ECE430
Exam #2
Fall 2006

Name Solution
(Print Name)

Section: (Circle One) 2 MWF 3 MWF

Problem 1 _______ Problem 2 _______ Problem 3 ______ TOTAL: ________

USEFUL INFORMATION

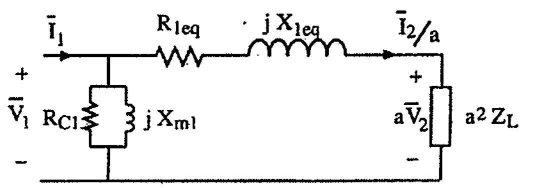
$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3}I_\phi$ (delta) $\quad$ 1 hp = 746 W $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

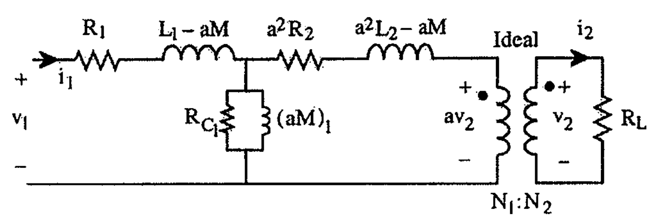
$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3}V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n}da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\lambda = Li = N\phi$ $\quad \phi = BA$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad a = N_1 / N_2$



Transformer Approximate Equivalent Circuit



Transformer Equivalent Circuit

For an ideal transformer: $N_1 i_1 = N_2 i_2$ and $v_1 / v_2 = N_1/N_2$

In phasor domain, $L_1 - aM \rightarrow jX_{\ell 1}$, $a^2L_2 - aM \rightarrow ja^2X_{\ell 2}$, $aM_1 \rightarrow jX_{m1}$

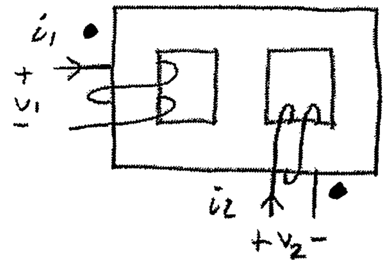
$$W_m = \int_0^\lambda i\,d\hat{\lambda} \qquad W_m' = \int_0^i \lambda\,d\hat{i} \qquad f^e = \frac{\partial W_m'(i,x)}{\partial x} = -\frac{\partial W_m(\lambda,x)}{\partial x} \qquad \underset{a\rightarrow b}{EFE} = \int_a^b i\,d\lambda \qquad \underset{a\rightarrow b}{EFM} = -\int_a^b f^e dx$$

$$\underset{a\rightarrow b}{EFE} + \underset{a\rightarrow b}{EFM} = W_{mb} - W_{ma} \qquad x \rightarrow \theta \quad and \quad f^e \rightarrow T^e \qquad W_m + W_m' = \lambda i$$

Problem 1 (30 pts.)

1a) A single-phase transformer is wound as shown below with the permeability of the iron equal to 1,000 times that of air:

a) Put the polarity dot markings on the two coils.
b) Do you think this transformer has a lot of leakage inductance, or a small amount (explain)?



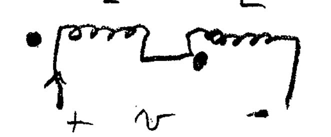
A lot of leakage inductance because the center leg will take a lot of the flux,

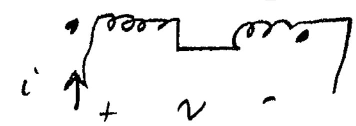
1b) Two identical but mutually coupled coils are connected in two ways, and inductance is measured in each case.

$k = \frac{.01}{\sqrt{.02 \times .02}} = 0.5$

a) In series with the undotted terminal of one coil connected to the dotted terminal of the other coil. Leq = 0.06 H
b) In series with the undotted terminal of one coil connected to the undotted terminal of the other coil. Leq = 0.02 H

Find L, M, and k $L = .02$





$2L - 2M = .02$

$2L + 2M = 0.06$ $M = .01$

$4L = .08$

$v = L\frac{di}{dt} + M\frac{di}{dt} + L\frac{di}{dt} + M\frac{di}{dt} = (2L + 2M)\frac{di}{dt}$

$v = L\frac{di}{dt} - M\frac{di}{dt} + L\frac{di}{dt} - M\frac{di}{dt} = (2L - 2M)\frac{di}{dt}$

1c) A single-phase, 60 Hz transformer is rated for 7,200 Volts (RMS) on the primary (source) side and 240 Volts (RMS) on the secondary (load) side. It has a power rating of 75 KVA. Neglect all resistance and the shunt magnetizing reactance in the transformer.

a) What are the rated currents on the primary and secondary sides?
b) What should the series equivalent reactance as seen from the primary side be in order to limit the short circuit current (under rated source voltage) to 7 times rated?

a) $i_{p_{rated}} = \frac{75000}{7200} = 10.4A$ $\qquad i_{s_{rated}} = \frac{75000}{240} = 10.4 \times 30 = 312A$

b) $7 i_{p_{rated}} = 72.8$ $\qquad x_{eq_p} = \frac{7200}{72.8} \approx 100\,\Omega$

Problem 2 (40 pts)

In class, generally we treated mutual inductances to be sinusoidal in rotational devices. In the system below, we have added a third harmonic (the 3θ terms). This gives the mutual inductance a more trapezoidal shape, which is common in some actual machines.

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & M(\cos\theta - 0.1\cos(3\theta)) \\ 0 & L_s & M(\sin\theta + 0.1\sin(3\theta)) \\ M(\cos\theta - 0.1\cos(3\theta)) & M(\sin\theta + 0.1\sin(3\theta)) & L_r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_r \end{bmatrix}$$

a) Find the co-energy in terms of the currents.

b) Find an expression for the energy stored in the coupling field in terms of currents.

c) Find the torque of electric origin T^e in terms of the currents.

d) Suppose $i_a = i_b = i_r = 1$ A, $L_s = L_r = 1$ H, $M = 0.9$ H, and $\theta = 60°$. What is the contribution to the torque due to including the third harmonic (3θ) mutual inductance terms?

a) $W_m' = \frac{1}{2} L_s i_a^2 + \frac{1}{2} L_s i_b^2 + m(\cos\theta - 0.1\cos 3\theta) i_a i_r + m(\sin\theta + 0.1\sin 3\theta) i_b i_r$
$+ \frac{1}{2} L_r i_r^2$

b) $W_m = W_m'$ since it is linear

c) $T^e = \frac{\partial W_m'}{\partial \theta} = -m\sin\theta\, i_a i_r + 0.3m \sin 3\theta\, i_a i_r$
$+ m\cos\theta\, i_b i_r + 0.3\, m \cos 3\theta\, i_b i_r$

d) $T^e\Big|_{\text{no } 3^{rd}} = -0.9\sin 60° + 0.9\cos 60°$

$T^e_{\text{with } 3^{rd}} = -0.9\sin 60° + 0.9\cos 60° + 0.27\overset{0}{\cancel{\sin 180°}} + 0.27\overset{-1}{\cancel{\cos 180°}}$

Contribution to torque is -0.27 Nm

Problem 3 (30 points)

An electric machine has the following flux linkage vs current characteristic:

$$\lambda_1 = 0.01 i_1 + 0.03\cos\theta\, i_2$$

$$\lambda_2 = 0.03\cos\theta\, i_1 + 0.02\, i_2$$

This machine is operated over a cycle while the current i_1 is kept constant at 3 Amps.

The cycle is:

1. Start at $\theta = 0$ and $i_2 = 0$.
2. Move from $\theta = 0$ to $\theta = 180$ degrees while i_2 remains at 0.
3. Move from $i_2 = 0$ to $i_2 = 5$ Amps while θ remains at 180 degrees.
4. Move from $\theta = 180$ degrees to $\theta = 360$ degrees while i_2 remains at 5 Amps.
5. Move from $i_2 = 5$ Amps to $i_2 = 0$ Amps while θ remains at 360 degrees.

a) What is the energy stored in the coupling field when $\theta = 180$ degrees and $i_2 = 5$ Amps?
b) How much energy is converted from electrical to mechanical form for each cycle.
c) Is this a motor or a generator?

a) $W_m' = 0.005\, i_1^2 + 0.03\cos\theta\, i_1 i_2 + 0.01\, i_2^2$ $\qquad W_m = W_m'$ because of linearity

$$W_m'\Big|_{\substack{\theta=180^\circ \\ i_2=5 \\ i_1=3}} = 0.005\times 9 - 0.03\times 15 + 0.01\times 25$$

$$= .045 - 0.45 + 0.25 = -0.2 + .045 = -0.155\ \text{J}$$

b) $EFM_{cycle} = -\int_{cycle} T^e\, d\theta \qquad T^e = -0.03\sin\theta\, i_1 i_2 = -.09\sin\theta\, i_2$

$$EFM = \cancel{\int_0^{\pi} .09\sin\theta \times 0\, d\theta} + \cancel{\int_{\pi}^{\pi} .09\sin\theta\, i_2\, d\theta} + \int_{\pi}^{2\pi} .45\sin\theta\, d\theta$$

$$= -.45\cos\theta\Big|_{\pi}^{2\pi} = -.45 - (-.45\cos\pi) = -0.9\ \text{J}$$

so $\boxed{EFE_{cycle} = +0.9\ \text{J}}$

c) $EFM_{cycle} < 0$ so motor

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