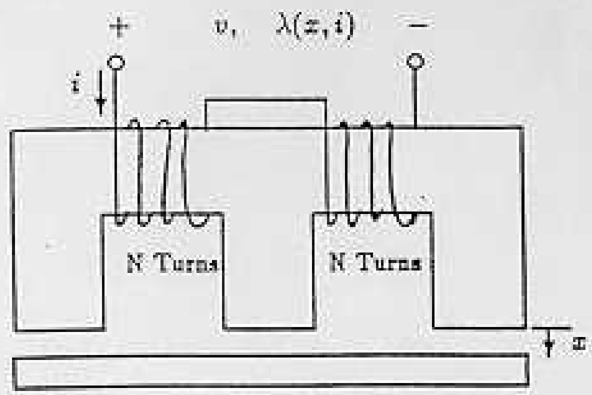
Name Solutions
(Print Name)

Section: M **Tu**
(Circle One)

ECE330 C&N
Fall 2002

Problem 1 (40 pts.)

1. The magnetic field system shown below consists of a fixed upper piece and a moveable lower piece. Both pieces are constructed from infinitely permeable material, and the permeability of the gaps is μ_0. The fixed and moveable pieces are separated by a distance x. The surface area of each face is A. The system is driven by two coils, each of N turns, connected in series.

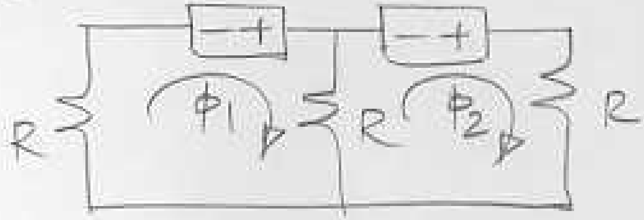


(a) Calculate the flux linkage $\lambda(x,i)$.

(b) Determine the co-energy $W_m'(x,i)$.

(c) Calculate the force of electric origin acting on the movable piece.

a) Equivalent circuit method:



Left Loop:

$Ni = R(\phi_1 - \phi_2) + R\phi_1$

$Ni = 2R\phi_1 - R\phi_2$ — (I)

Right Loop:

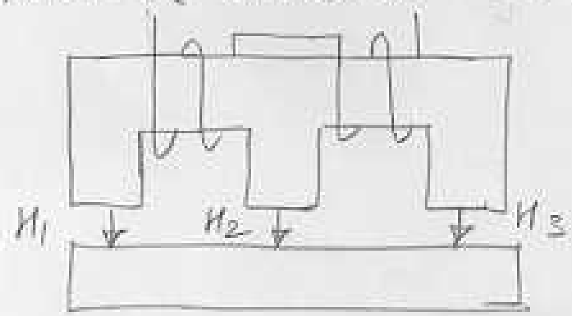
$Ni = R\phi_2 + R(\phi_2 - \phi_1)$

$Ni = -R\phi_1 + 2R\phi_2$ — (II)

$\Rightarrow \phi_1 = \phi_2 = \dfrac{Ni}{R}$

$$\lambda(x,i) = N\phi_1 + N\phi_2 = \frac{2N^2 i}{R} = \frac{2N^2 i}{x/\mu_0 A} = \boxed{2\mu_0 A N^2 \frac{i}{x}}$$

ACL & Gauss' Law method:



$$\left.\begin{aligned} H_1 x - H_2 x &= -Ni \\ H_2 x - H_3 x &= -Ni \end{aligned}\right\} \text{ACL}$$

$$H_1 + H_2 + H_3 = 0 \Leftarrow \text{Gauss}$$

Solve for $H_1, H_2, H_3 \Rightarrow H_2 = 0$, $H_3 = -H_1 = \frac{Ni}{x}$

$$\lambda = N\phi_1 + N\phi_2 = N\left(\frac{Ni}{x}\mu_0 A\right) + N\left(\frac{Ni}{x}\mu_0 A\right)$$

$$= \boxed{\frac{2\mu_0 N^2 i A}{x}}$$

b) $$W_m'(x,i) = \int_0^i \lambda(i', x)\,di' = \int_0^i 2\mu_0 A N^2 \frac{i'}{x}\,di'$$

$$= \boxed{\mu_0 A N^2 \frac{i^2}{x}}$$

c) $$f^e = \frac{\partial W_m'}{\partial x} = \boxed{-\mu_0 A N^2 \frac{i^2}{x^2}}$$

Problem 2 (30 points)

Write the following equation in state space form by defining $\theta = x_1$ and $\dot{\theta} = x_2$. Then numerically integrate for 2 time steps using $\Delta t = .01$ sec., i.e., compute θ and $\dot{\theta}$ at $t = .01$ and .02 sec. The initial conditions are $\theta(0) = 0.5$ rad, $\dot{\theta}(0) = 0$.

$$\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} - 10\theta^3 = 0$$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 10x_1^3 - 10x_2 \end{aligned} \quad ; \quad x(0) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \ \Delta t = 0.01 \text{ sec.}$$

$$x_1(0.01) = x_1(0) + \Delta t\, x_2(0)$$
$$= \boxed{0.5}$$
$$x_2(0.01) = x_2(0) + \Delta t \left[10x_1(0)^3 - 10x_2(0)\right]$$
$$= \boxed{0.0125}$$
$$x_1(0.02) = x_1(0.01) + \Delta t\, x_2(0.01)$$
$$= \boxed{0.500125}$$
$$x_2(0.02) = x_2(0.01) + \Delta t\left[10x_1(0.01)^3 - 10x_2(0.01)\right]$$
$$= \boxed{0.02375}$$

Problem 3 (30 points total)

Assume the state space equations for an electromechanical system are

$$\dot{x}_1 = x_2 \qquad = f_1(x_1, x_2)$$
$$\dot{x}_2 = \sin(x_1) - 0.866 - x_2 \qquad = f_2(x_1, x_2)$$

(10 pts) a) Write the linearized form of the above equations (i.e., $\Delta\dot{\mathbf{x}} = \mathbf{A}\,\Delta\mathbf{x}$).

(10 pts) b) This system has two equilibrium points. What are they?

(10 pts) c) Determine the eigenvalues of each of the equilibrium points. Tell whether or not each equilibrium point is stable.

a) $$A = \begin{bmatrix} \partial f_1/\partial x_1 & \partial f_1/\partial x_2 \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \cos x_1 & -1 \end{bmatrix}$$

ie $$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ \cos x_1^e & -1 \end{bmatrix}}_{A} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

b) $$x^e = \begin{bmatrix} 60^\circ \\ 0 \end{bmatrix}, \begin{bmatrix} 120^\circ \\ 0 \end{bmatrix}$$

c) for $x^e = \begin{bmatrix} 60^\circ \\ 0 \end{bmatrix}$; $A = \begin{bmatrix} 0 & 1 \\ 0.5 & -1 \end{bmatrix}$

$\Rightarrow \lambda = 0.366, -1.366$

unstable

for $x^e = \begin{bmatrix} 120^\circ \\ 0 \end{bmatrix}$; $A = \begin{bmatrix} 0 & 1 \\ -0.5 & -1 \end{bmatrix}$

$\Rightarrow \lambda = -0.5 \pm 0.5j$

stable