ECE 430 Exam #1, Spring 2010 Name: Solution

90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. 25 / 25 2. 25 / 25

3. 25 / 25 4. 25 / 25 Total 100 / 100

first two

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^*$

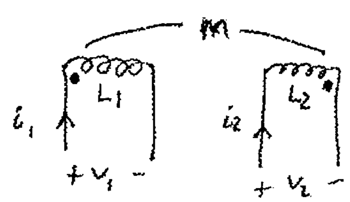
$\bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $I_L = \sqrt{3} I_\phi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

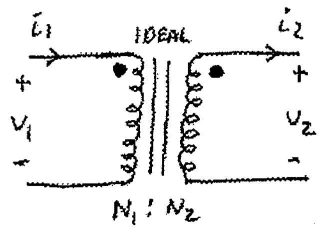
$-180^\circ < \theta < 0$ (lead) $V_L = \sqrt{3} V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\Re = \frac{l}{\mu A}$ $MMF = Ni = \phi \Re$

$\phi = BA$ $\lambda = Li = N\phi$ $k = \frac{M}{\sqrt{L_1 L_2}}$ 1 hp = 746 Watts

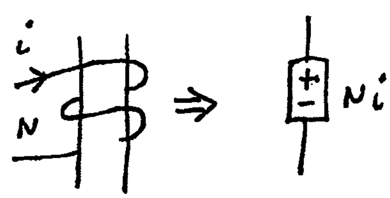


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



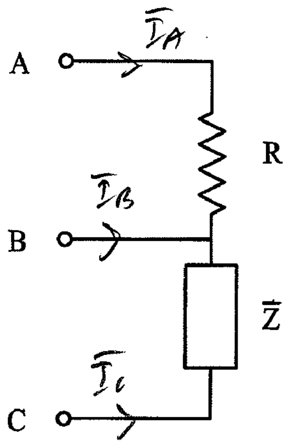
$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

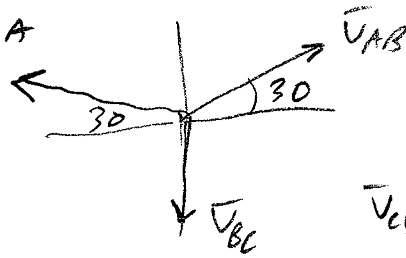
$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1. (25 points)

A balanced, symmetrical 208 V (RMS Line to Line), 3-phase, 60 Hz source with ABC sequence is supplying the three-wire (not balanced 3-phase) load shown below.





Assume $\angle \bar{V}_{AB} = 30°$

$\bar{I}_A = \frac{208\angle 30}{100} = 2.08\angle 30°$

$\bar{V}_{CB} = -\bar{V}_{BC} = 208\angle 90°$

$\bar{I}_C = \frac{\bar{V}_{CB}}{100\bar{k}}$

Resistor R is fixed at 100 Ohms. Impedance $\bar{Z}$ is specified as a multiple $\bar{k}$R, where $\bar{k}$ could be positive, negative, or even complex.

a) If $\bar{k} = 1$, what are the three line current magnitudes, and what is the total three-phase real power consumed by this load? (12 points)

If $\bar{k} = 1$, $\boxed{\bar{I}_C = \frac{208\angle 90}{100} = 2.08\angle 90°}$ $\boxed{\bar{I}_A = 2.08\angle 30 \text{ (above)}}$

$\bar{I}_B = -\bar{I}_A - \bar{I}_C = -2.08\angle 30 - 2.08\angle 90 = -1.801 - j1.04 - j2.08 = -1.801 - j3.12$

$\boxed{\bar{I}_B = 3.60\angle 240°}$ $\boxed{P = \frac{208^2}{100} + \frac{208^2}{100} = 865 \text{ W}}$

b) What value of $\bar{k}$, if any, will produce balanced three-phase line currents with this connection? (13 points)

Using $\angle \bar{V}_{AB} = 30°$, $\bar{I}_A = 2.08\angle 30°$.

Or, $\bar{k} = 1\angle -60°$ will also shift $\bar{I}_C$ to $2.08\angle 150°$. This would be ABC sequence currents.

From a), we found that $\bar{k} = 1$ gives $\bar{I}_C = 2.08\angle 90°$. So, if we use $\bar{k} = -1$ then $\bar{I}_C$ will be $2.08\angle -90°$ and that will give $\bar{I}_B = 2.08\angle 150°$ which is a balanced 3φ set (ACB seq.)

Problem 2. (25 pts)

Three loads are connected in parallel across a 60Hz, 3-phase source at 208 Volts (RMS Line-Line).

Load #1: Delta-connected load with 12 Amps phase current at 0.9 power factor lag
Load #2: Wye-connected load with 20 Ohms per phase pure resistance
Load #3: Delta-connected load with 3,000 Watts plus 1,500 Vars of power (3-phase)

a) Find the line current magnitude for each of the three loads (8 points)
b) Find the source line current magnitude (8 points)
c) Find the value of capacitive VARS (3-phase) that should be added in parallel to these three loads to make the overall power factor 0.95 lag (9 points).

a) $I_{\ell_1} = \sqrt{3}\, I_{\phi_1} = \boxed{20.784\,A}$ $\quad I_{\ell_2} = \dfrac{208/\sqrt{3}}{20} = \boxed{6\,A}$

$I_{\ell_3} = \dfrac{\sqrt{3000^2 + 1500^2}}{\sqrt{3} \times 208} = \boxed{9.31\,A}$

b) $\bar{S}_{TOT} = 3 \times 12 \times 208 \angle 25.8^\circ + 3 \times 120 \times 6 + 3000 + j1500$

$= 6742 + j3259 + 2160 + 3000 + j1500 = 11902 + j4759 = 12818 \angle 21.8^\circ$

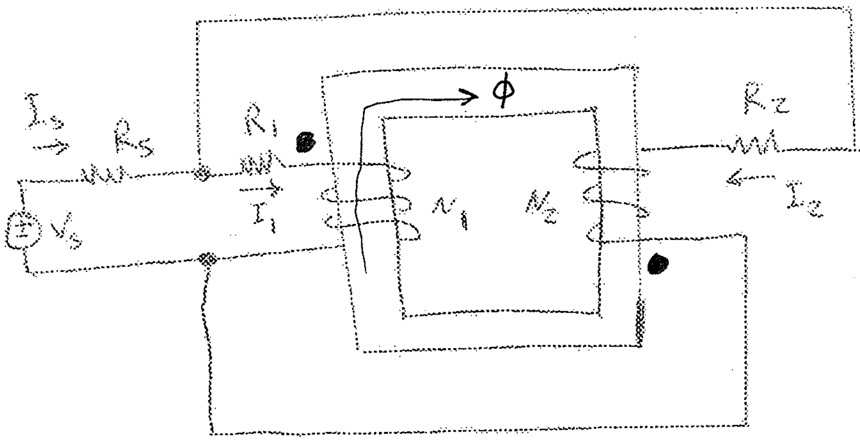
$12818 = \sqrt{3} \times 208 \times I_\ell \quad \boxed{I_\ell = 35.58\,A}$

c) $\bar{S}_{3\phi\,NEW} = \dfrac{11902}{0.95} \angle +\cos^{-1} .95 = 11902 + j3912$

$\boxed{Q_{add} = 3912 - 4759 = -847 \text{ VARS}}$

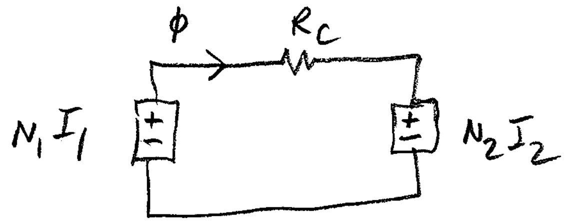
Problem 3. (25 points)

A magnetic core with two coils is shown below. Magnetic core reluctance, R_C, electrical resistances, R_S, R_1, R_2, and winding turns, N_1, N_2 are known.



a) Label the dots on both coils (4 points) (see figure)

b) Draw the magnetic equivalent circuit (Neglect leakage flux) (4 points)



$$-N_1 I_1 + \phi R_c + N_2 I_2 = 0$$

$$\phi = \frac{N_1 I_1 - N_2 I_2}{R_c}$$

c) Solve for inductances, L_1, L_2, mutual inductance, M, and coupling factor, k (9 points)

$$v_1 \text{ (+ on top)} = N_1 \frac{d\phi}{dt} = \frac{N_1^2}{R_c}\frac{dI_1}{dt} - \frac{N_1 N_2}{R_c}\frac{dI_2}{dt}$$

$$v_2 \text{ (+ on top)} = -N_2 \frac{d\phi}{dt} = -\frac{N_1 N_2}{R_c}\frac{dI_1}{dt} + \frac{N_2^2}{R_c}\frac{dI_2}{dt}$$

$$L_1 = \frac{N_1^2}{R_c} \qquad L_2 = \frac{N_2^2}{R_c}$$

$$M = \frac{N_1 N_2}{R_c}$$

$$k = \frac{N_1 N_2 / R_c}{\sqrt{\frac{N_1^2}{R_c}\frac{N_2^2}{R_c}}} = 1$$

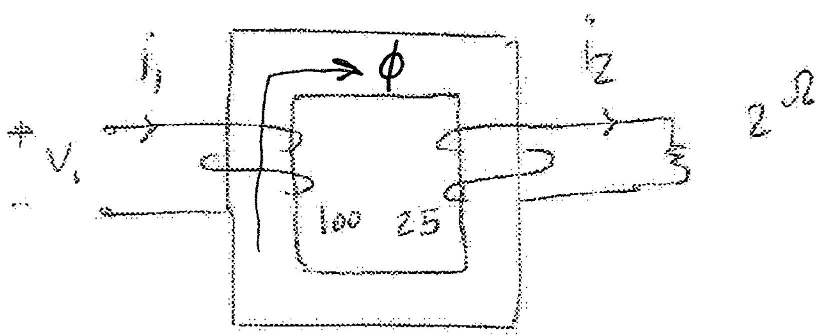
d) Write the two equations for V_S in terms of I_1 and I_2 (8 points)

$$V_S = (I_1 + I_2) R_S + R_1 I_1 + v_1 \text{ (above)}$$

$$V_S = (I_1 + I_2) R_S + R_2 I_2 + v_2 \text{ (above)}$$

Problem 4. (25 points)

A two-coil system is shown below. You can neglect the leakage flux, but the core has some reluctance that you need to compute.



System parameters are:

$$\begin{cases} N_1 = 100 \quad N_2 = 25 \\ Mean\ \ length = 0.4m \\ Area = 0.005\ m^2 \\ \mu_r = 1000 \\ Coil\ resistances\ are\ negligible \end{cases}$$

$v_1 = 120\sqrt{2}\cos(2\pi 60 t)$

If a 120 V (RMS), 60 Hz sinusoidal voltage source is applied to coil 1, and a 2 ohm resistor is connected across the terminals of coil 2. Find the following quantities in time domain:

a) The magnetic flux, ϕ, in the iron core (5 points) use cw flux

$v_1 = N_1 \frac{d\phi}{dt} = 120\sqrt{2}\cos(2\pi 60t)$ $\phi = \frac{120\sqrt{2}}{100 \times 2\pi 60} \sin(2\pi 60 t)$

$\boxed{\phi = .0045 \sin 2\pi 60 t \text{ webers}}$

$\boxed{i_1 = 4.26\sqrt{2}\cos(2\pi 60 t - 28^\circ)}$

b) The magnetic flux density, B, in the iron core (5 points)

$\bar{I}_1 = \frac{2.86}{\sqrt{2}}\angle -90^\circ + \frac{5.3}{\sqrt{2}}\angle 0$

$B = \frac{\phi}{A} = \frac{.0045}{.005} \sin 2\pi 60 t = 0.9 \sin 2\pi 60 t$ Tesla (or webers/m²)

c) The secondary coil current, i_2 (7 points)

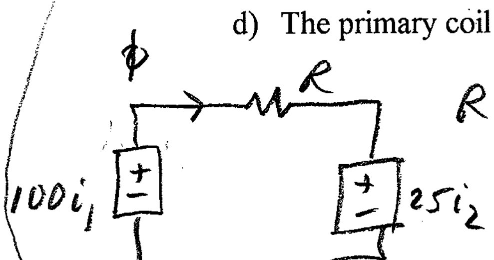
$= 3.75 - j2.02$

$= 4.26\angle -28^\circ$

v_2 (+ on top) $= 25 \frac{d\phi}{dt} = 25 \times .0045 \times 2\pi 60 \cos 2\pi 60 t$

$v_2 = 42.42 \cos 2\pi 60 t = 2 i_2$

d) The primary coil current, i_1 (8 points)



$R = \frac{0.4}{4\pi \times 10^{-7} \times 1000 \times .005}$

$= 63662$

$\boxed{i_2 = 21.21 \cos 2\pi 60 t}$

$100 i_1 = 63662 \times .0045 \sin 2\pi 60 t$
$+ 25 \times 21.21 \cos 2\pi 60 t$

$i_1 = 2.86 \sin 2\pi 60 t + 5.3 \cos 2\pi 60 t$