ECE430
Spring 2006
FINAL
May 10, 2006

Name Solutions

Section (C for Kimball MWF, F for Tate TR)____________

1:	
2:	
3:	
4:	
5:	
6:	
Total:	

Equations:

$$\bar{S}_{1\phi} = \bar{V}\bar{I}^* = \frac{|\bar{V}|^2}{\bar{Z}^*} = |\bar{I}|^2 \bar{Z}$$

$$\bar{S}_{3\phi} = 3\bar{V}_\phi \bar{I}_\phi^* = \sqrt{3} V_L I_L \measuredangle \theta$$

$$P_{3\phi} = \sqrt{3} V_L I_L \cos\theta$$

$$Q_{3\phi} = \sqrt{3} V_L I_L \sin\theta$$

$$pf = \cos\left(\measuredangle \bar{V} - \measuredangle \bar{I}\right)$$

$$\theta > 0 \rightarrow lagging, \theta < 0 \rightarrow leading$$

$$P^2 + Q^2 = S^2$$

$$X_c = -\frac{1}{\omega C}$$

$$X_L = \omega L$$

$$wye, abc\ sequence: \bar{V}_L = \bar{V}_\phi\left(\sqrt{3}\measuredangle 30^\circ\right), \bar{I}_\phi = \bar{I}_L$$

$$delta, abc\ sequence: \bar{V}_\phi = \bar{V}_L, \bar{I}_L = \bar{I}_\phi\left(\sqrt{3}\measuredangle -30^\circ\right)$$

$$\bar{Z}_\Delta = 3\bar{Z}_Y$$

$$\bar{Z}_1 \,||\, \bar{Z}_2 = \left(\bar{Z}_1^{-1} + \bar{Z}_2^{-1}\right)^{-1}$$

$$\Re = \frac{l}{\mu A}$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$\lambda = N\Phi = Li$$

$$L = N^2 \wp = \frac{N^2}{\Re}$$

$$mmf\left(source\right) = Ni$$

$$mmf\left(drop\right) = \Phi\Re$$

$$\sum mmf = 0\ around\ loop$$

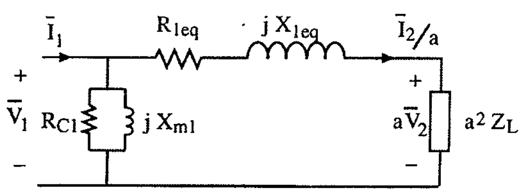
$$\oint \bar{H}\bullet d\bar{l} = \int \bar{J}\bullet\hat{n}\,da$$

$$\oint \bar{E}\bullet d\bar{l} = -\frac{\partial}{\partial t}\int \bar{B}\bullet\hat{n}\,da$$

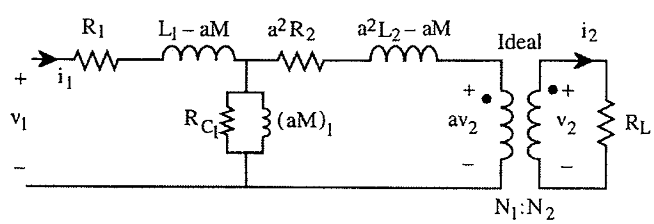
$$\oint \bar{B}\bullet\hat{n}\,da = 0$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$v = \frac{d\lambda}{dt}$$



Transformer Approximate Equivalent Circuit



Transformer Equivalent Circuit

$$W_m = \int_0^{\lambda} i\,d\hat{\lambda}$$

$$W_m' = \int_0^{i} \lambda\,d\hat{i}$$

$$T^e = \frac{\partial W_m'}{\partial \theta} = -\frac{\partial W_m}{\partial \theta}$$

$$f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$$

$$\underset{a\to b}{EFE} = \int_a^b i\,d\lambda$$

$$\underset{a\to b}{EFM} = -\int_a^b f^e\,dx$$

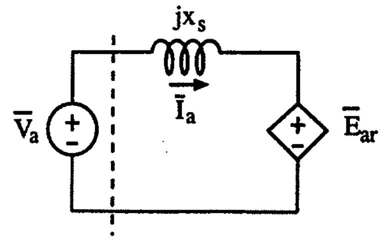
$$\boxed{\begin{array}{c}\text{For } \dot{x}_1 = f_1(x_1,x_2) \text{ and } \dot{x}_2 = f_2(x_1,x_2), \\ \begin{bmatrix}\Delta\dot{x}_1 \\ \Delta\dot{x}_2\end{bmatrix} \approx \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x=x^e} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x=x^e} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x=x^e} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x=x^e} \end{bmatrix} \begin{bmatrix}\Delta x_1 \\ \Delta x_2\end{bmatrix}\end{array}}$$

$$x(t_0+\Delta t) \approx x(t_0) + \Delta t \cdot \left.\frac{dx}{dt}\right|_{t=t_0}$$

For $\underline{\dot{x}} = \underline{A}\underline{x}$, the eigenvalues λ of the system are given by $|\lambda \underline{I} - \underline{A}| = 0$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Synchronous machines



Synchronous machine equivalent circuit

$$P_T = P_m = \frac{-3E_{ar}V_a \sin(\delta)}{x_s}$$

$$\omega_m = \frac{2}{p}\omega_s$$

$$E_{ar} = \frac{\omega_s M I_r}{\sqrt{2}}$$

$$T^e \omega_m = P_m$$

Induction machines

$$T^e = \frac{P_m}{\omega_m}$$

$$P_{ag} = 3\left|\bar{I}_r'\right|^2 \frac{R_r'}{s}$$

$$P_m = 3\left|\bar{I}_r'\right|^2 R_r' \left(\frac{1-s}{s}\right)$$

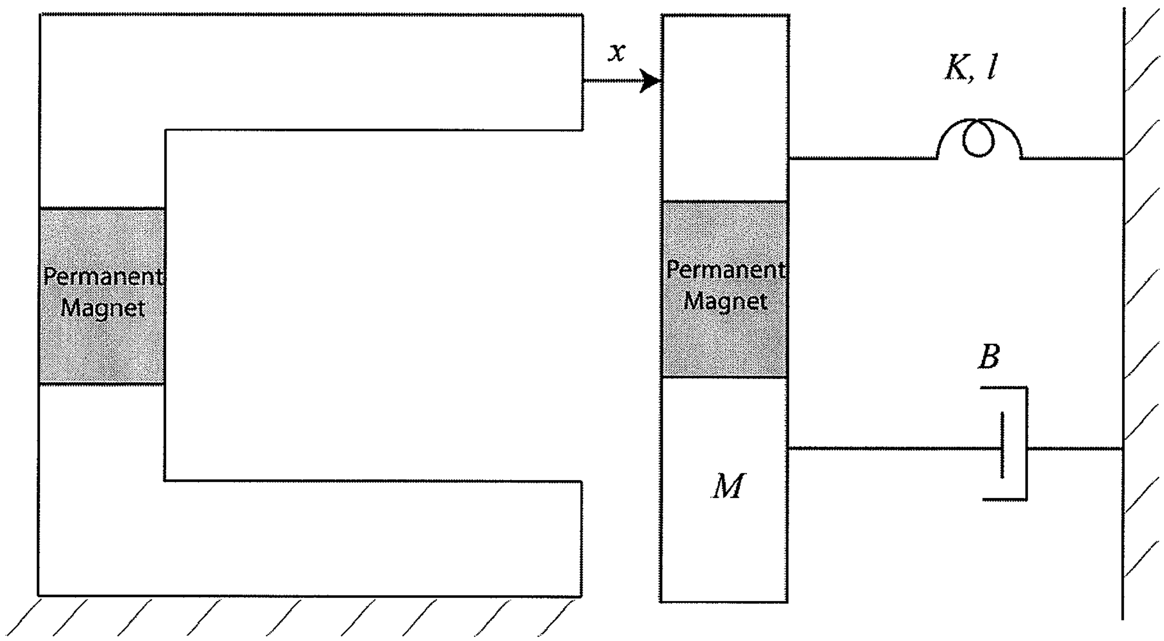
$$\omega_s = \omega_m\left(\frac{p}{2}\right) + \omega_r$$

$$s = \frac{\omega_r}{\omega_s} = \frac{N_s - N}{N_s}$$

$$s_{\max T} = \pm \frac{R_r'}{\sqrt{R_s^2 + (X_{ls} + X_{lr}')^2}}$$

$$N_s = f_s \frac{120}{p}$$

Problem 1 (25 points)



The permanent magnet / mechanical system shown above has the following co-energy expression:

$$W_m' = \ln(x)$$

The spring constant $K = 1$ N/m, and the zero-force length l of the spring is 2 m (i.e., for $x = 2$ m, the spring exerts no force on the mass). The dashpot damping coefficient $B = 2$ N/(m/s). The mass of the movable member (M) is 1 kg.

a) Determine the force exerted by the permanent magnet on the movable member
b) Write down the differential equations for this system in state space form
c) Find all equilibrium points for the system
d) Write the differential equations for the system after linearization at each equilibrium point
e) Determine whether or not each equilibrium point is stable

Problem 1

(2 pts.) a) $f^e = \frac{\partial}{\partial x} W_M' = \frac{\partial}{\partial x} \ln(x) = \boxed{\frac{1}{x}}$

(10 pts.) b) $M\ddot{x} = \frac{1}{x} - K(x-\ell) - B\dot{x}$

$$\ddot{x} = \frac{1}{x} - (x-2) - 2\dot{x}$$

Let $y_1 = x$, $y_2 = \dot{x}$

$$\boxed{\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \frac{1}{y_1} - (y_1 - 2) - 2y_2 \end{aligned}}$$

(4 pts.) c) $0 = y_2^e$

$0 = \frac{1}{y_1^e} - (y_1^e - 2) - 2y_2^e \Rightarrow 0 = 1 - y_1^{e^2} + 2y_1^e \Rightarrow 0 = -y_1^{e^2} + 2y_1^e + 1$

$$y_1^e = \frac{-2 \pm \sqrt{4+4}}{-2} = \frac{-2 \pm 2\sqrt{2}}{-2} = 1 \mp \sqrt{2}$$

But x must be >0, so only equil. is

$\boxed{1+\sqrt{2}} = 2.414$

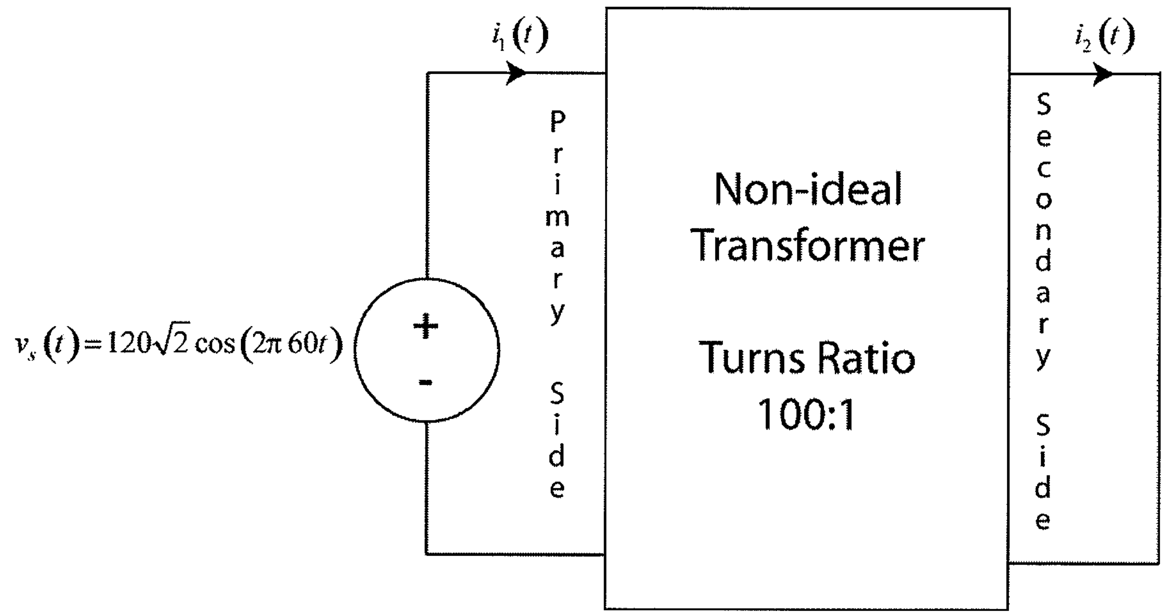
(5 pts.) d) $A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{y_1^{e^2}} - 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{(1+\sqrt{2})^2} - 1 & -2 \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ \underset{\sim\, 2\sqrt{2}-4}{-1.1716} & -2 \end{bmatrix}$

$\Delta\dot{y} = A\Delta y$

(4 pts.) e) eigenvalues of A: $-1 \pm j0.4142$

$$\boxed{\text{Yes, } (\mathrm{Re}\{\lambda\}) < 0 \text{ for all } \lambda \text{ stable}}$$

Problem 2 (25 points)

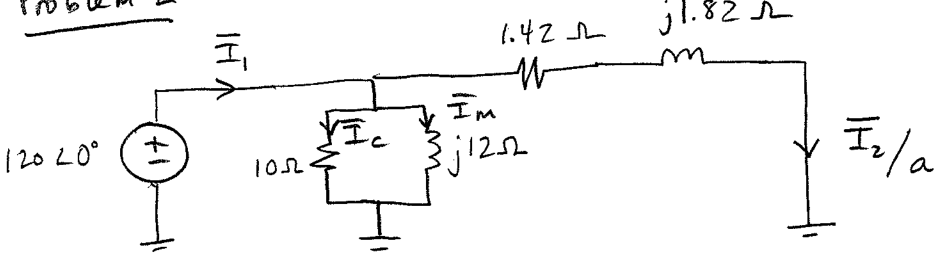


The figure above shows a single-phase voltage source hooked up through a non-ideal transformer to a short. The non-ideal transformer is to be modeled using the approximate transformer model, with $R_{1eq} = 1.42\ \Omega$, $X_{1eq} = 1.82\ \Omega$, $R_{C1} = 10\ \Omega$, and $X_{m1} = 12\ \Omega$.

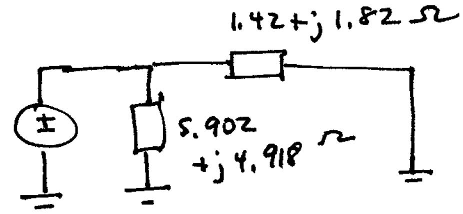
Find:

a) The time-domain expression for the current through the short ($i_2(t)$)
b) The time-domain expression for the current leaving the source ($i_1(t)$)
c) Average (real) power losses in the transformer

Problem 2



$\Rightarrow$



(8 pts.) a) $$\frac{\overline{I}_2}{a} = \frac{120\angle 0^\circ}{1.42 + j1.82} = 31.977 - j40.985 = 51.984\angle -52.078^\circ$$

$$\overline{I}_2 = (100)\cdot(31.977 - j40.985) = 3197.7 - j4098.5$$
$$= 5198.36\angle -52.04^\circ$$

$$\boxed{\begin{aligned} i_2(t) &= 5198.36\sqrt{2}\cos(2\pi 60t - 52.04^\circ) \\ &= 7351.59\cos(2\pi 60t - 52.04^\circ) \end{aligned}}$$

b) $$\overline{I}_c = \frac{120\angle 0^\circ}{10\angle 0^\circ} = 12\angle 0^\circ \text{ A} \qquad \overline{I}_m = \frac{120\angle 0^\circ}{12\angle 90^\circ} = 10\angle -90^\circ = -j10 \text{ A}$$

(12 pts.)

$$\overline{I}_1 = \overline{I}_c + \overline{I}_m + \overline{I}_2/a = 12 - j10 + 31.977 - j40.985$$
$$= 43.9772 - j50.985$$
$$= 67.331\angle -49.22^\circ$$

$$\boxed{\begin{aligned} i_1(t) &= 67.331\sqrt{2}\cos(2\pi 60t - 49.22^\circ) \\ &= 95.220\cos(2\pi 60t - 49.22^\circ) \end{aligned}}$$

(5 pts.) c) $$P_c = \text{Re}\{(120\angle 0^\circ)(12\angle 0^\circ)\} = 1440 \text{ W}$$

$$P_R = \left|\overline{I}_2/a\right|^2 (1.42) = 3837.26 \text{ W}$$

$$P_c + P_R = 5277.26 \text{ W} = P_{loss}$$

Also,

$$\overline{S}_{in} = \overline{V}_{in}\,\overline{I}_{in}^* = (120\angle 0^\circ)(67.331\angle 49.22^\circ) = 5277 + j6118 \text{ VA}$$

$$P_{loss} = 5277.26 \text{W}$$

✓

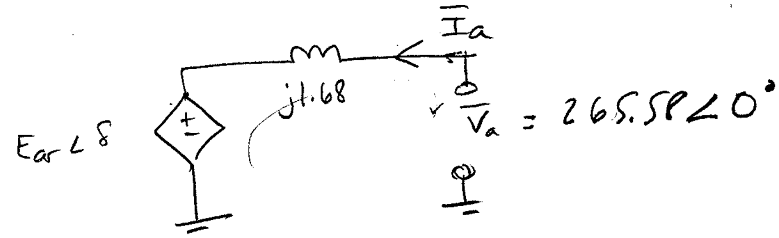
Problem 3 (25 points)

A 60-Hz, three-phase, 4-pole synchronous motor is observed to have a terminal voltage of 460 V (line-line) and a terminal current of 120 A at a power factor of 0.95 lagging. The field current under this operating condition is 47 A. The machine synchronous reactance is equal to 1.68 Ω. Assume the armature resistance to be negligible.

Calculate:

a) The torque angle δ
b) The magnitude of the field-to-armature mutual inductance
c) The electrical power input to the motor and the torque on the shaft

Problem 3



a) $\bar{I}_a = 120\angle -\cos^{-1}(0.95)$

$\bar{I}_a = 120\angle -18.195°$

10 pts.

$-E_{ar}\angle\delta - (\bar{I}_a)(1.68\angle 90°) + \left(\frac{460}{\sqrt{3}}\angle 0°\right) = 0$

$E_{ar}\angle\delta = \left(\frac{460}{\sqrt{3}}\angle 0°\right) - (120\angle -18.195°)(1.68\angle 90°)$

$E_{ar}\angle\delta = 278.82\angle -43.39°$

$\delta = -43.39°$

b) $278.82 = \frac{\omega_s M I_r}{\sqrt{2}} \rightarrow M = \frac{\sqrt{2}\cdot 278.82}{(2\pi 60)(47)} = 0.022254\ H$ or $22.254\ mH$

5 pts.

c) $P_T = 3\,\mathrm{Re}\{\bar{V}_a\,\bar{I}_a^*\} = 3\cdot\mathrm{Re}\left\{\left(\frac{460}{\sqrt{3}}\angle 0°\right)(120\angle 18.195°)\right\} = 3\cdot\mathrm{Re}\{30276 + j\,9951.32\}$

$= 3\cdot 30276 = 90828.7W$

10 pts. -or-

$$P_T = \frac{-3\cdot 278.82\cdot\left(\frac{460}{\sqrt{3}}\right)\cdot\sin(-43.39°)}{1.68}$$

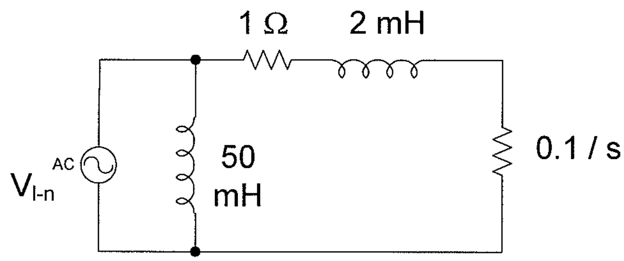
$= 90828.7\ W$

$\omega_M = \frac{2}{P}\cdot 2\pi 60 = \frac{2}{4}\cdot 2\pi 60 = 188.495\ rad/s$

$T^e\omega_m = P_M \rightarrow T^e = \frac{90828.74\ W}{(188.495\ rad/s)} = \boxed{481.86\ N\cdot m}$

Problem 4 (25 points)

A four-pole induction motor with the approximate equivalent circuit shown below is operated from 240 V (line-to-line), 60 Hz, three-phase power. Rated speed is 1760 RPM.

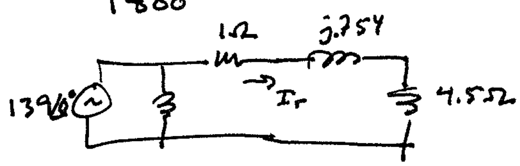


(a) Find mechanical power at rated slip, in horsepower (1 hp = 746 W) (5 pts).
(b) Find the slip for maximum torque (5 pts).
(c) Find the maximum torque (5 pts).

Next, the machine is shipped to Europe, where it is operated from 220 V (line-to-line), **50** Hz three-phase power.

(d) Find the slip for maximum torque (5 pts).
(e) Find the maximum torque (5 pts).

a) $s = \dfrac{1800 - 1760}{1800} = 0.02222$ ①



$I_r = \dfrac{139\angle 0^\circ}{1+4.5+j.754} = 24.96\angle -7.806^\circ$ ②

$P_m = 3(24.96)^2(0.1)\left(\dfrac{1-.02222}{0.02222}\right)$

$P_m = 8224\,W = 11.02\,hp$ ②

b) $s_{maxT} = \pm\dfrac{R_r'}{\sqrt{R_s^2+(X_{ls}+X_{lr}')^2}} = \pm\dfrac{0.1}{\sqrt{1^2+0.754^2}} = 0.07985$ ⑤

c) $P_{maxT} = 3\,|I_r'|^2 R_r'\left(\dfrac{1-s_{maxT}}{s_{maxT}}\right)$ $\qquad \dfrac{R_r'}{s_{maxT}} = \dfrac{0.1}{.07985} = 1.252$

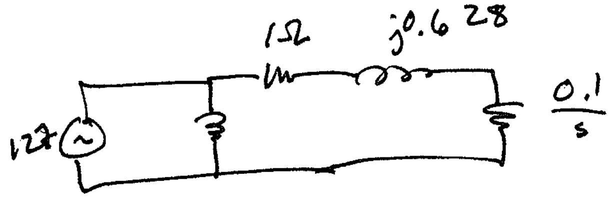
$I_r' = \dfrac{139\angle 0^\circ}{2.252+j.754} = 58.53\angle -18.51^\circ$ ②

$P_{maxT} = 3(58.53)^2(0.1)\left(\dfrac{1-0.07985}{0.07985}\right) = 11.84\,kW$ ①

$\omega_{m\,maxT} = (1-s_{maxT})\left(\dfrac{2}{P}\right)(377) = 173.4\,rad/s$ ①

$T_{max} = \dfrac{P}{\omega_m} = 68.26\,N\cdot m$ ①

d)



(3)

$$s_{maxT} = \frac{0.1}{\sqrt{1^2 + 0.628^2}} = 0.08469 \quad (2)$$

e)

$$\bar{I}_r' = \frac{127\angle 0^\circ}{1 + \frac{0.1}{0.08469} + j0.628} = 55.96\ \angle -16.06^\circ \quad (2)$$

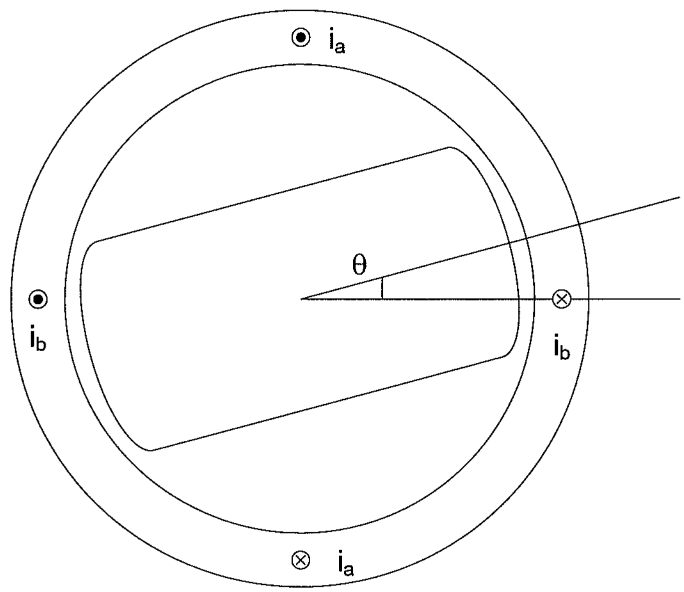
$$P = 3\,(55.96)^2\,(0.1)\left(\frac{1-0.08469}{0.08469}\right) = 10.15\,\text{kW} \quad (1)$$

$$\omega_m = (1 - s_{maxT})\left(\frac{2}{P}\right)\omega_s = 143.8\ \text{rad/s} \quad (1)$$

$$T_{max} = \frac{10.15\,\text{kW}}{143.8\,\text{rad/s}} = 70.60\ \text{N}\cdot\text{m} \quad (1)$$

Problem 5 (25 points)

The motor shown below is a synchronous reluctance machine—essentially, a synchronous machine with no field winding. Assume that all inductances are zero, constant, or vary as $L_0 + L_1\cos\phi$, where ϕ is some function of θ. For example, adding or subtracting 90° or 180° could convert the cosine to a sine or change the plus to a minus. Express all answers in terms of θ.



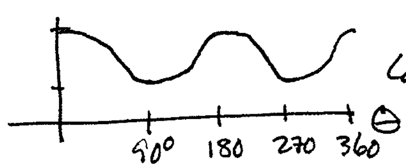
(a) Determine the flux linkage matrix relating $\begin{bmatrix} \lambda_a \\ \lambda_b \end{bmatrix}$ to $\begin{bmatrix} i_a \\ i_b \end{bmatrix}$ (10 pts).

(b) Determine the co-energy W_m' (8 pts).

(c) Determine the torque of electric origin $T^e(i_a, i_b, \theta)$ (7 pts).

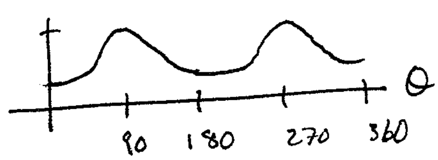
a) $\begin{bmatrix} \lambda_a \\ \lambda_b \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} \\ L_{ba} & L_{bb} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$ $\quad L_{ab} = L_{ba}$

L_{aa}:



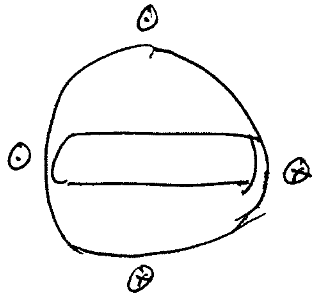
$L_0 + L_1\cos 2\theta$

L_{bb}:



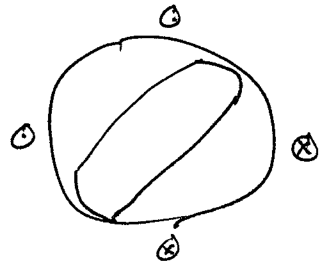
$L_0 - L_1\cos 2\theta$

L_{ab}:



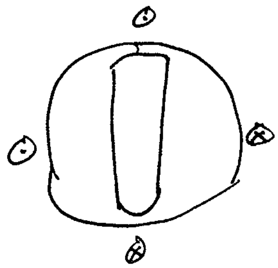
$L_{ab} = 0$

$0, 180°$



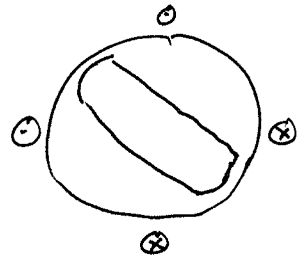
$L_{ab} = +\max$

$45°, 225°$



$L_{ab} = 0$

$90°, 270°$



$L_{ab} = -\max$

$135°, 315°$

$$M \sin 2\theta$$

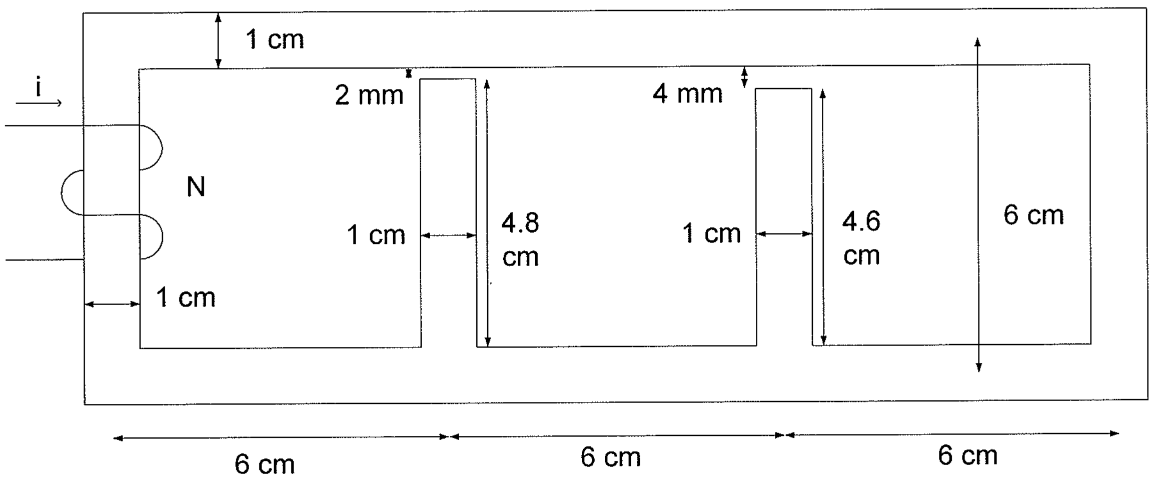
$$\begin{bmatrix} \lambda_a \\ \lambda_b \end{bmatrix} = \begin{bmatrix} \overset{3}{L_0 + L_1 \cos 2\theta} & \overset{2}{M \sin 2\theta} \\ \overset{2}{M \sin 2\theta} & \overset{3}{L_0 - L_1 \cos 2\theta} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

b) $$W_m' = \underset{3}{\tfrac{1}{2}(L_0 + L_1 \cos 2\theta)\, i_a^2} + \underset{3}{M \sin 2\theta\, i_a i_b} + \underset{2}{\tfrac{1}{2}(L_0 - L_1 \cos 2\theta)\, i_b^2}$$

c) $$T^e = \frac{\partial W_m'}{\partial \theta} = \underset{2}{-L_1 \sin 2\theta\, i_a^2} + \underset{3}{2M \cos 2\theta\, i_a i_b} + \underset{2}{L_1 \sin 2\theta\, i_b^2}$$

Problem 6 (25 points)

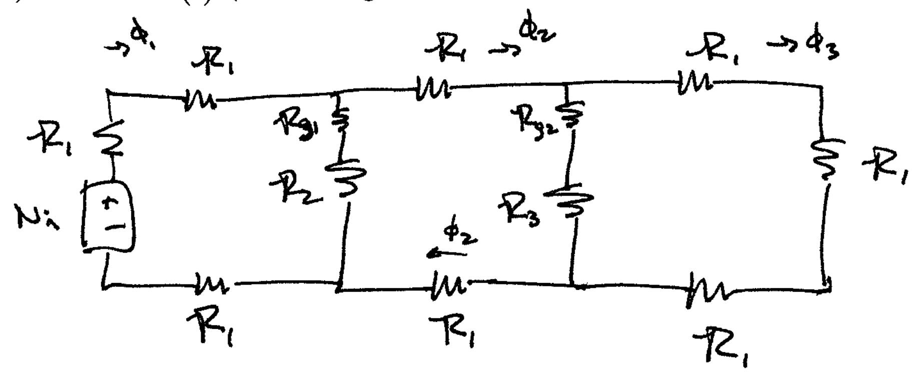
Consider the magnetic structure below. N = 100. Depth into the page is 2 cm. Include the effects of fringing. Use the lengths provided for reluctance paths. Relative permeability of the iron is $\mu_r = 1000$.



(a) Draw the magnetic equivalent circuit (12 pts).

(b) Find $\lambda(i)$ (flux linkage in the coil as a function of current) (13 pts).

a)



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$$\mathcal{R}_1 = \frac{l_1}{\mu A_1} = \frac{0.06}{1000(4\pi\times10^{-7})(0.01)(0.02)} = 238.7e3$$ 2

$$\mathcal{R}_2 = \frac{0.048}{1000\mu_0(.01)(.02)} = 191\,e3$$ 2

$$\mathcal{R}_3 = \frac{.046}{1000\mu_0(.01)(.02)} = 183e3$$ 2

$$\mathcal{R}_{g1} = \frac{.002}{\mu_0(.012)(.022)} = 6.027e6$$ 2

$$\mathcal{R}_{g2} = \frac{.004}{\mu_0(.014)(.024)} = 9.471e6$$ 2

b)

$$\left.\begin{aligned}
&Ni - 3\phi_1 R_1 - (\phi_1-\phi_2)(R_{g1}+R_2) = 0 \\
&(\phi_2-\phi_1)(R_{g1}+R_2) + 2R_1\phi_2 + (\phi_2-\phi_3)(R_{g2}+R_3) = 0 \\
&(\phi_3-\phi_2)(R_{g2}+R_3) + 3R_1\phi_3 = 0
\end{aligned}\right\} \quad 8$$

$$(3R_1 + R_{g1} + R_2)\,\phi_1 + (-R_{g1} - R_2)\,\phi_2 + 0\,\phi_3 = Ni$$

$$(-R_{g1} - R_2)\,\phi_1 + (2R_1 + R_{g1} + R_2 + R_{g2} + R_3)\,\phi_2 + (-R_{g2} - R_3)\,\phi_3 = 0$$

$$0\phi_1 + (-R_{g2} - R_3)\,\phi_2 + (3R_1 + R_{g2} + R_3)\,\phi_3 = 0$$

$$\phi_3 = \frac{R_{g2}+R_3}{3R_1 + R_{g2} + R_3}\,\phi_2 = 0.9309\,\phi_2$$

$$\cancel{\phi_2 = (R_{g2}+R_3)\phi_3 + (R_{g1}}$$

$$(-R_{g1}-R_2)\phi_1 + \left(2R_1 + R_{g1} + R_2 + R_{g2} + R_3 + (-R_{g2}-R_3)(0.9309)\right)\phi_2 = 0$$

$$\phi_2 = \frac{R_{g1}+R_2}{(\qquad\qquad)}\,\phi_1 = 0.8446\,\phi_1$$

$$Ni = \phi_1\left(3R_1 + R_{g1} + R_2 - 0.8446(R_{g1}+R_2)\right)$$

$$Ni = \phi_1(1.682e6)$$

$$\phi_1 = (59.44e{-}6)\,i \quad 3$$

$$\lambda = N\phi_1 = (5.944\,\text{mH})\,i \quad 2$$