Problem 1. (25 points)

A rotating electrical machine has the following electric terminal relationships for the stator and rotor flux linkages:

$$\lambda_s = L_s[2.5 + \cos(2\theta)]\, i_s + M\sin(\theta)\, i_r$$
$$\lambda_r = M\sin(\theta)\, i_s + L_r\, i_r$$

Where θ is the angle of the rotor, i_s is the current in the stator winding and i_r is the current in the rotor winding. L_s, L_r, and M are constant inductance terms.

a) Find the energy stored in the coupling field as a function of θ, i_s, i_r, L_s, L_r, and M
b) Find the torque of electric origin acting on the rotor in terms of θ, i_s, i_r, L_s, L_r, and M
c) If the machine is operated with $i_r = I_{dc}$ (i.e. a constant dc current) and $i_s = 0$, and spun at a constant speed $\omega = d\theta/dt$, what is the stator terminal voltage, $v_s(t)$, in terms of the currents, the inductances, time, and ω? (Assume $\theta = \theta_0$ at time t=0, and neglect all resistances).

a) $W_m = W_m' = L_s[2.5 + \cos(2\theta)]\dfrac{i_s^2}{2} + M\sin(\theta)\cdot i_s i_r + L_r\dfrac{i_r^2}{2}$

b) $T^e = \dfrac{\partial W_m'}{\partial\theta} = -L_s\sin(2\theta)\cdot i_s^2 + M\cos(\theta)\cdot i_s\cdot i_r$

c) $\theta = \omega t + \theta_0$

$$v_s(t) = \cancel{i_s R}^{\,0} + \frac{d\lambda_s}{dt} = \frac{\partial\lambda_s}{\partial\theta}\cdot\frac{d\theta}{dt} + \frac{\partial\lambda_s}{\partial i_s}\cdot\cancel{\frac{di_s}{dt}}^{\,0} + \frac{\partial\lambda_s}{\partial i_r}\cdot\cancel{\frac{di_r}{dt}}^{\,0}$$

$$= M\cos(\theta)\cdot I_{dc}\cdot\omega$$

$$= M I_{dc}\cdot\omega\cos(\omega t + \theta_0)$$

Problem 2

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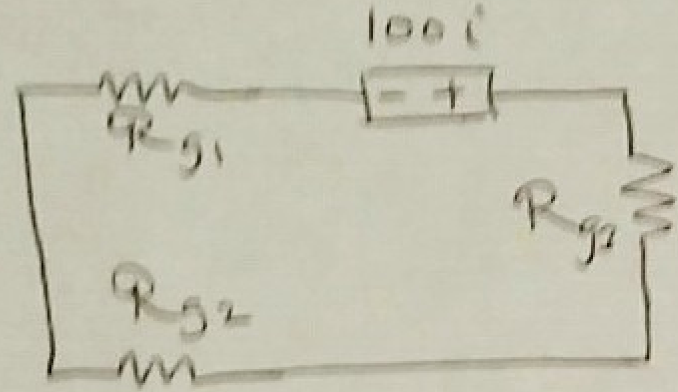
6 pts

a)



$$R_{g_1} = R_{g_2} = \frac{x}{\mu_0(0.02 \times 0.01)} = \frac{5000x}{\mu_0}$$

$$R_{g_3} = \frac{0.001}{\mu_0(0.02 \times 0.02)} = \frac{2.5}{\mu_0}$$

$$R_{total} = R_{g_1} + R_{g_2} + R_{g_3} = \frac{10{,}000x + 2.5}{\mu_0} = 7.96 \times 10^9 x + 1.99 \times 10^6 \ (H^{-1})$$

b) $$\phi = \frac{Ni}{R_T} = \frac{100i}{R_T}$$

$$\lambda = N\phi = \frac{N^2 i}{R_T} = \frac{10^4 i}{R_T} = \frac{i}{7.96 \times 10^5 x + 1.99 \times 10^2}$$

c) $$v = \frac{d\lambda}{dt} + iR = \frac{\partial \lambda}{\partial i} \cdot \frac{di}{dt} + \frac{\partial \lambda}{\partial x} \cdot \frac{dx}{dt} + iR$$

$$= iR + \frac{1}{7.96 \times 10^5 x + 1.99 \times 10^2} \frac{di}{dt} - \frac{7.96 \times 10^5 i}{(7.96 \times 10^5 x + 1.99 \times 10^2)} \cdot \frac{dx}{dt}$$

d) $$W_m' = \frac{1}{2} \cdot \frac{i^2}{7.96 \times 10^5 x + 1.99 \times 10^2}$$

$$f^e = \frac{\partial W_m'}{\partial x} = \frac{-3.98 \times 10^5 i^2}{\cancel{2}(7.96 \times 10^5 x + 1.99 \times 10^2)^2}$$

Problem 3. (25 points.)

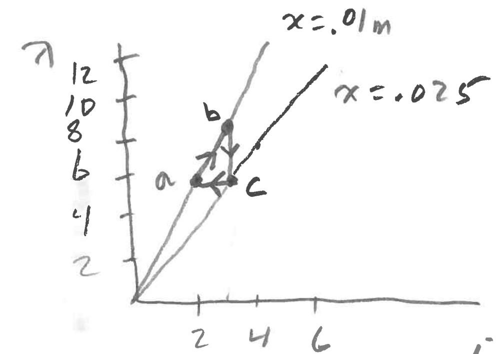
An electromechanical system has the following flux linkage-current relationship:

$$\lambda = \frac{0.09}{(0.02+x)}\, i \qquad W_m' = \frac{0.045}{(.02+x)}\, i^2 \qquad f^e = -\frac{0.045\, i^2}{(.02+x)^2}$$

In the following questions, EFE stands for "Energy From the Electrical system into the coupling field", and EFM stands for "Energy From the Mechanical system into the coupling field".

Consider the following to be point a: x = .01 meters, i = 2 Amps, λ = 6 Wb-Tns.

a) Find the EFE and EFM as the system is moved along constant x from point a to point b which has i = 3 Amps.
b) Find the EFE and EFM as the system is moved along constant current from point b to point c which has x = 0.025 meters.
c) Find W_m at point c
d) Find the EFE and EFM as the system is moved along constant λ from point c to point a which has x = .01 meters.
e) For this cycle, is this a motor or a generator? Explain why.



a) $$EFE_{a-b} = \int_6^9 \frac{\lambda}{3}\, d\lambda = \frac{\lambda^2}{6}\Big|_6^9 = \frac{81}{6} - \frac{36}{6} = \frac{45}{6} = 7.5\text{ J}$$

$$W_{m_b} = W'_{m_b} = \frac{1}{2}\, 9\times 3 = 13.5\text{ J}$$

$$W_{m_a} = W'_{m_a} = \frac{1}{2}\, 6\times 2 = 6\text{ J}$$

$$W_{m_b} - W_{m_a} = 7.5 = 7.5 + EFM_{a-b}$$

$$\boxed{EFM_{a-b} = 0}$$

b) $$EFE_{b-c} = \int_9^6 3\, d\lambda = 3\lambda\Big|_9^6 = 18 - 27 = -9\text{ J}$$

$$W_{m_c} = W'_{m_c} = \frac{1}{2}\, 6\times 3 = 9\text{ J}$$

$$W_{m_c} - W_{m_b} = 9 - 13.5 = -9 + EFM_{b-c}$$

$$\boxed{EFM_{b-c} = 4.5\text{ J}}$$

c) $\boxed{W_{m_c} = 9\text{ J above}}$

d) $$EFE_{c-a} = 0$$

$$W_{m_a} - W_{m_c} = 6 - 9 = -3 = 0 + EFM_{a-c}$$

$$\boxed{EFM_{a-c} = -3\text{ J}}$$

e) $$EFE_{cycle} = 7.5 - 9 + 0 = -1.5\text{ J}$$

Generator

$$EFM_{cycle} = 0 + 4.5 - 3 = +1.5\text{ J}$$

Problem 4. (25 points.)

An electro-mechanical device has the following nonlinear dynamic model:

$$0.04\frac{d^2\delta}{dt^2} = 2-3E\sin\delta-0.01\frac{d\delta}{dt}$$

a) Write this model in standard state-space form using δ and $\omega = \frac{d\delta}{dt}$ as dynamic states.

b) Find all of the equilibrium points in the interval with δ between -180 degrees and + 180 degrees if the value of $E = 1.0$.

c) Pick one of the equilibrium points as the initial value of the dynamic states and use Euler's method with time step $\Delta t = 0.01\text{s}$ to estimate the values of the two state variables at $t = 0.01\text{s}$ and $t = 0.02\text{s}$ if the value E is changed to 0.4 at time equals zero.

a) $x_1 = \delta \quad x_2 = \frac{d\delta}{dt} = w \qquad \dot{x}_1 = x_2$

$\dot{x}_2 = 25\left[2 - 3E\sin x_1 - .01x_2\right]$

b) $0 = x_{2e}$

$0 = 2 - 3E\sin x_{1e} \quad E=1 \quad x_{1e} = 41.81^\circ$ or $138.2^\circ \quad x_{2e} = 0$

c) Pick $x_{1e} = 41.81^\circ \quad x_{2e} = 0 \quad (x_{1e} = 0.7295 \text{ rad}, x_{2e} = 0 \text{ r/s})$

$E = 0.4$

$x_1(.01) = 0.7297 + 0 \times .01 = 0.7297 \text{ rad}$

$x_2(.01) = 0 + 25\left[2 - 0.8 - 0\right] \times .01 = 0.3 \text{ r/s}$

$x_1(.02) = 0.7297 + 0.3 \times .01 = 0.7327 \text{ rad} \quad (41.98^\circ)$

$x_2(.02) = 0.3 + 25\left[2 - 0.8027 - .003\right] \times .01 = 0.5986 \text{ r/s}$