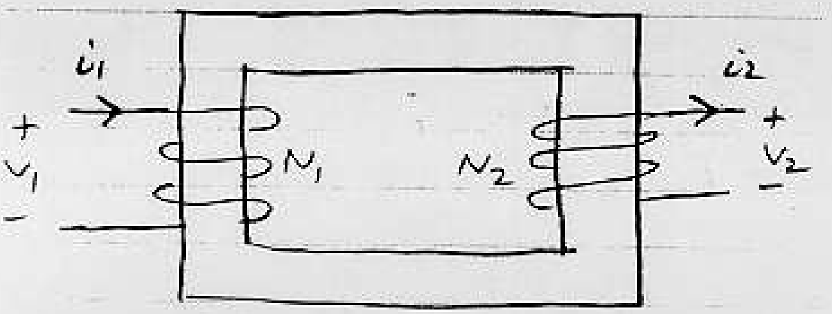
Problem 2 (20 pts.)



Given: cross-section area = 10^{-3} m^2

Length of main flux path = 0.2 m

$v_1 = 170 \cos 377t$ volts

(a) Find N_1 so that the flux density in the iron is limited to a peak value of 1.0 tesla in steady state with $i_2 = 0$. (Assume infinite permeability for the iron.)

(b) Using these turns, what will be the peak value of i_1 if the iron premeability is 500 μ_0 and $i_2 = 0$? (Neglect leakage flux) $\left(\mu_0 = 4\pi \times 10^{-7}\, H/m\right)$

(c) What is the peak value of i_1 if N_2 = 900 turns and an 80 Ω resistor is added on the "2" side? (Neglect leakage flux)

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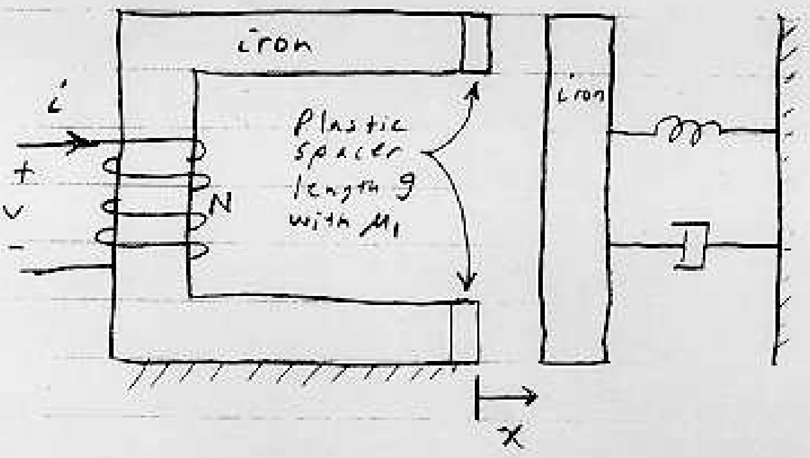
Problem 3 (20 pts.)



All materials have width **w** and depth **d**. $\mu_{iron} = \infty$. Consider only one flux path.

(a) (8 pts.) Find an expression for the force of electrical origin in the positive x direction when $x = 0$, as a function of current i.

(b) (8 pts.) Find the energy transferred from the electrical system into the coupling field when x moves from 0 to x_1 while i is held constant at I_o.

(c) (2 pts.) Is the answer to (b) a positive number or negative?

(d) (2 pts.) In one sentence, give a physical explanation for the sign on the force of electrical origin in part (a).

Problem 4 (20 pts.)

For the following state space model

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2x_3 - x_1^3 + x_1$$
$$\dot{x}_3 = (-2x_2x_3 - x_3 + 5)$$

the initial conditions are $x_1(0) = 1$, $x_2(0) = 0.5$ and $x_3(0) = 10$ at $t = 0$.

a) (10 pts.) With a time step of $\Delta t = 0.1$ sec, integrate the system to obtain x_1, x_2, x_3 at $t = 0.1$ sec.
b) (5 pts.) Find the equilibrium points.
c) (5 pts.) Find the linearized model, i.e., matrix A at any one of the equilibrium points. (No eigenvalue computation needed.)

a) $x_1(0.1) = x_1(0) + \Delta t\, \dot{x}_1\big|_0 = 1 + 0.1 \times 0.5 = \boxed{1.05}$

$x_2(0.1) = x_2(0) + \Delta t\, \dot{x}_2\big|_0 = 0.5 + 0.1\left(-0.1 \times 0.5 \times 10 - 1^3 + 1\right) = \boxed{0.45}$

$x_3(0.1) = x_3(0) + \Delta t\, \dot{x}_3\big|_0 = 10 + 0.1\left(-2 \times 0.5 \times 10 - 10 + 5\right) = \boxed{8.5}$

b) $\begin{cases} 0 = x_2 \\ 0 = -0.1\, x_2 x_3 - x_1^3 + x_1 \\ 0 = -2x_2x_3 - x_3 + 5 \end{cases} \Rightarrow \quad x_2 = 0, \quad x_1^3 - x_1 = 0 \Rightarrow x_1 = 0, 1, -1, \quad x_3 = 5$

3 equilibrium points: $\underset{\#1}{(0,0,5)}$, $\underset{\#2}{(1,0,5)}$, and $\underset{\#3}{(-1,0,5)}$

c) Linearized model

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3x_1^2+1 & -0.1x_3 & -0.1x_2 \\ 0 & -2x_3 & -2x_2-1 \end{bmatrix}$$

$\#1 \begin{bmatrix} 0 & 1 & 0 \\ 1 & -0.5 & 0 \\ 0 & -10 & -1 \end{bmatrix}$, $\#2 \begin{bmatrix} 0 & 1 & 0 \\ -2 & -0.5 & 0 \\ 0 & -10 & -1 \end{bmatrix}$, $\#3 \begin{bmatrix} 0 & 1 & 0 \\ -2 & -0.5 & 0 \\ 0 & -10 & -1 \end{bmatrix}$

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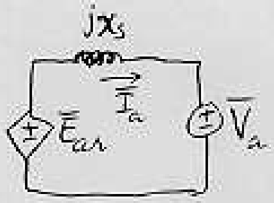
Problem 5 (20 pts.)

A three-phase, 60 Hz, 6 pole, 550 V (line-line) wye connected synchronous generator is delivering 250 kW at rated voltage. The excitation voltage E_{ar} is 460 V per phase and the synchronous reactance $x_s = 1.2\ \Omega$ (Neglect stator resistance.)

a) (7 pts.) Calculate the torque angle δ and the torque of electrical origin (specify + or –).
b) (5 pts.) Find the generator current magnitude, power factor and reactive generation Q.
c) (5 pts.) For the same terminal voltage and active power generation P determine the excitation voltage E_{ar} to deliver power with a power factor of 0.8 lagging. What is the new value of the torque angle δ in this case?
d) (3 pts.) How many poles should an induction motor have to turn at 900 RPM at no load when served by this generator?

a) $\bar{V}_a = \frac{550}{\sqrt{3}} = 317.5\angle 0^\circ$ V



$P = \frac{3 E_{ar} V_a \sin\delta}{x_s}$

$\sin\delta = \frac{P x_s}{3 E_{ar} V_a} = \frac{250\times10^3\times1.2}{3\times460\times317.5} = 0.6846$

$\boxed{\delta = 43.2^\circ}$, $T^e = \frac{P}{\omega_m} = \frac{250\times10^3}{2\pi\left(\frac{60}{6/2}\right)} = \boxed{1989.4 \text{ N-m}}$

b) $\bar{I}_a = \frac{\bar{E}_{ar} - \bar{V}_a}{j x_s} = \frac{460\angle 43.2^\circ - 317.5\angle 0^\circ}{j1.2} = 262.8\angle -3.24^\circ$ A

$\boxed{I_a = 262.8 \text{ A}}$, $PF = \cos 3.24^\circ = \boxed{0.998}$, $Q = P\tan(3.24^\circ) = 14.15$ kVAr

c) $\bar{S}_{new} = \frac{P/3}{0.8}\angle\cos^{-1}0.8 = 104.2\angle 36.9^\circ$ kVA

$\bar{I}_{a,new} = \frac{\bar{S}^*_{new}}{\bar{V}^*_a} = \frac{104.2\angle -36.9}{317.5} = 0.328\angle -36.9$ kA

$\bar{E}_{ar,new} = 317.5\angle 0^\circ + (j1.2)(328\angle -36.9) = 637\angle 29.6^\circ$ V

$\boxed{\delta_{new} = 29.6^\circ}$

d) $\frac{P}{2} N_m = N_s$

$\frac{P}{2} 900 = 3600$

$\boxed{P = 8}$