ECE 330 Exam #1, Fall 2016 Name: Solution
90 Minutes

Section (Check One) MWF 9 am _______ MWF 10 am _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\quad \bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

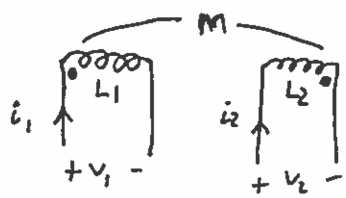
$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3} I_\phi$ (delta) $\quad \bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3} V_\phi$ (wye)

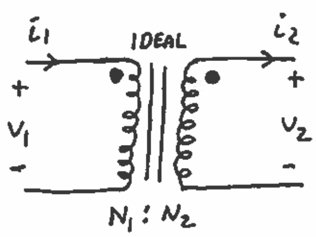
ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\varphi = B A$ $\quad \lambda = N\varphi = Li$ (if linear) $\quad v = d\lambda/dt$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

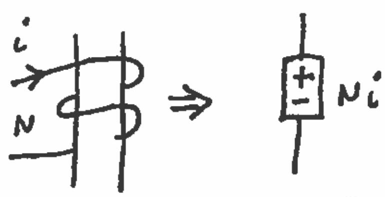


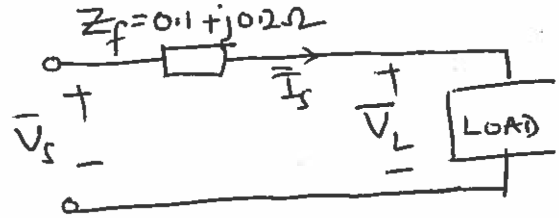
$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$





Problem 1. (25 points)

A feeder with an impedance of 0.1+*j*0.2 Ohms supplies a single-phase 20kW, 0.85 lagging power factor load. The voltage across the load is $v(t) = \sqrt{2}\,(120)\,\sin(377t + \pi/3)$V Calculate:

a) The source current phasor $\bar{I}_s$ and the instantaneous $i_s(t)$
b) The source (sending end) voltage phasor $\bar{V}_s$
c) The power factor angle at the sending end
d) The total complex power supplied by the source, $\bar{S}_{total}$
e) The magnitude of the voltage at the receiving end if the load is removed (open circuited)

a) $\bar{V}_L = 120\angle 60^\circ - 90^\circ \text{ V} = 120\angle -30^\circ \text{ V}$

$\bar{S} = \frac{20k}{0.85}\angle \cos^{-1}(0.85) \text{ VA} = 23529\angle 31.79^\circ \text{ VA}$

$\bar{I}_s = \bar{I}_L = \left(\frac{\bar{S}_L}{\bar{V}_L}\right)^* = \frac{23529\angle -31.79^\circ}{120\angle 30^\circ} = 196.1\angle -61.79^\circ \text{ A}$

$i_s(t) = 196.1\sqrt{2}\,\cos(377t - 61.79^\circ) \text{ A}$

b) $\bar{V}_s = \bar{V}_L + \bar{I}_s \bar{Z}_f = 120\angle -30^\circ + (196.1\angle -61.79^\circ)(0.1 + j0.2)$
$= 159\angle -21.68^\circ \text{ V}$

c) p.f. angle $= \theta_{V_s} - \theta_{I_s} = -21.68^\circ + 61.79^\circ = 40.1^\circ$

d) $\bar{S}_{total} = \bar{V}_s \cdot \bar{I}_s^* = (159\angle -21.68^\circ)(196.1\angle -61.79^\circ)$
$= 31176\angle 40.1^\circ \text{ VA}$

e) $\bar{I}_s = 0 \Rightarrow \bar{V}_{L_{new}} = \bar{V}_s = 159\angle -21.68^\circ \text{ V}$.

(extra paper at the end)

Problem 2. (25 points)

A balanced 3-phase, 208 Volt (line-line), Wye-connected source serves a balanced, 3-phase, Wye-connected, lagging power factor load. A variable 3-phase capacitor bank is connected across the load in a Delta configuration. Measurements of the source line current for various values of capacitor Vars (3-phase) give the following test results for tests T1 to T8:

	T1	T2	T3	T4	T5	T6	T7	T8
Capacitor Vars (3-phase):	0	200	400	600	800	1,000	1,200	1,400
Source line current:	3.75	3.43	3.17	3.00	2.92	2.95	3.07	3.29

a) By just looking at the numbers in the table above, about how many Vars (3-phase) would you say the original load (without the capacitors) consumes?
b) Approximately how many Watts (3-phase) would you say the original load (without the capacitors) consumes?
c) What is the exact value of the original load P + jQ (3-phase --- without the capacitors)?
d) What would the source line current be in test T4 if the same capacitors used in test T4 were connected in a Wye rather than a Delta?

a) minimum line current means unity power factor for constant P. So 2.92 A is min, so $\boxed{Q = 800 \text{ VARS}}$

b) $\sqrt{3} \times 208 \times 2.92 = \boxed{1052\text{W} = P}$

c) $|P + jQ| = \sqrt{3} \times 208 \times 3.75 = \sqrt{P^2 + Q^2} = 1351$

$|P + j(Q - 600)| = \sqrt{3} \times 208 \times 3 = \sqrt{P^2 + (Q-600)^2} = 1081$

(OR OTHER TEST)

$P^2 + Q^2 = 1351^2 \text{ VA}$ $\quad P^2 + Q^2 - 1200Q + 360K = 1081^2 \text{ VA}$

$0 - 1200Q + 360K = 1081^2 - 1351^2 = -656{,}640$

$-1200Q = -1{,}016{,}640$ $\quad \boxed{Q = 847 \text{ VARS}}$

$\boxed{P = 1052 \text{W}}$

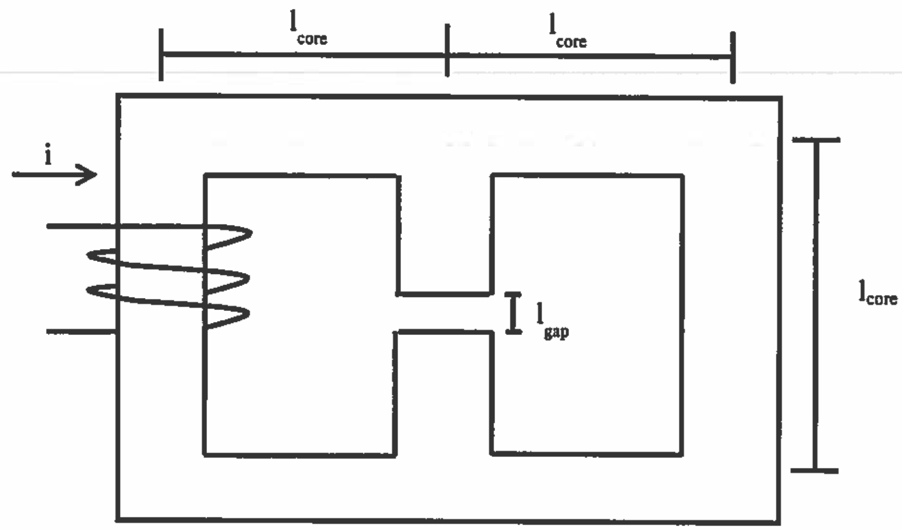
d) $Q_{cap} = 600/3 = 200$ VARS

$\boxed{\text{So } I_L = 3.43\text{A}}$

(extra paper at the end)

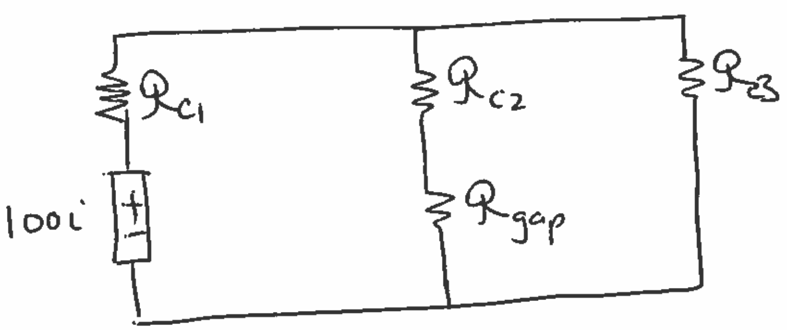
Problem 3. (25 points)

Consider the iron geometry given in the figure below. Assume μ_r of the iron core = 1000, l_{core} = 10 cm, l_{gap} = 0.1 cm, the cross section of the core is 1cm by 1cm, and number of turns is 100. Account for air gap fringing in the following calculations.



a) Draw the equivalent magnetic circuit and calculate and label all reluctances.
b) Calculate the inductance of the coil.
c) Find the current (assume dc) needed to generate a flux density in the left leg of 0.5 Tesla.
d) With this same current, what is the flux density in the center and right legs?

a)



$$\mathcal{R}_{c1} = \mathcal{R}_{c3} = \frac{0.3}{1000\mu_0 \times 1\times10^{-4}} = 2.387\times10^{6} \ H^{-1}$$

$$\mathcal{R}_{c2} = \frac{(0.1-0.001)}{1000\mu_0\times1\times10^{-4}} = 7.878\times10^{5} \ H^{-1}$$

Accounting for fringing,

$A_{gap} = 1.1\times1.1 = 1.21 \ cm^2$.

$$\mathcal{R}_g = \frac{0.001}{\mu_0\times1.21\times10^{-4}} = 6.577\times10^{+6} \ H^{-1}$$

(extra paper at the end)

b) $L_{coil} = \frac{N^2}{\mathcal{R}_T}$

Total reluctance, $\mathcal{R}_T = \mathcal{R}_{c1} + (\mathcal{R}_{c2} + \mathcal{R}_{gap}) \parallel \mathcal{R}_{c3}$

$= 4.19 \times 10^6 \ H^{-1}$

$L_{coil} = 100^2 / 4.19 \times 10^6 = 2.387 \ mH$

c) Flux through left leg, $\phi_1 = BA = 0.5 \times 10^{-4}$ Wb

$100 i = \mathcal{R}_T \times \phi_1 \Rightarrow i = 2.095 \ A$

d) Flux throug right leg $= \frac{100 i - \phi_1 \mathcal{R}_{c1}}{\mathcal{R}_{c3}} = 3.78 \times 10^{-5}$ Wb

Flux through middle leg $= \phi_1 - 3.78 \times 10^{-5} = 1.22 \times 10^{-5}$ Wb

$B_{right} = \frac{3.78 \times 10^{-5}}{10^{-4}} = 0.378 \ T$

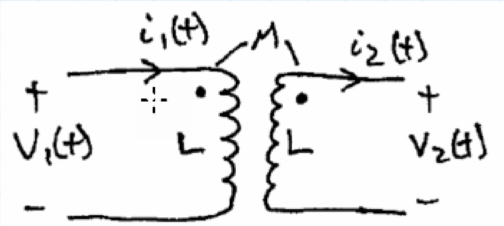
$B_{center} = \frac{1.22 \times 10^{-5}}{10^{-4}} = 0.122 T$

Problem 4. (25 points)

Two identical coils (each with zero resistance) are located near each other.

When a 60Hz sinusoidal voltage of 120 Volts (RMS) is applied to coil #1, the coil #1 current is 0.5 Amps (RMS) and the voltage measured on the open-circuited coil #2 is 60 Volts (RMS).

(a) What are the self inductances of coil #1 and #2 in Henries?



In this problem $L_1 = L_2$.

Some important consts and units:

$f := 60\ Hz$

$\omega := 2 \cdot \pi \cdot f$ $\quad \frac{Wb}{A} = 1\ H$

$120\ \pi = 376.991$

$H \cdot A = 1\ Wb$

$V \cdot s = 1\ Wb$ $\quad j := \sqrt{-1}$

$i_{1rms} := 0.5\ A$ $\quad i_{1peak} := i_{1rms}\ \sqrt{2} = 0.707\ A$

$v_{1rms} := 120\ V$ $\quad v_{1peak} := v_{1rms}\ \sqrt{2} = 169.706\ V$

$v_1(t) = 120\ \sqrt{2} \cdot \cos(120\ \pi \cdot Hz \cdot t) \cdot V$ $\quad i_1(t) = \frac{\sqrt{2}}{2} \cdot \cos(120\ \pi \cdot t) \cdot A$

$v_2(t) = 60\ \sqrt{2} \cdot \cos(120\ \pi \cdot Hz \cdot t) \cdot V$

$V_1 := 120\ V$

$V_2 := 60\ V$

Equations for the two loops are given in **(1)** and **(2)**

(1) $v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) - M \cdot \frac{d}{dt} i_2(t)$ $\quad$ **(2)** $v_2(t) = -L_2 \cdot \frac{d}{dt} i_2(t) + M \cdot \frac{d}{dt} i_1(t)$

This can be solved by integration but assume steady state and use phasors.

$$120 \cdot V = j \cdot 120\ \pi \cdot Hz \cdot L_1 \cdot \frac{1}{2}\ A \xrightarrow{solve, L_1} -\frac{2i \cdot V}{\pi \cdot A \cdot Hz}$$

Note j gives 90 degree phase shift, it is not included in the magnitude of L.

$L_1 := \frac{2 \cdot V}{\pi \cdot A \cdot Hz} = 0.637\ H$ $\quad \boxed{L_2 := L_1 = 0.6366\ H}$

(b) What is the magnitude of the mutual inductance between coil #1 and coil #2 in Henries?

$$60\ V = j \cdot 120\ \pi \cdot Hz \cdot M \cdot \frac{1}{2}\ A \xrightarrow{solve, M} undefined$$

Note j gives 90 degree phase shift, it is not included in the magnitude of M.

$M := \frac{V \cdot 1}{\pi \cdot A \cdot Hz} = 0.318\ H$ $\quad \boxed{M = 0.318\ H}$

(c) What is the coefficient of coupling for these two coils?

$k := \frac{M}{\sqrt{L_1 \cdot L_2}} = 0.5$ $\quad \boxed{k = 0.5}$

(d) What are the current magnitudes in coil #1 and #2 if a resistive load of 10 Ohms is placed across coil #2 while the given voltage is applied across coil #1?

When the load is added to the secondary side, there are two loops, two equations, two unknowns.

$$v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) - M \cdot \frac{d}{dt} i_2(t)$$

$$v_2(t) = -L_2 \cdot \frac{d}{dt} i_2(t) + M \cdot \frac{d}{dt} i_1(t)$$ Due to the load $v_2(t) = i_2(t) \cdot R$

$$V_1 = j \cdot 120\ \pi \cdot L_1 \cdot I_1 - j \cdot 120\ \pi \cdot M \cdot I_2$$
$$I_2 \cdot 10 = -j \cdot 120\ \pi \cdot L_2 \cdot I_2 + j \cdot 120\ \pi \cdot M \cdot I_1$$

substituions: $120\ \pi \cdot L_1 = 240\ H$, $120\ \pi \cdot L_2 = 240\ H$, $120\ \pi \cdot M = 120\ H$

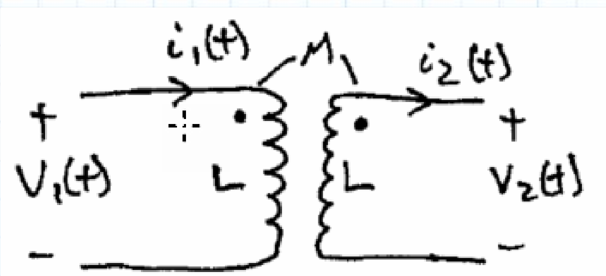
$$120 = 240\ I_1 - 120 \cdot I_2 \quad \textbf{(1)}$$
$$0 = 120 \cdot I_1 - 240 \cdot I2 - 10\ I_2 \quad \textbf{(2)}$$

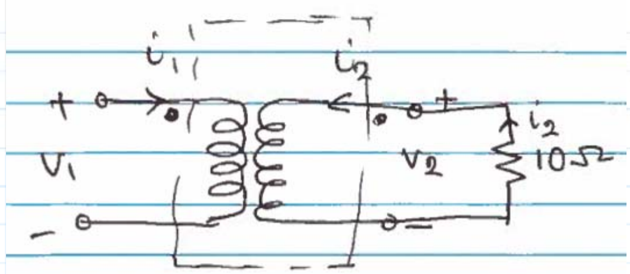
$$I_{matrix} := \text{rref}\left(\begin{bmatrix} 240 & -120 & 120 \\ 120 & -250 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0.658 \\ 0 & 1 & 0.316 \end{bmatrix}$$

$$I := \text{submatrix}\left(I_{matrix}, 1, 2, 3, 3\right) \cdot A = \begin{bmatrix} 0.658 \\ 0.316 \end{bmatrix} A$$

$$I_1 = 0.658\ A \qquad I_2 = 0.316\ A$$

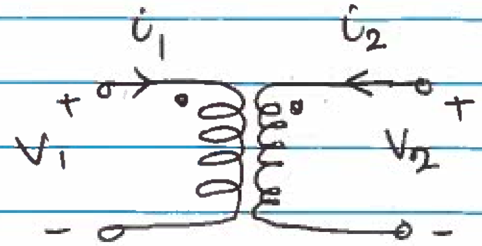
See the next 3 pages for an alternate approach. **But note the solutions are the same!** See the models below. Can you spot the difference?





(4)

(a)



$V_1 = 120\ V$ (rms)
$i_1 = 0.5A$ (rms)
$V_2 = 60\ V$

$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{--- (1)}$$

$$V_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt} \quad \text{--- (2)}$$

Winding 2 open circuit $\Rightarrow$
$i_2 = 0$

(1) Boils down to

$$\therefore \quad V_1 = L_1 \frac{d}{dt} i_1$$

in phasors

$$\bar{V}_1 = j\omega L_1 \bar{I}_1$$

took magnit

$$\therefore \quad L_1 = \left| \frac{\bar{V}_1}{j\omega \bar{I}_1} \right| = \left| \frac{120}{2\pi \times 60 \times 0.5} \right|$$

$$\boxed{L_1 = 0.6366\ H}$$

$$\boxed{L_2 = 0.6366\ H}$$

$L_1 = L_2$ because the coils are identical.

(b) . 2 Boils down to .

$$V_2 = M\frac{di_1}{dt}$$ (as winding 2 is still open circuit)

In Phasor form

$$\bar{V}_2 = j\omega M \bar{I}_1$$

$$M = \left|\frac{\bar{V}_2}{j\omega \bar{I}_1}\right| = \left|\frac{60}{j2\pi \times 60 \times 0.5}\right|$$

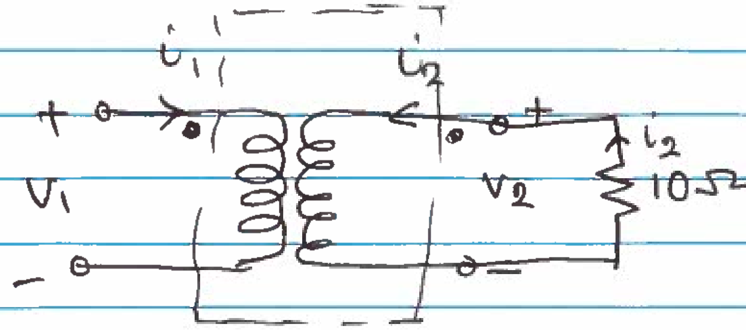
$$\boxed{M = 0.318\ H}$$

(c) Coefficient of coupling,

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.318}{\sqrt{0.6366 \times 0.6366}}$$

$$\boxed{K = 0.5}$$

d



$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{--- } (1)$$

$$V_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt} \quad \text{--- } (2)$$

and $V_2 = -10\, i_2$ — (3)

Use (2) and (3)

$$\boxed{\begin{aligned} -10\, i_2 &= M\frac{di_1}{dt} + L_2\frac{di_2}{dt} \quad \text{--- } (4) \\ V_1 &= L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{--- } (5) \end{aligned}}$$

Substitute $\frac{d}{dt}$ with $j\omega$.
for phasors.

(5) Boils down to

$$\bar{V}_1 = j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2$$

(4) Boils down to

$$j\omega L_2 \bar{I}_2 + j\omega M \bar{I}_1 + 10\,\bar{I}_2 = 0$$