

Section (Check One)

MWF 10am

TTh 12:30pm

1. _____ / 25 2. _____ / 25

3. _____ / 25 4. _____ / 25 Total _____ / 100

Useful information

$$\sin(x) = \cos(x - 90^\circ)$$

$$\bar{V} = \bar{Z}\bar{I}$$

$$\bar{S} = \bar{V}\bar{I}^*$$

$$\bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle \theta$$

$$0 < \theta < 180^\circ \text{ (lag)}$$

$$I_L = \sqrt{3}I_\phi \text{ (delta)}$$

$$\bar{Z}_Y = \bar{Z}_\Delta / 3$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$$

$$-180^\circ < \theta < 0 \text{ (lead)}$$

$$V_L = \sqrt{3}V_\phi \text{ (wye)}$$

$$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot n d\mathbf{a}$$

$$\int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot n d\mathbf{a}$$

$$\mathfrak{R} = \frac{l}{\mu A}$$

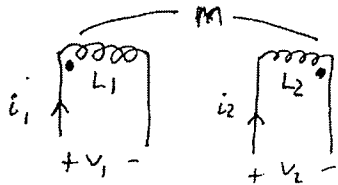
$$MMF = Ni = \phi \mathfrak{R}$$

$$\phi = BA$$

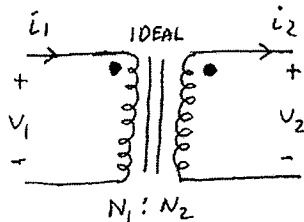
$$\lambda = N\phi$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$1 \text{ hp} = 746 \text{ Watts}$$

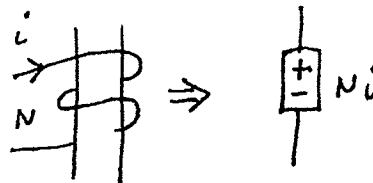


$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$



$$a = \frac{N_1}{N_2} \quad N_1 i_1 = N_2 i_2$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$



Problem 1. (25 points)

A single-phase load of 10 kW at the PF of 0.707 lagging is supplied from a in-series feeder whose impedance is $0.1 + j0.1 \Omega$. The voltage across the load is $v(t) = \sqrt{2} (120) \sin(337t + 120^\circ)$.

Calculate the following:

- The source current phasor \bar{I}_s and the instantaneous $i_s(t)$ (7 points);
- The source voltage phasor \bar{V}_s across the load and feeder (5 points);
- Sketch the two source phasors in the complex plane and indicate the PF angle (5 points);
- The complex power consumed by the load \bar{S} and the total complex power supplied by the source \bar{S}^{total} (8 points).

$$a) \quad \bar{V} = 120 \angle 120^\circ - 90^\circ = 120 \angle 30^\circ \text{ V}, \quad \bar{S} = \frac{10 \text{ k}}{0.707} \angle \cos^{-1}(0.707) \text{ kVA}$$

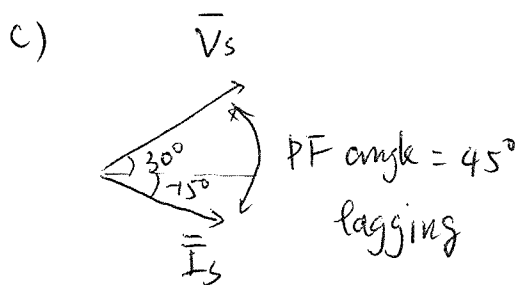
$$\bar{I}_s = \left(\frac{\bar{S}}{\bar{V}} \right)^* = \left(\frac{14.14 \angle 45^\circ}{120 \angle 30^\circ} \right)^* = 117.87 \angle -15^\circ \text{ A}$$

$$b) \quad \bar{V}_s = \bar{V} + \bar{I}_s \cdot \bar{Z}_{feeder} \quad i_s(t) = 117.87\sqrt{2} \cos(337t - 15^\circ) \text{ A}$$

$$= 120 \angle 30^\circ + 117.87 \angle -15^\circ \times (0.1 + j0.1)$$

$$= 120 \angle 30^\circ + 16.67 \angle 30^\circ$$

$$= 136.67 \angle 30^\circ \text{ V}$$



$$d) \quad \bar{S} = 14.14 \angle 45^\circ \text{ kVA} = 10 + j10 \text{ kVA}$$

$$\bar{S}^{total} = \bar{V}_s \bar{I}_s^*$$

$$= 16.11 \angle 30^\circ - (-15^\circ) \text{ kVA}$$

$$= 16.11 \angle 45^\circ \text{ kVA}$$

$$= 16.11 + j16.11 \text{ kVA}$$

Problem 2. (25 points)

The following three-phase, balanced loads are connected across a three-phase, wye-connected source of 480 Volts (RMS – line to line):

- Load #1: Wye-connected load with line current of 58 Amps (RMS) at 0.85 PF lag;
 Load #2: Delta-connected load with 80 KW (3-phase) at 0.8 PF lag;

Calculate the following:

- The total complex power (3-phase) consumed by both loads (5 points)
- Total source line current RMS magnitude (5 points)
- The phase current RMS magnitude for each load. (10 points)
- A delta-connected capacitor bank is added in parallel to make the overall power factor equal to unity. Determine the required VARs per phase. (5 points)

$$\bar{S}_1 = \sqrt{3} 480 \text{ V} \times 58 \text{ A} \angle +\cos^{-1}(0.85) = 48.22 \angle +31.79^\circ \text{ kVA} = 40.99 + j25.40 \text{ kVA}$$

$$\bar{S}_2 = \frac{80 \text{ k}}{0.8} \angle +\cos^{-1}(0.8) \text{ VA} = 100 \text{ k} \angle 36.87^\circ \text{ kVA} = 80 + j60 \text{ kVA}$$

$$\bar{S} = \bar{S}_1 + \bar{S}_2 = 120.99 + j85.40 \text{ kVA} = 148.09 \angle 35.2^\circ \text{ kVA}$$

$$I_L = \frac{|\bar{S}|}{\sqrt{3} V_L} = \frac{148.09 \text{ k}}{\sqrt{3} \times 480} = 178.13 \text{ A}$$

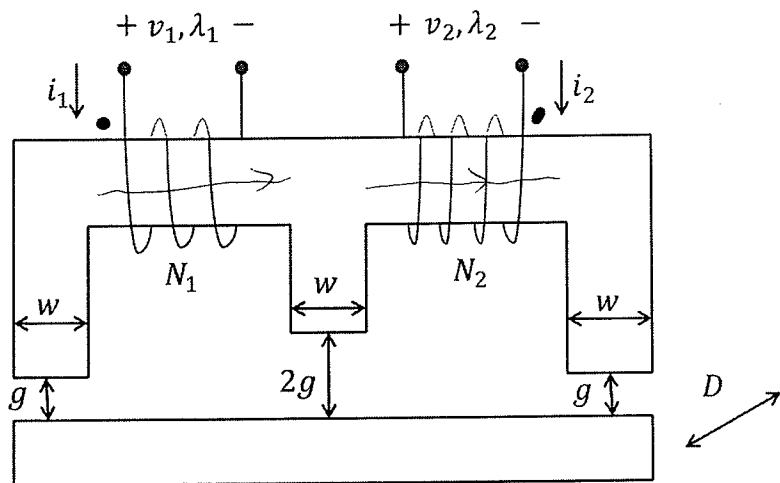
$$I_{\phi,1} = I_{L,1} = 58 \text{ A}$$

$$I_{\phi,2} = \frac{1}{\sqrt{3}} I_{L,2} = \frac{1}{\sqrt{3}} \frac{S_2}{\sqrt{3} V_L} = \frac{100 \text{ k}}{3 \times 480} = 69.44 \text{ A}$$

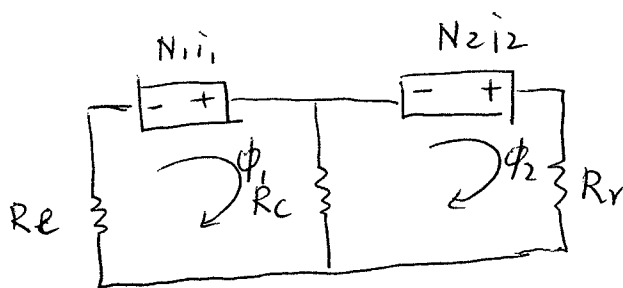
$$\begin{aligned} \bar{S} + 3jQ_{\phi}^c &= P \Rightarrow |Q_{\phi}^c| = \left| -\frac{Q}{3} \right| \\ &= \left| -\frac{85.40}{3} \right| = 28.47 \text{ kVAR/phase} \end{aligned}$$

Problem 3. (25 points)

The following figure shows an electric device, of which both the upper and lower pieces are composed of infinitely permeable magnetic material. It has depth D into the paper, and all other lengths are indicated in the figure. The distance values are: $D = 3\text{cm}$, $w = 2\text{cm}$, and $g = 2\text{mm}$. There is no fringing or flux leakage.



- a) Given the dot marking for coil 1, finish the dot marking for coil 2 in the figure (5 points);
- b) Draw the magnetic equivalent circuit, and find each reluctance value. Write two “loop” equations that you could use to find the flux through the two coils in terms of the two currents – do not simplify or solve (10 points);
- c) Suppose the self inductances of coils 1 and 2 are L_1 and L_2 , respectively, and the mutual inductance between the two is $M > 0$. Use these inductance values to write the equations for obtaining v_1 and v_2 in terms of the two currents (10 points).



$$R_e = R_r = \frac{g}{\mu_0 w D} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = 2.65 \times 10^6 \frac{\text{A-turns}}{\text{Wb}}$$

$$R_c = \frac{2g}{\mu_0 w D} = 5.31 \times 10^6 \frac{\text{A-turns}}{\text{Wb}}$$

$$N_1 i_1 = R_c (\phi_1 - \phi_2) + R_e \phi_1$$

$$N_2 i_2 = R_r \phi_2 + R_c (\phi_2 - \phi_1)$$

c).
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

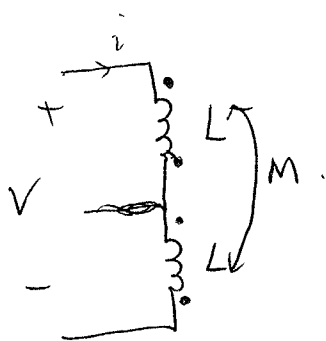
Problem 4. (25 points)

- a) Two identical but mutually coupled lossless ($R=0$) coils are connected in two ways, and the inductance as seen by the combination is measured in each case.

In series with the undotted terminal of one coil connected to the undotted terminal of the other coil. $L_{eq} = 0.08$ H.

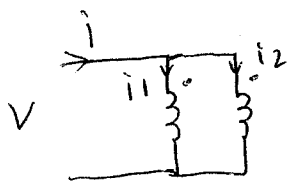
In parallel with the dotted terminals connected together and the undotted terminals connected together. $L_{eq} = 0.05$ H.

Find the self inductance of each coil (L), the mutual inductance between the two coils (M), and the coefficient of coupling k (12 points).



$$V = L \frac{di}{dt} + L \frac{di}{dt} - 2M \frac{di}{dt}$$

$$\Rightarrow L_{eq} = 2L - 2M$$



$$V = L \frac{di_1}{dt} + M \frac{di_2}{dt} = L \frac{di_2}{dt} + M \frac{di_1}{dt}$$

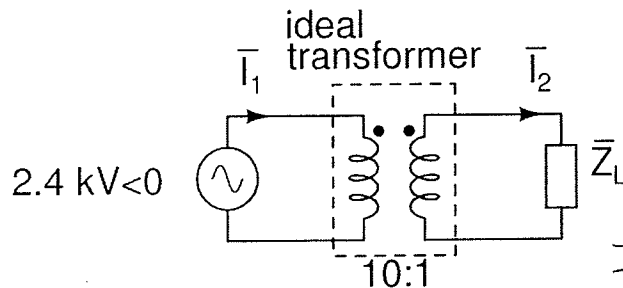
$$2V = (L + M) \frac{d(i_1 + i_2)}{dt} \Rightarrow V = \frac{L+M}{2} \frac{di}{dt}$$

$$L_{eq} = \frac{L+M}{2}$$

$$\left\{ \begin{array}{l} 2(L - M) = 0.08 \\ \frac{L+M}{2} = 0.05 \end{array} \right\} \Rightarrow \begin{array}{l} L = 0.07 \text{ H} \\ M = 0.03 \text{ H} \end{array}$$

- b) An ideal single-phase 60 Hz transformer as shown below is used to supply power to a complex impedance load drawing 5 kW at 0.9 power factor lagging.

$$k = \frac{0.03}{\sqrt{0.07^2}} = \frac{3}{7}$$



$$\begin{aligned} \bar{S} &= \frac{5 \text{ kW}}{0.9} \angle +\cos^{-1}(0.9) \\ &= 5.56 \angle 25.84^\circ \text{ kVA} \end{aligned}$$

$$I_1 = \frac{S}{V_1} = \frac{5.56 \text{ k}}{2.4 \text{ k}} = 2.32 \text{ A}$$

Find the magnitudes of currents I_1 and I_2 (13 points).

$$I_2 = \frac{N_1}{N_2} I_1 = 10 I_1 = 23.2 \text{ A}$$