ECE330
Exam #2
Spring 2004

Name Solution
(Print Name)

Section: **(Circle One)** **10 MWF (Sauer)** **2 MWF (Kimball)**

Problem 1 _______ **Problem 2** _______ **Problem 3** _______ **Problem 4** _______

TOTAL: ________

USEFUL INFORMATION

$$\oint_c \underline{H} \cdot \underline{dl} = \int_s \underline{J} \cdot \underline{n}\, da \qquad \oint_c \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\int_s \underline{B} \cdot \underline{n}\, da \qquad \oint_s \underline{B} \cdot \underline{n}\, da = 0$$

$$mmf = Ni = \phi R \qquad R = \frac{l}{\mu A} \qquad \phi = BA \qquad B = \mu H \qquad \lambda = N\phi$$

$$W_m = \int i \Big|_{x = const} d\lambda \qquad W_m' = \int \lambda \Big|_{x = const} di \qquad W_m + W_m' = \lambda i$$

$$f^e = -\frac{\partial W_m}{\partial x} \qquad f^e = \frac{\partial W_m'}{\partial x}$$

For rotation, $x \rightarrow \theta$
$f \rightarrow T$

$$\underset{a-b}{EFE} = \int_{\lambda_a}^{\lambda_b} i \Big|_{path} d\lambda \qquad \underset{a-b}{EFM} = -\int_{x_a}^{x_b} f^e \Big|_{path} dx$$

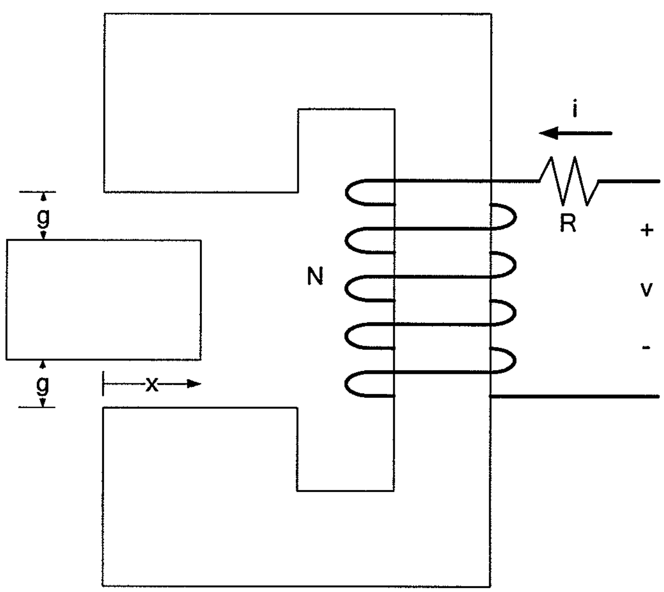
$$W_{m_b} - W_{m_a} = \underset{a-b}{EFE} + \underset{a-b}{EFM} \qquad x(t_0+\Delta t) \approx x(t_0) + \frac{dx}{dt}\Big|_{t_0} \Delta t$$

Problem 1 (25 pts.)

For the structure drawn below, the movable member is constrained to move left and right only as indicated in the figure where "x" is the distance to the right edge of the movable member. The large member with the coil is fixed, and the depth into the page for both members is 2cm. The gap g is 1mm, and the number of turns N =100. Find:

a. Total reluctance of the magnetic circuit.
b. Flux linkage, λ. (defined for the voltage polarity shown)
c. Co-energy, W_m'.
d. Force of electrical origin, f^e.
e. An expression for the voltage, v.

Express all of these as functions of current and/or position and/or velocity and/or time as appropriate. You may neglect fringing in the gap, and you may assume the iron is infinitely permeable.



a) $\mathcal{R} = \frac{2g}{\mu_0 d x} = \frac{.002}{4\pi \times 10^{-7} \times .02 x}$

$= \frac{7.96 \times 10^4}{x}$

b) $\phi \mathcal{R} = 100 i$

$\phi = \frac{100 i x}{7.96 \times 10^4}$

$\lambda = \frac{i x}{7.96}$

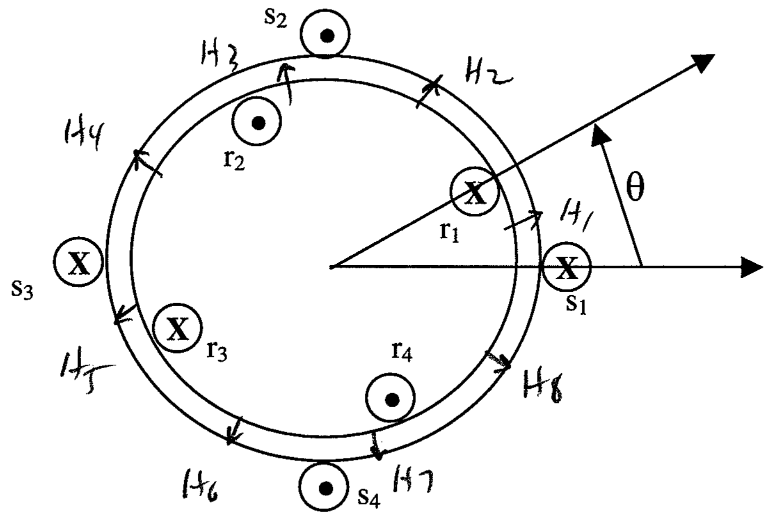
c) $w_m' = \frac{x i^2}{15.92}$

d) $f^e = \frac{i^2}{15.92}$

e) $v = iR + \frac{x}{7.96}\frac{di}{dt} + \frac{i}{7.96}\frac{dx}{dt}$

Problem 2 (25 pts)

A single-phase, four-pole machine is shown below. There are only two sources involved - the stator current goes into s_1, then out s_2, (N_s times), then into s_3, and out s_4 (N_s times). The stator voltage terminals are s_1 (+) and s_4 (-). The same notation is used for the rotor coil where there are N_r turns for each part of the rotor coil. The air gap is uniform with distance g. The iron is infinitely permeable. The mean radius is r, and depth into the paper is ℓ.



a) Assign the 8 different magnetic field intensity vectors H_1 to H_8 and write the eight equations that you would need to compute each H in terms of the currents i_s and i_r.
b) Write the equation you need to compute the stator flux linkage in terms of the different H quantities and angle theta. The stator terminal voltage (with the polarity given) is equal to the time derivative of this stator flux linkage.

a) $H_1 g - H_8 g = N_s i_s$ $\quad$ $H_1 g - H_4 g = N_s i_s$

$H_1 g - H_7 g = N_s i_s - N_r i_r$ $\quad$ $H_1 g - H_3 g = N_s i_s - N_r i_r$

$H_1 g - H_6 g = -N_r i_r$ $\quad$ $H_1 g - H_2 g = -N_r i_r$

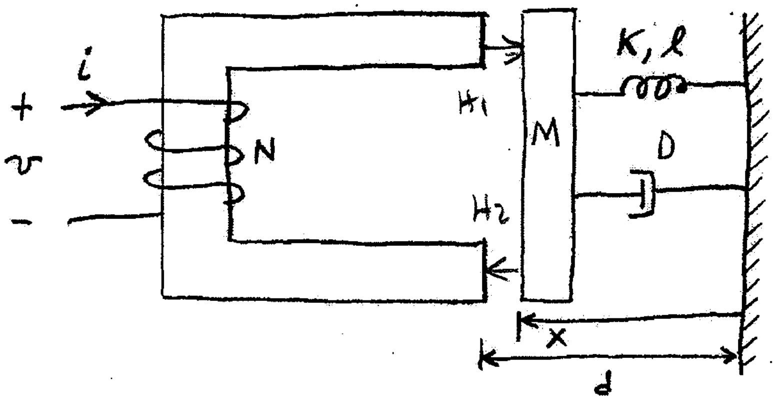
$H_1 g - H_5 g = 0$

$$\mu_0 H_1 \theta r \ell + \mu_0 H_2 (\tfrac{\pi}{2} - \theta) r \ell + \mu_0 H_3 \theta r \ell + \mu_0 H_4 (\tfrac{\pi}{2} - \theta) r \ell + \mu_0 H_5 \theta r \ell + \mu_0 H_6 (\tfrac{\pi}{2} - \theta) r \ell + \mu_0 H_7 \theta r \ell + \mu_0 H_8 (\tfrac{\pi}{2} - \theta) r \ell = 0$$

b) $\lambda_s = N_s \mu_0 H_1 \theta r \ell + N_s \mu_0 H_2 (\tfrac{\pi}{2} - \theta) r \ell + N_s \mu_0 H_5 \theta r \ell + N_s \mu_0 H_6 (\tfrac{\pi}{2} - \theta) r \ell$

Problem 3 (25 pts.)

Given the electromechanical relay shown below with the typical parameters as indicated. The rectangular pieces are iron with infinite permeability and cross sectional area A. The spring zero-force distance (script ℓ) is calibrated to the distance x.



a) Find an expression for the force of electrical origin acting on the mass M in terms of the various parameters in the figure and the coil current.
b) Write the complete dynamic model for this device in state space form (3 ordinary differential equations). Assume inputs are voltage and external force. Add a resistor to the coil.
c) If the mass is held fixed at an initial position x_0 while the coil is energized to a current value of i_0 and a flux linkage value of λ_0, find an expression for the energy transferred from the electrical system into the coupling field.
d) If the flux linkage is then held constant at λ_0 while the mass is moved from position x_0 to x_1, find an expression for the energy transferred from the mechanical system into the coupling field.

a) $H_1(d-x) + H_2(d-x) = Ni \qquad \mu_0 H_1 A_1 - \mu_0 H_2 A_2 = 0 \qquad A_1 = A_2 \quad H_1 = H_2$

$$H_1 = \frac{Ni}{2(d-x)} \qquad B_1 = \frac{\mu_0 N i}{2(d-x)} \qquad \phi_{up} = \frac{\mu_0 A N i}{2(d-x)} \qquad \lambda = \frac{\mu_0 A N^2 i}{2(d-x)}$$

$$W_m' = \frac{\mu_0 A N^2 i^2}{4(d-x)} \qquad f^e = \frac{\mu_0 A N^2 i^2}{4(d-x)^2}$$

b) $$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = \frac{1}{m}\left[f_{ext} + \frac{\mu_0 A N^2 i^2}{4(d-x)^2} - k(x-\ell) - Dv\right]$$

$$v = iR + \frac{\mu_0 A N^2}{2(d-x)}\frac{di}{dt} + \frac{\mu_0 A N^2 i}{2(d-x)^2} v \qquad \frac{di}{dt} = \frac{2(d-x)}{\mu_0 A N^2}\left[v - iR - \frac{\mu_0 A N^2 i v}{2(d-x)^2}\right]$$

c) $$\underset{a-b}{EFE} = \int_{path} i \, d\lambda = \int_0^{\lambda_0} \frac{2(d-x_0)}{\mu_0 A N^2} \lambda \, d\lambda = \frac{(d-x_0)\lambda_0^2}{\mu_0 A N^2}$$

d) $$\underset{b-c}{EFM} = -\int_{path} f^e \, dx = -\int_{x_0}^{x_1} \frac{\mu_0 A N^2 i^2}{4(d-x)^2} \Bigg|_{\lambda=\lambda_0} dx$$

$$= -\int_{x_0}^{x_1} \frac{\mu_0 A N^2}{\cancel{4(d-x)^2}} \left(\frac{\cancel{2(d-x)}\lambda_0}{\mu_0 A N^2} \right)^2 dx$$

$$= -\int_{x_0}^{x_1} \frac{\lambda_0^2}{\mu_0 A N^2} dx = \frac{\lambda_0^2 (x_0 - x_1)}{\mu_0 A N^2}$$

Problem 4 (25 pts.)

In the following state-space model the initial conditions are $X_1(0) = 1$, $X_2(0) = 0.5$, and $X_3(0) = 5$.

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -0.1X_2X_3 - X_1^3 + X_1X_3$$

$$\dot{X}_3 = -2X_2X_3 - X_3 + 4$$

a) With a time step of 0.1 seconds, use Euler's method to find X_1, X_2, and X_3 at t=0.1 seconds and 0.2 seconds.

b) Find all possible static equilibrium points.

a) $x_1(.1) = 1 + 0.5 \times .1 = 1.05$

$x_2(.1) = 0.5 + [(-0.1 \times 0.5 \times 5) - 1^3 + 1 \times 5].1 = 0.875$

$x_3(.1) = 5 + [-2 \times 0.5 \times 5 - 5 + 4].1 = 4.4$

$x_1(.2) = 1.05 + 0.875 \times .1 = 1.1375$

$x_2(.2) = 0.875 + [-0.1 \times .875 \times 4.4 - 1.05^3 + 1.05 \times 4.4].1 = 1.183$

$x_3(.2) = 4.4 + [-2 \times .875 \times 4.4 - 4.4 + 4).1 = 3.59$

b) $x_2^e = 0 \qquad x_3^e = 4$

$x_1^e = 0$ OR $x_1^e = +2$ OR $x_1^e = -2$