1. ________/25

2. ________/25

3. ________/25

4. ________/25

TOTAL ________/100

USEFUL INFORMATION

$\sin(x) = \cos(x - 90^\circ)$

$V = \bar{Z}I$

$S = \bar{V} \bar{I}^* = P + jQ$

$\bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle \theta$

$0 < \theta < 180^\circ$ (lag)

$I_L = \sqrt{3}I_\phi$ (delta)

$\bar{Z}_Y = \bar{Z}_\Delta / 3$

$-180^\circ < \theta < 0$ (lead)

$V_L = \sqrt{3}V_\phi$ (wye)

$\mu_0 = 4\pi \times 10^{-7}$ H/m

ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int H \cdot dl = \int J_f \cdot \hat{n} dA$

$\int E \cdot dl = -\frac{d}{dt} (\int B \cdot \hat{n} dA)$

$R = \frac{1}{\mu A}$

$N_i = R\phi$

$\phi = BA$

$\lambda = N\phi = Li$ (if linear)

$V = \frac{d\lambda}{dt}$

$k = \frac{M}{\sqrt{L_1L_2}}$

1hp = 746 W

$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

$\frac{a}{N} = \frac{N_1}{N_2}$

$N_1i_1 = N_2i_2$
Problem 1 (25 Points)
4 single-phase loads are connected in parallel to a generator that supplies 480 V. The loads are
given as:
1) A purely resistive load that consumes 50 kW
2) A load that consumes 10 kVA at a power factor of 0.707 leading
3) A load with impedance $Z = 120 + j50 \Omega$
4) A purely inductive load of 30 kVAR

a) What is the total power supplied by the source? (12 points)
\[ \overline{S_{1}} = 50 + j0 \text{ kVA} \]
\[ \theta_1 = -\cos^{-1}(0.707) = -45^\circ \]
\[ \overline{S_{2}} = 10 \angle -45^\circ \text{ kVA} \]
\[ = 7.071 - j7.071 \text{ kVA} \]
\[ \overline{S_{3}} = 120 + j50 \Omega \]
\[ = 130/22.628 \Omega \]
\[ \overline{S_{4}} = \frac{V^2}{Z} \angle 22.628^\circ \]
\[ \overline{S_{5}} = 1.772 \angle 22.628^\circ \text{ kVA} \]
\[ = 163 + j30.682 \text{ kVA} \]
\[ \overline{S_{6}} = 0 + j50 \text{ kVA} \]
\[ \overline{S_{\text{Tot}}} = \overline{S_{1}} + \overline{S_{2}} + \overline{S_{3}} + \overline{S_{4}} + \overline{S_{5}} + \overline{S_{6}} \]
\[ = 58.706 + j23.610 \text{ kVA} \]
\[ = 63.276 \angle 21.908^\circ \text{ kVA} \]

b) What is the source current? (5 points)
\[ \overline{I_{\text{ tot}}} = \frac{\overline{S_{\text{ tot}}}}{V} \]
\[ \overline{I} = \left( \frac{S_{\text{ tot}}}{V} \right) \]
\[ \overline{I} = 31.324 \angle -21.908^\circ \text{ A} \]

C) A capacitor is connected in parallel with the loads to make the power factor 0.995 lagging. What is the magnitude of the new source current? (8 points)
\[ \theta_6 = \cos^{-1}(0.995) \]
\[ = 5.732^\circ \]
\[ Q_6 = \frac{\overline{I}_{\text{ tot}} (\theta)}{\sin \theta} \]
\[ Q_6 = 893 \text{ kVAR} \]
\[ S_6 = \sqrt{P^2 + Q_6^2} \]
\[ \overline{S_6} = 59.001 \text{ kVA} \]
\[ \overline{I_6} = \frac{S_{\text{ tot}}}{V} \]
\[ \overline{I_6} = 122.919 \text{ A} \]
Problem 2 (25 points)
Three 3-phase loads are connected in parallel to a wye-connected source with 208 V line to line. The loads are given as:

1) A wye connected load consuming 10 kVA at a 0.95 leading power factor
2) A wye connected load consuming 80 kW at a 0.975 lagging power factor
3) A delta connected load with impedance \( Z = 15 + j8 \) \( \Omega \)

a) What is the source line current? (13 points)

\[
\begin{align*}
\mathbf{S}_1 &= 10 \angle 0.95^\circ \text{kVA} \\
\mathbf{S}_2 &= 80 \angle -0.975^\circ \text{kW} \\
\mathbf{S}_3 &= (15 + j8) \angle 0^\circ \text{kVA}
\end{align*}
\]

\[
\mathbf{S}_{\text{tot}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3
\]

\[
= 96.257 + j18.703 \text{kVA}
\]

\[
= 98.057 \angle 10.99^\circ \text{kVA}
\]

\[
\mathbf{I}_L = \left( \frac{\mathbf{S}_{\text{tot}}}{\sqrt{3} \mathbf{V}} \right)^*
\]

\[
\mathbf{I}_L = 277.124 \angle -10.99^\circ \text{A}
\]

b) A delta connected capacitor bank is added in parallel to the three loads to make the overall power factor 0.995 lagging. What is the new source line current? (12 points)

\[
\begin{align*}
\mathbf{Q}_1 &= 9.460 \text{kVAR} \\
\mathbf{Q}_2 &= 0 \\
\mathbf{Q}_3 &= 3 \left( \frac{120^\circ}{\sqrt{3}} \right) \text{kVAR}
\end{align*}
\]

\[
\begin{align*}
\mathbf{S}'_1 &= 96.257 + j18.703 \text{kVA} \\
\mathbf{S}'_2 &= 96.720 \angle -7.93^\circ \text{kVA}
\end{align*}
\]

\[
\mathbf{I}'_L = \left( \frac{\mathbf{S}'_{\text{tot}}}{\sqrt{3} \mathbf{V}} \right)^*
\]

\[
\mathbf{I}'_L = 268.48 \angle -7.93^\circ \text{A}
\]
Problem 3 (25 Points)

A coil is wound 250 times around an iron core with infinite permeability (dimensions given above) that has an air gap of 5 mm and a depth into the page of 3 cm. Fringing effects are included.

With the voltage polarity and current direction as defined above:

a) What is the area of the air gap (include fringing)? (3 points)

\[ A = \left( \frac{2}{\pi} + \frac{5}{\pi} \right) \left( \frac{3}{\pi} + \frac{5}{\pi} \right) \]

\[ A = 0.000875 \text{ m}^2 \]

b) Draw the magnetic equivalent circuit and determine the reluctance values for the air gap. (4 points)
c) What is the inductance of the coil? (10 points)

\[ N_0 = \phi(3 R_0) \]
\[ \Phi = \frac{N_0}{3 R_0} \cdot i \]
\[ \lambda = N_0 \Phi \Rightarrow \lambda = \frac{N_0^2}{3 R_0} \cdot i = Li \]
\[ L = \frac{N_0^2}{3 R_0} \]
\[ L = 0.00458 \, \text{H} \]

d) What is the inductance of the coil if fringing is neglected? (8 points)

\[ A = 0.00160 \]
\[ R_0 = \frac{3}{\mu_0 A} = 4.631 \times 10^6 \, \text{A} \cdot \text{m} / \text{H} \]
\[ L = \frac{N_0^2}{3 R_0} \]
\[ L = 0.00314 \, \text{H} \]
Two coils are wrapped around an iron core with infinite permeability as shown. Coil 1 has 400 turns while coil 2 has 100 turns. The air gap $g$ is 2.5 mm and the depth into the page is 2 cm. Neglect fringing.

a) Put the dot markings on the two coils. (5 points)

b) What is the self-inductance and the mutual inductance of the two coils? (5 points each=15 total points)
c) What is the RMS magnitude of the open circuit voltage on coil 2 if \( i_1(t) = 10\cos(377t) \)?

(5 points)

\[
V_i = \frac{dl_1}{dt} = L_1 \frac{dl_1}{dt} - M \frac{dl_2}{dt} \\
V_e = \frac{dl_2}{dt} = -M \frac{dl_2}{dt} + L_2 \frac{dl_2}{dt}
\]

Open Circuit: \( i_1 = 0 \)

\[
\frac{dl_1}{dt} = -10(377)\sin(377t) \\
= 10(377)\sin(-377t) \\
= 10(377)\cos(-377t+90) \\
= 10(377)\cos(377t+90)
\]

\[
V_2 = -M \frac{dl_2}{dt} \\
V_2 = -9.096\cos(377t+90) \text{ V} \\
V_6 = 9.096\cos(377t+90) \text{ V}
\]

\[
V_{rms} = \frac{9.096}{\sqrt{2}} \\
V_{rms} = 6.4152 \text{ V}
\]
(Blank page for extra work)