

ECE330
Exam #1
Fall 2003

Name Solution
(Print Name)

Section: (Circle One) 10 MWF (Sauer) 10TT (Krein)

Problem 1 _____ Problem 2 _____ Problem 3 _____ TOTAL: _____

USEFUL INFORMATION

$$\sin x = \cos(x - 90^\circ)$$

$$\bar{z}_y = \frac{1}{3} \bar{z}_\Delta$$

$$\bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle \theta$$

$$I_L = \sqrt{3} I_\phi (\angle + \alpha)$$

$$\oint_C \underline{H} \cdot d\underline{\ell} = \int_S \underline{J} \cdot \underline{n} \, dA$$

$$\oint_C \underline{E} \cdot d\underline{\ell} = - \frac{d}{dt} \int_S \underline{B} \cdot \underline{n} \, dA$$

$$\oint_S \underline{B} \cdot \underline{n} \, dA = 0$$

$$R = \frac{l}{\mu A}$$

$$\text{mmf} = Ni = \phi R$$

$$\lambda = N\phi = Li$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

Problem 1 (42 pts.) (No partial credit - 6 points each)

- a) A single-phase load has a voltage of $157 \cos(377t + 15^\circ)$ Volts with a current into the positive terminal of $12 \sin(377t + 70^\circ)$ Amps.

$$\bar{V} = 111.03 \angle 15^\circ \quad \bar{I} = 8.49 \angle -20^\circ$$

$$\bar{S} = 942 \angle 35^\circ$$

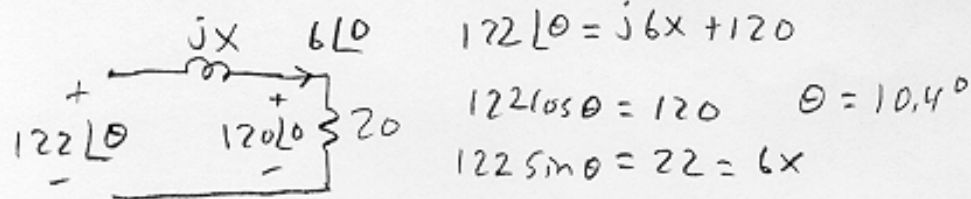
The real power absorbed by this load is 772 Watts.

- b) Two loads in parallel consume the complex powers of $100 + j100$ kVA and $50 - j20$ kVA.

$$\bar{S} = 150k + j80k = 170k \angle 28^\circ$$

The power factor of the total load is 0.88 (specify lead lag circle one)

- c) A single-phase load resistor of 20 Ohms is being served through a line with an inductive reactance of value X. The source voltage magnitude is 122 Volts and the load voltage magnitude is 120 Volts.



The value of X is 3.67 Ohms.

- d) A 3 phase, delta connected load has a line to line voltage of 480 V. The complex power per phase is $2,000 + j700$ VA.

$$\bar{S}_{3\phi} = 6000 + j2100 = 6357 \angle 19^\circ = \sqrt{3} \times 480 I_L \angle \theta$$

The magnitude² of the line current is 7.65 A.

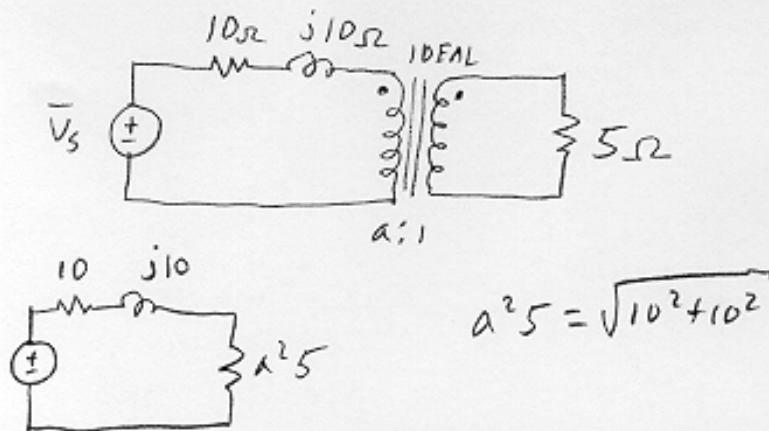
- e) A coil of 300 turns is wound on an iron core whose cross sectional area is 0.002 square meters. The applied voltage is $120\sqrt{2} \cos(2\pi 60t)$ Volts.

$$120\sqrt{2} \cos(2\pi 60t) = 300 \frac{d\phi}{dt} \quad \phi = \frac{120\sqrt{2}}{300 \times 2\pi 60} \sin 2\pi 60t$$

$$B = \frac{\phi}{A} = \frac{120\sqrt{2}}{0.002 \times 300 \times 2\pi 60} \sin 2\pi 60t$$

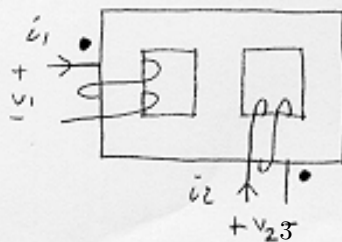
The peak value of the magnetic flux density is 0.75 Tesla.

- f) Find the turns ratio "a" to maximize the power absorbed by the 5Ω load in the impedance matching circuit shown below:



The turns ratio "a" should be 1.68.

- g) Put the polarity dot markings on the two coils shown below.



Problem 2 (29 pts)

The following three-phase balanced loads are connected in parallel across a three-phase wye-connected, 60 Hz source of 4,160 V (line to line)

- Load #1 120 kVA at 0.8 PF lag (Wye connected)
 Load #2 180 kW at 0.7 PF lag (Wye connected)
 Load #3 13 Amps phase current, unity power factor (Delta connected)

- a) Find the total complex power consumed by the three loads
 b) Find the total source line current (magnitude).
 c) Find the CAPACITANCE needed per phase (for a delta connection) so that the overall power factor is 0.95 lag.
 d) Find the new source line current with the 3-phase bank of capacitors installed.

$$\begin{aligned}
 a) \quad \bar{S}_{3\phi} &= 120k \angle 37^\circ + \frac{180k}{0.7} \angle 46^\circ + 3 \times 13 \times 4160 \angle 0 \\
 &= 95.8k + j72.2k \\
 &\quad + 180k + j185k \\
 &\quad + 162k \\
 &\quad \boxed{438k + j257k \text{ VA}}
 \end{aligned}$$

$$b) \quad \sqrt{(438k)^2 + (257k)^2} = \sqrt{3} \times 4160 I_L \quad \boxed{I_L = 70 \text{ Amps}}$$

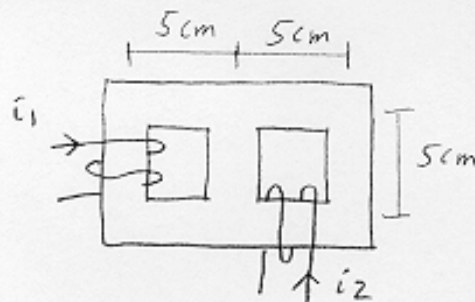
$$c) \quad \frac{438k}{0.95} = 461k \text{ VA} \quad \bar{S}_{\text{new}} = 461k \angle 18^\circ \quad Q_{\text{new}} = 142 \text{ kVAR}$$

$$Q_{\text{add}} = (257 - 142)k = 115k = 3 \times 4160^2 \times 2\pi 60 C$$

$$\boxed{C = 5.88 \mu\text{F}}$$

$$d) \quad 461k = \sqrt{3} \times 4160 I_L \quad \boxed{I_L = 64 \text{ Amps}_{\text{new}}}$$

Problem 3 (29 pts)



$$\mu = 1000 \mu_0$$

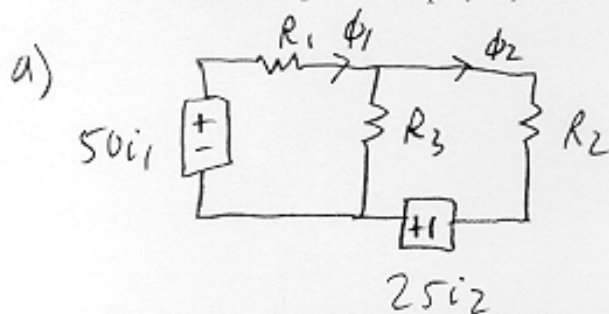
$$N_1 = 50 \text{ turns}$$

$$N_2 = 25 \text{ turns}$$

$$A = 10 \text{ cm}^2 \text{ everywhere}$$

Neglect leakage flux, coil resistance and core losses.

- Find the coil self inductances (L_1 and L_2) plus the mutual inductance (M)
- Find the flux density (B) in each vertical leg if the current $i_1 = 5$ Amps (DC), and the current $i_2 = -10$ Amps (DC).



$$R_1 = R_2 = \frac{.15}{1000 \times 4\pi \times 10^{-7} \times .001}$$

$$= 119,366$$

$$R_3 = \frac{R_1}{3} = 39,789$$

$$-50i_1 + 119,366\phi_1 + 39,789(\phi_1 - \phi_2) = 0$$

$$-25i_2 + 119,366\phi_2 + 39,789(\phi_2 - \phi_1) = 0$$

$$\begin{bmatrix} 159,155 & -39,789 \\ -39,789 & 159,155 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 50i_1 \\ 25i_2 \end{bmatrix}$$

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$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \frac{\begin{bmatrix} 159,155 & 39,789 \\ 39,789 & 159,155 \end{bmatrix} \begin{bmatrix} 50i_1 \\ 25i_2 \end{bmatrix}}{2.375 \times 10^{10}}$$

$$\phi_1 = 335 \times 10^{-6} i_1 + 41.8 \times 10^{-6} i_2$$

$$\phi_2 = 83.8 \times 10^{-6} i_1 + 167 \times 10^{-6} i_2$$

$$\lambda_1 = 50\phi_1 = 0.01675 i_1 + 0.0021 i_2$$

$$\lambda_2 = 25\phi_2 = 0.0021 i_1 + 0.004175 i_2$$

$$\boxed{\begin{array}{l} L_1 = 0.01675 \text{ H} \\ L_2 = 0.004175 \text{ H} \\ M = 0.0021 \text{ H} \end{array}}$$

$$b) \quad B_1 = \frac{\phi_1}{A} = \frac{335 \times 10^{-6} \times 5}{.001} - \frac{41.8 \times 10^{-6} \times 10}{.001} = 1.258 \text{ T}$$

$$B_2 = \frac{\phi_2}{A} = \frac{83.8 \times 10^{-6} \times 5}{.001} - \frac{167 \times 10^{-6} \times 10}{.001} = -1.251 \text{ T}$$

$$B_3 = B_1 - B_2 = 2.5 \text{ T}$$