ECE 330 Exam #2, Fall 2015 Name: SOLUTIONS
90 Minutes (PREPARED BY ADG)

Section (Check One) MWF 10am ________ MWF 2pm ________

TTH

1. ________ / 25 2. ________ / 25

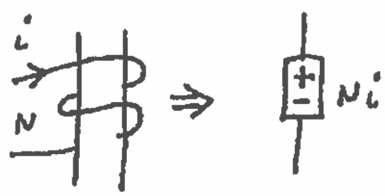
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^*$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $B = \mu H$ $\phi = BA$ $\lambda = N\phi$ $\lambda = Li$ (if linear)



$W_m = \int_0^\lambda i \, d\hat{\lambda}$ $W_m' = \int_0^i \lambda \, d\hat{i}$ $W_m + W_m' = \lambda i$ $f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $x \to \theta$

$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i \, d\lambda$ $\underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\lambda = \frac{\partial W_m'}{\partial i}$ $i = \frac{\partial W_m}{\partial \lambda}$

Problem 1. (25 points)

The co-energy of a device is given by

$$W_m'(i,x) = \frac{i^3}{6x} + \frac{i}{x}$$

Find:

a) The expression for flux linkage (λ) as a function of i and x. ✓
b) The force of electrical origin $f^e(i,x)$
c) The energy stored in the coupling field W_m as a function of i and x.

(a) $$\lambda(i,x) = \frac{\partial W_m'(i,x)}{\partial i} \longrightarrow$$

$$\longrightarrow \lambda(i,x) = \frac{3i^2}{6x} + \frac{1}{x} = \frac{1}{2}\cdot\frac{i^2}{x} + \frac{1}{x}$$

(b) $$f^e(i,x) = \frac{\partial W_m'(i,x)}{\partial x}$$

$$f^e(i,x) = -\frac{1}{x^2}\left(\frac{i^3}{6} + i\right)$$

(c) $$W_m = \lambda \cdot i - W_m'$$

$$W_m(i,x) = \lambda(i,x)\cdot i - W_m'(i,x)$$

$$= \left(\frac{1}{2}\frac{i^2}{x} + \frac{1}{x}\right)\cdot i - \left(\frac{i^3}{6x} + \frac{i}{x}\right)$$

$$= \frac{i^3}{2x} + \cancel{\frac{i}{x}} - \frac{i^3}{6x} + \cancel{\frac{i}{x}}$$

$$= \frac{2i^3}{6x} = \frac{i^3}{3x}$$

Problem 2. (25 points)

An electric machine (1 = stator, 2 = rotor) has the following linear flux linkage vs current characteristic:

$$\lambda_1 = 0.2 i_1 + 0.1 \sin\theta \, i_2$$

$$\lambda_2 = 0.1 \sin\theta \, i_1 + 0.3 \, i_2$$

a) What is the energy stored in the coupling field when $\theta = 90$ degrees, $i_1 = 3$ Amps, and $i_2 = 5$ Amps?

b) How much energy is given to the coupling field by the mechanical system if θ is changed from 90 degrees to 60 degrees while the two currents remain constant?

c) How much energy is given to the coupling field by the electrical system during that same path from θ equals 90 degrees to 60 degrees while the two currents remain constant?

Since the system is linear, $W_m = W_m'$; thus, I will work with the coenergy to avoid inverting the flux-current relations above.

a) $$W_m'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1(\tilde{i}_1, 0, \theta)\, d\tilde{i}_1 + \int_0^{i_2} \lambda_2(i_1, \tilde{i}_2, \theta)\, d\tilde{i}_2$$

$$= \frac{1}{2} 0.2 i_1^2 + 0.1 \sin\theta \cdot i_1 i_2 + \frac{1}{2} 0.3 i_2^2$$

$$\boxed{W_m'\left(i_1 = 3A, i_2 = 5A, \theta = \frac{\pi}{2}\right) = \frac{1}{2} 0.2 \cdot 9 + 0.1 \cdot 1 \cdot 5 \cdot 3 + \frac{1}{2} 0.3 \cdot 25 = 6.15 J = W_m\left(3A, 5A, \frac{\pi}{2}\right)}$$

b) $\cancel{W_m\left(3,5,\frac{\pi}{2}\right) - W_m\left(3,5,\frac{\pi}{3}\right) = EFC + EFM}$

$\cancel{= W_m'\left(3,5,\frac{\pi}{2}\right) - W_m'\left(3,5,\frac{\pi}{3}\right)}$

$$\underset{a \to b}{EFM} = -\int_{\pi/2}^{\pi/3} T^e(i_1 = 3, i_2 = 5, \theta)\, d\theta = -\int_{\pi/2}^{\pi/3} \frac{\partial W_m'(i_1 = 3, i_2 = 5, \theta)}{\partial \theta}$$

$$= -\int_{\pi/2}^{\pi/3} 0.1 \cos\theta \cdot 3 \cdot 5 \, d\theta = -0.1 \cdot 3 \cdot 5 \cdot \sin\theta \Big|_{\pi/2}^{\pi/3} = 0.2.$$

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(c)

$$\underset{a\to b}{EFE} = \int_{\lambda_1^{(a)}}^{\lambda_1^{(b)}} \underbrace{i_1(\lambda_1, \lambda_2)}_{3} d\lambda_1 + \int_{\lambda_2^{(a)}}^{\lambda_2^{(b)}} \underbrace{i_2(\lambda_1, \lambda_2)}_{5} d\lambda_2$$

$$= \int_{1.1}^{1.033} 3\, d\lambda_1 + \int_{4.5}^{0.26} 5\, d\lambda_2 = -0.4\,J$$

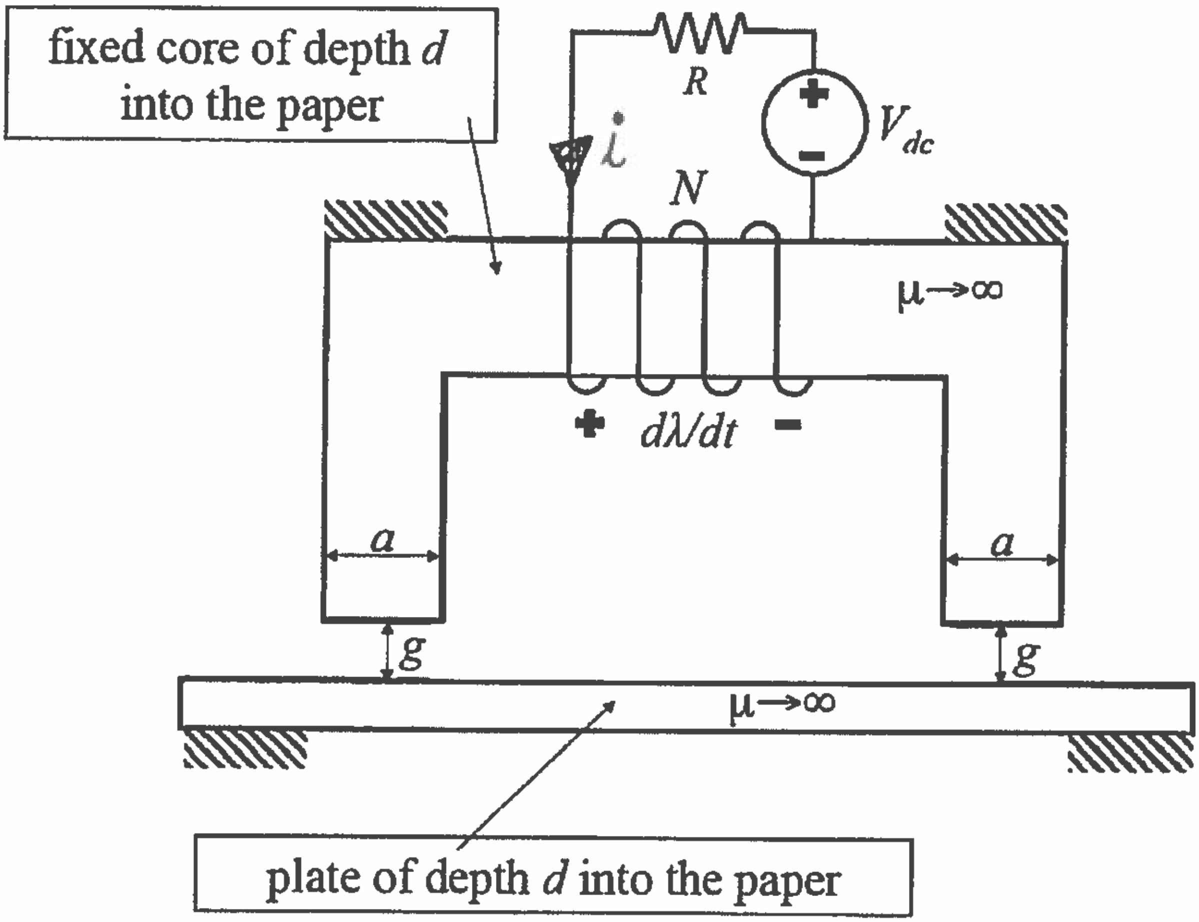
$$\lambda_1^{(a)} = 0.2\cdot 3 + 0.1\cdot \sin\frac{\pi}{2}\, 5 = 0.6 + 0.5 = 1.1$$

$$\lambda_1^{(b)} = 0.2\cdot 3 + 0.1\cdot \sin\frac{\pi}{3}\cdot 5 = 1.033$$

$$\lambda_2^{(a)} = 0.2\cdot 5 + 0.1\cdot \sin\frac{\pi}{2}\cdot 5 = 4.5$$

$$\lambda_2^{(b)} = 0.2\cdot 5 + 0.1\cdot \sin\frac{\pi}{6}\cdot 5 = 0.26$$

Problem 3. (25 points.)



For the system above, assume that a steady-state operating point has been reached; then:

a) Find an expression for the flux linkage, λ, that includes g and V_{dc}. *(6pt)*

For the i in the figure, we have that

$$\lambda(i,g) = \frac{\mu_0 \cdot N^2 \cdot a \cdot d}{2 \cdot g}\, i$$

KVL : $V_{dc} = R \cdot i + \frac{d\lambda}{dt}$

In steady state, $\frac{d\lambda}{dt} = 0 \longrightarrow i = \frac{V_{dc}}{R}$;

Thus $\lambda(V_{dc}, g) = \frac{\mu_0 N^2 a d}{2g \cdot R} V_{dc}$

b) Find an expression for the energy stored in the coupling magnetic field that includes g and V_{dc}. (6pt)

From the flux relation $\lambda(i, g)$ and since the system is linear, we can work with the coenergy:

$$W_m'(i, g) = \int_0^i \frac{\mu_0 N^2 a d}{2g} \tilde{i}\, d\tilde{i} = \frac{\mu_0 N^2 a d}{\cancel{4}g} i^2$$

Since $i = \frac{V_{dc}}{R}$, $W_m'(g, V_{dc}) = \frac{\mu_0 N^2 a d}{4gR^2} V_{dc}^2$

c) Find an expression for the co-energy that includes g and V_{dc} (6pt)

$$W_m'(g, V_{dc}) = W_m(g, V_{dc})$$

$$= \frac{\mu_0 N^2 a d}{4gR^2} V_{dc}^2$$

d) Find an expression for the force of electrical origin that tries to reduce the airgap that includes g and V_{dc}. (7pt)

$$f^e(i, g) = \frac{\partial W_m'(i, g)}{\partial g} = -\frac{\mu_0 N^2 a d\, i^2}{4g^2}$$

Again, since $i = \frac{V_{dc}}{R}$,

We have that

$$f^e(V_{dc}, g) = -\frac{\mu_0 N^2 a d}{4g^2R^2} V_{dc}^2$$

OR could compute magnitude only (no minus sign)

Problem 4. (25 points.)

The rotor angle dynamics of a synchronous generator are described by the following second-order non-linear differential equation:

$$M\, d^2\delta/dt^2 + B\, d\delta/dt = P - K \sin \delta,$$

where M, B, P, and K are positive constants.

a) If you were to write the above differential equation in state-space form, which variables would you take as states? (5pt)

$$\left.\begin{aligned} x_1 &:= \delta \\ x_2 &:= \frac{d\delta}{dt} \end{aligned}\right\} \longrightarrow x = [x_1, x_2]^T$$

b) For the states you chose in a), write the state-space model equations. (10pt)

$$\frac{dx_1}{dt} = \frac{d\delta}{dt} = x_2 \longrightarrow \boxed{\frac{dx_1}{dt} = x_2}$$

$$\frac{dx_2}{dt} = \frac{d^2\delta}{dt^2} = \frac{1}{M}\left(P - K\sin\delta - B\frac{d\delta}{dt}\right) \longrightarrow$$

$$\longrightarrow \boxed{\frac{dx_2}{dt} = -\frac{B}{M}\cdot x_2 + \frac{1}{M}(P - K\sin x_1)}$$

c) Find the equilbrium points of the state-space model in b) that correspond to values of δ between 0 and π. (10pt)

Eq. points:

$$\frac{dx_1}{dt} = 0 \Rightarrow x_2^{eq} = 0$$

$$\frac{dx_2}{dt} = 0 \longrightarrow 0 = -\cancel{\frac{B}{M} x_2^{eq}} + \frac{1}{M}(P - K\sin x_1)$$

$$x_1^{eq(1)} = \arcsin\left(\frac{P}{K}\right) \longrightarrow$$

$$x_1^{eq(2)} = \pi - \arcsin\left(\frac{P}{K}\right)$$