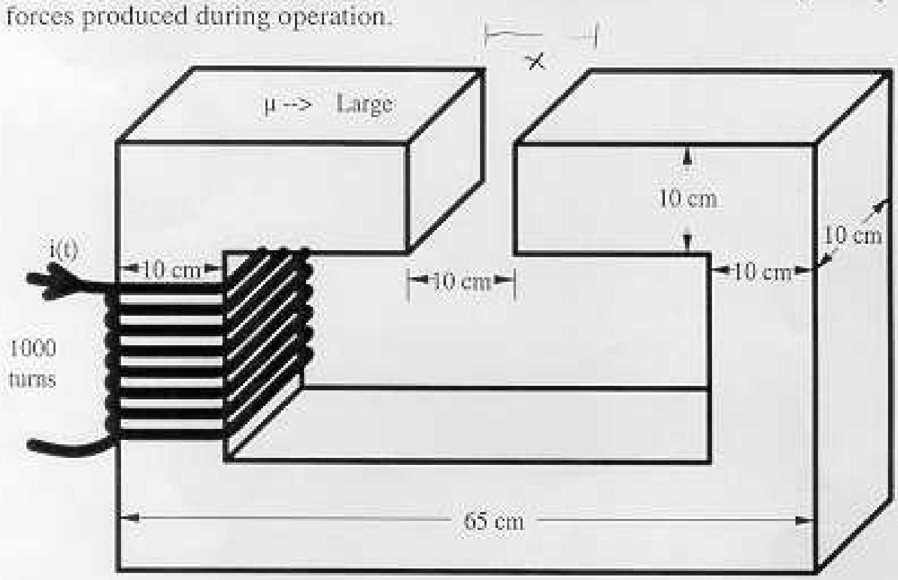
Problem 1 (25 pts.)

In a certain physics experiment, it is desired to generate a strong magnetic field in a significant volume to test a sample. The core structure shown in the figure is proposed to do this. The core material is a special cobalt alloy that can handle about 2.7 T without saturation. ~~The field is supplied as a pulse~~. It is a concern as to whether the material is strong enough to withstand the forces produced during operation.



a) When a 200 Amp current is applied, what mechanical force is experienced on the cobalt core? In what direction? You may give your answer either in N or in lb.

$\lambda = 1000\phi \qquad \phi = B \times .1^2 = .01B \qquad B = 4\pi\times10^{-7} H$

$1000i = Hx \qquad H = \frac{1000i}{x} \qquad \text{so } B = \frac{4\pi\times10^{-4} i}{x} \qquad \text{so } \phi = \frac{4\pi\times10^{-6} i}{x}$

$\text{so } \lambda = \frac{4\pi\times10^{-3}}{x} i \qquad w_m' = \frac{2\pi\times10^{-3}}{x} i^2 \qquad f^e = \frac{\partial w_m'}{\partial x} = -\frac{2\pi\times10^{-3} i^2}{x^2}$

$= 25{,}133 \text{ N}$

b) As the current is brought up from 0 to 200 A, how much electrical energy must be delivered to the coil?

$EEE = \int_0^{\lambda_A} i\,d\lambda \qquad \lambda_A = \frac{4\pi\times10^{-3}\times200}{.1} = 25.13 \qquad i = \frac{.1\lambda}{4\pi\times10^{-3}}$

$= 7.96\lambda$

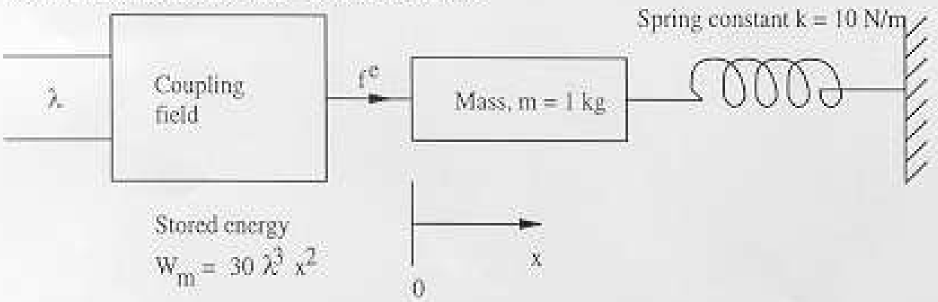
$= \int_0^{25.13} 7.96\lambda\,d\lambda = 3.98\times25.13^2 = 2513 \text{ J}$

Problem 2 (25 pts)

In a certain electromechanical system, the stored energy in the coupling field is given by

$W_m(\lambda,x) = 30\lambda^3x^2$ (units of joules)

A block diagram of the system is shown below. The spring is set up to generate 0 force when $x=0$. Constraints are that $-2 \text{ m} < x < 2 \text{ m}$.



a) Write the equations of mechanical motion for this system in state space form.

$$\frac{dx}{dt} = v \qquad\qquad \frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -10x + f^e \qquad\qquad \frac{dv}{dt} = -10x - 60\lambda^3 x$$

$$f^e = -60\lambda^3 x$$

b) The system starts from a stationary position, with $\lambda(0)= 1$ Wb and $x(0) = 1$ m. The electrical input is operating to maintain λ constant. Provide an estimate of the velocity dx/dt at time $t = 0.02$ s. (Hint: Use Euler's method with a step size of 0.01 s)

$$x(.01) = 1 + 0\times.01 = 1 \text{ m}$$

$$v(.01) = 0 + (-10\times1 - 60\times1\times1).01 = -0.7 \text{ m/s}$$

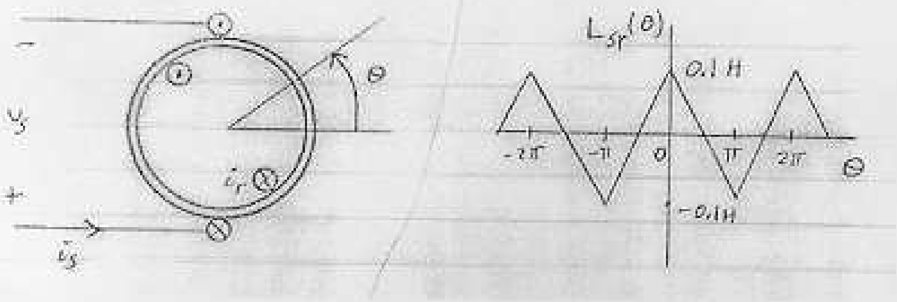
$$x(.02) = 1 + (-0.7)\times.01 = 0.993 \text{ m}$$

$$v(.02) = -0.7 + (-10\times0.993 - 60\times0.993).01 = -1.4 \text{ m/s}$$

(-1.395) m/s

Problem 3 (25pts)

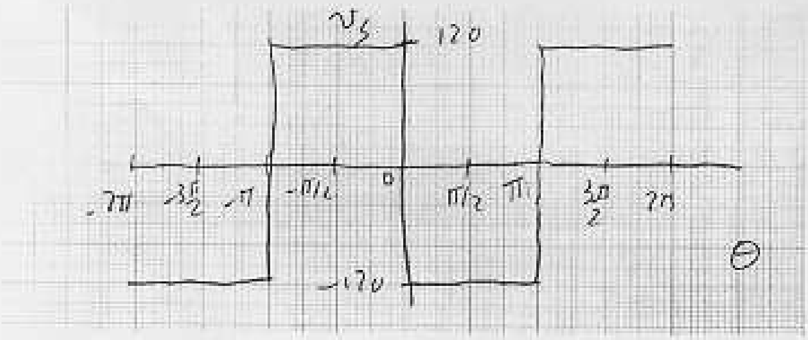
A single-phase generator consists of a coil on the stator and a coil on the rotor with a mutual inductance variation with θ as shown in the figure below. The rotor is being driven at a constant speed of 377 radians per second and the rotor coil has a constant dc current $i_r = 5$ A. The stator coil is open circuited ($i_s = 0$).



(a) Plot the open circuit voltage as a function of θ (label all points)

$v_s = \frac{d}{dt}(L_{sr}(\theta)\times 5)$ $\quad 0<\theta\le\pi \; \left(L_{sr}(\theta) = -\frac{0.2}{\pi}\theta\right)$

$v_s = -\frac{1}{\pi}\omega = -\frac{377}{\pi} = -120v$



$i_s = 10$ Amps

(b) What is the torque of electrical origin when θ = 45°?

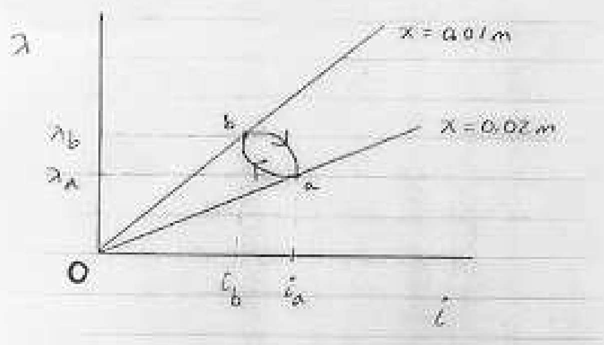
$W_m' = \frac{1}{2}L_{ss}i_s^2 + L_{sr}(\theta)i_s i_r + \ldots i^2$

$T^e = i_s i_r \frac{\partial L_{sr}(\theta)}{\partial \theta}$

$= 10\times 5\times\left(-\frac{0.2}{\pi}\right) = -\frac{10}{\pi}$ Nm

Problem 4 (25pts)

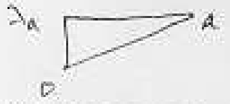
An electromechanical system has a linear flux-linkage vs current relationship for constant position x, as shown in the figure below for two values of x. The system goes through one complete cycle from point a to point b and back to point a as shown in the path on the figure.



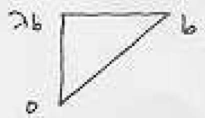
$W_m = \int i\,d\lambda \quad x = \text{const}$

$EFE = \int_{\lambda_a}^{\lambda_b} i\,d\lambda \quad \text{path}$

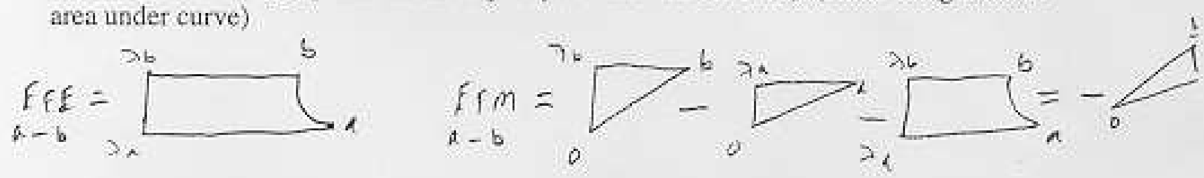
(a) Find the energy stored in the coupling field at point a (use a well-labeled graphical area to give your answer). (Hint: Integration is area under curve)



(b) Find the energy stored in the coupling field at point b (use a well-labeled graphical area to give your answer). (Hint: Integration is area under curve)



(c) Find the EFE (energy input from the electrical terminals) and the EFM (energy input from the mechanical terminals) in the case when the system moves from point a to point b (use well-labeled graphical areas to give your answers for each). (Hint: Integration is area under curve)



(d) Find EFE and the EFM for the movement of the system from point b back to point a (use well-labeled graphical areas to give your answers for each). (Hint: Integration is area under curve)