ECE 330 Exam 2: Spring 2021

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USEFUL INFORMATION

sin(x)=cos(x-90°) $\quad \bar{V} = \bar{Z}\bar{I} \quad \bar{S} = \bar{V}\bar{I}^* = P + jQ \quad \bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

0<θ<180° (lag) $\quad I_L = \sqrt{3}I_\varphi$ (delta) $\quad \bar{Z}_Y = \bar{Z}_\Delta/3$

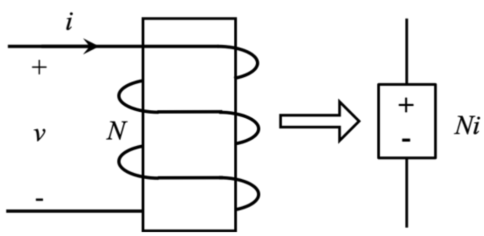
-180°<θ<0 (lead) $\quad V_L = \sqrt{3}V_\varphi$ (wye) $\quad \mu_0 = 4\pi \times 10^{-7}$H/m

ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA \qquad \int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right) \qquad \mathcal{R} = \frac{l}{\mu A} \qquad Ni = \mathcal{R}\varphi$

$\varphi = BA \qquad \lambda = N\varphi = Li$ (if linear) $\qquad v = \frac{d\lambda}{dt} \qquad k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W

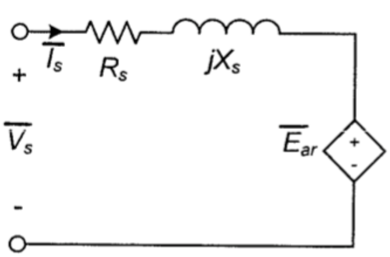


$W_m = \int_0^\lambda i d\hat{\lambda} \qquad W_m' = \int_0^i \lambda d\hat{\imath} \qquad W_m + W_m' = i\lambda \qquad f^e = -\frac{\partial W_m}{\partial x} = \frac{\partial W_m'}{\partial x}$

$x \to \theta, f^e \to T^e$

$EFE_{a\to b} = \int_a^b i d\lambda \qquad EFM_{a\to b} = \int_a^b -f^e dx \qquad EFE_{a\to b} + EFM_{a\to b} = W_{mb} - W_{ma}$

$i = \frac{\partial W_m}{\partial \lambda} \qquad \lambda = \frac{\partial W_m'}{\partial i}$



$N_s = \frac{120 f_s}{P}$

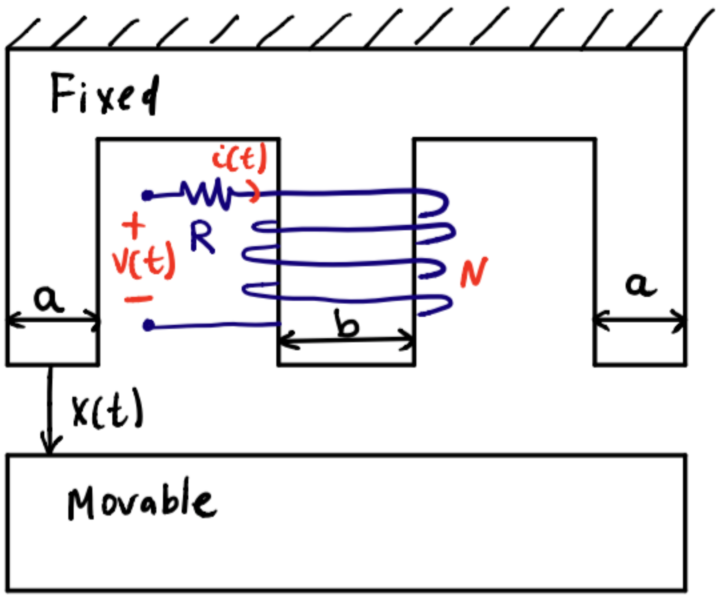
$\overline{E_{ar}} = \frac{\omega_s M I_r}{\sqrt{2}} \angle\delta \qquad \delta < 0 \to$ motor

For $R_s = 0$: $\quad P_T = \frac{3V_s|\overline{E_{ar}}|}{X_s}\sin(-\delta) \qquad Q_T = \frac{3V_s^2}{X_s} - \frac{3V_s|\overline{E_{ar}}|}{X_s}\cos(\delta)$

$T^e = \frac{P_m}{\omega_m} \qquad \omega_m = \frac{2}{P}\omega_s \qquad$ (for synchronous machine, $P_m = P_T$ in formulas above)

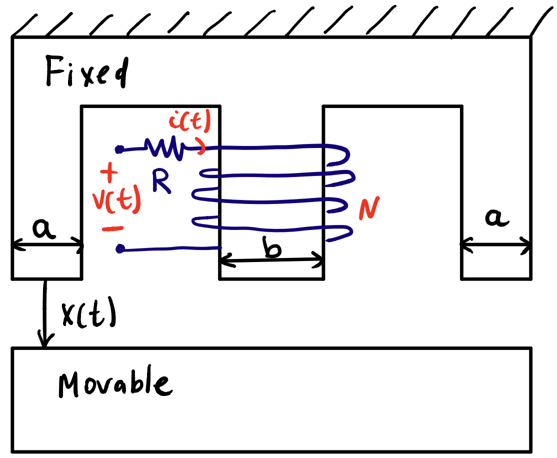
Problem 1 (25 Points)

For the structure shown below, the movable member is constrained to move up and down only as indicated, and the air gap between the fixed and movable member is x(t). The width a = 2 cm, the width b = 4 cm, number of turns N = 200, R = 1 Ω. Depth into the page for all members d = 4 cm. Neglect fringing and assume that the permeability of the structure to be infinite. Note: permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ H/m.



a) Find the flux linkage λ of the coil in terms of x(t) and i(t)?
b) Find an expression for the voltage, v(t) in terms of x(t), i(t) and time t.

Pr 1, solution



$R = 1\Omega$ $\quad d = 4cm$

$N = 200$ $\quad a = 2cm$

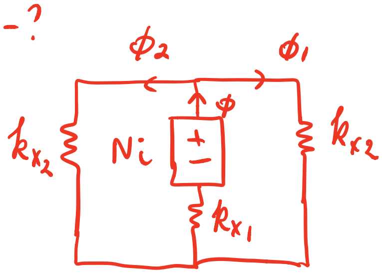
$b = 4cm$

Neglect fringing,

$\mu_{iron} = \infty$

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m

a) $\lambda - ?$



$$R_{x_1} = \frac{x}{\mu_0 A_1}$$

$$R_{x_2} = \frac{x}{\mu_0 A_2} \Rightarrow R_{x_2} = 2R_{x_1}$$

$$A_1 = 2A_2$$

$$\phi = \phi_1 + \phi_2$$

$$\phi_1 = \phi_2$$

KVL: $Ni = \phi_2 R_{x_2} + \phi R_{x_1} =$

$= \phi_2 R_{x_2} + 2\phi_2 R_{x_1} =$

$= \phi_2 (R_{x_2} + 2R_{x_1}) =$

$= \phi_2 4 R_{x_1}$

$$\phi_2 = \frac{Ni}{4R_{x_1}}$$

$$\phi = \frac{2Ni}{4R_{x_1}} = \frac{Ni}{2R_{x_1}}$$

$\lambda = N\phi =$

$$= \frac{N^2 i}{2R_{x_1}} = \frac{N^2 i \mu_0 A_1}{2x} = \frac{200^2 \cdot 4\pi \cdot 10^{-7} \cdot 16 \cdot 10^{-4}}{2x} = \frac{4.02 \cdot 10^{-5}}{x} i$$

$L(x)$

b) $v(t)$ - ?

$$v = i \cdot R + \frac{d\lambda}{dt} = i \cdot R + L(x)\frac{di}{dt} + i\frac{\partial L}{\partial x}\frac{dx}{dt} =$$

$$= i + \frac{4.02 \cdot 10^{-5}}{x}\frac{di}{dt} - i \cdot \frac{4.02 \cdot 10^{-5}}{x^2}\frac{dx}{dt}$$

Problem 2 (25 Points)

A certain electromechanical system is described as

$$\lambda_1 = \frac{1}{x - 0.5} i_1 - \frac{0.25}{x - 0.5} i_2$$

$$\lambda_2 = -\frac{0.25}{x - 0.5} i_1 + \frac{0.5}{x - 0.5} i_2$$

where λ_1 and i_1 are the flux linkage and current for coil 1, λ_2 and i_2 are the flux linkage and current for coil 2, and x is the length of the air gap inside the iron core.

a) What is the energy stored in the coupling field when $x = 1.5\ m, i_1 = 10\ A$, and $i_2 = 5\ A$?
b) What is the force of electric origin when $x = 1.5\ m, i_1 = 10\ A$, and $i_2 = 5\ A$?
c) What does the sign of the force of electric origin mean for the physical system?

Problem 2:

$$\lambda_1 = \frac{1}{x-0.5} i_1 - \frac{0.25}{x-0.5} i_2$$

$$\lambda_2 = \frac{-0.25}{x-0.5} i_1 + \frac{0.5}{x-0.5} i_2$$

a) Because system is Linear, $W_m = W_m'$

$$W_m' = \int_0^{i_1} \lambda_1(\hat{i}_1, i_2=0, x)\, d\hat{i}_1 + \int_0^{i_2} \lambda_2(i_1, \hat{i}_2, x)\, d\hat{i}_2$$

$$W_m' = \frac{1}{2}\left(\frac{1}{x-0.5}\right) i_1^2 + \frac{1}{2}\left(\frac{0.5}{x-0.5}\right) i_2^2 - \frac{0.25}{x-0.5} i_1 i_2$$

$$W_m' = 43.75\,J$$

$$\boxed{W_m = 43.75\,J}$$

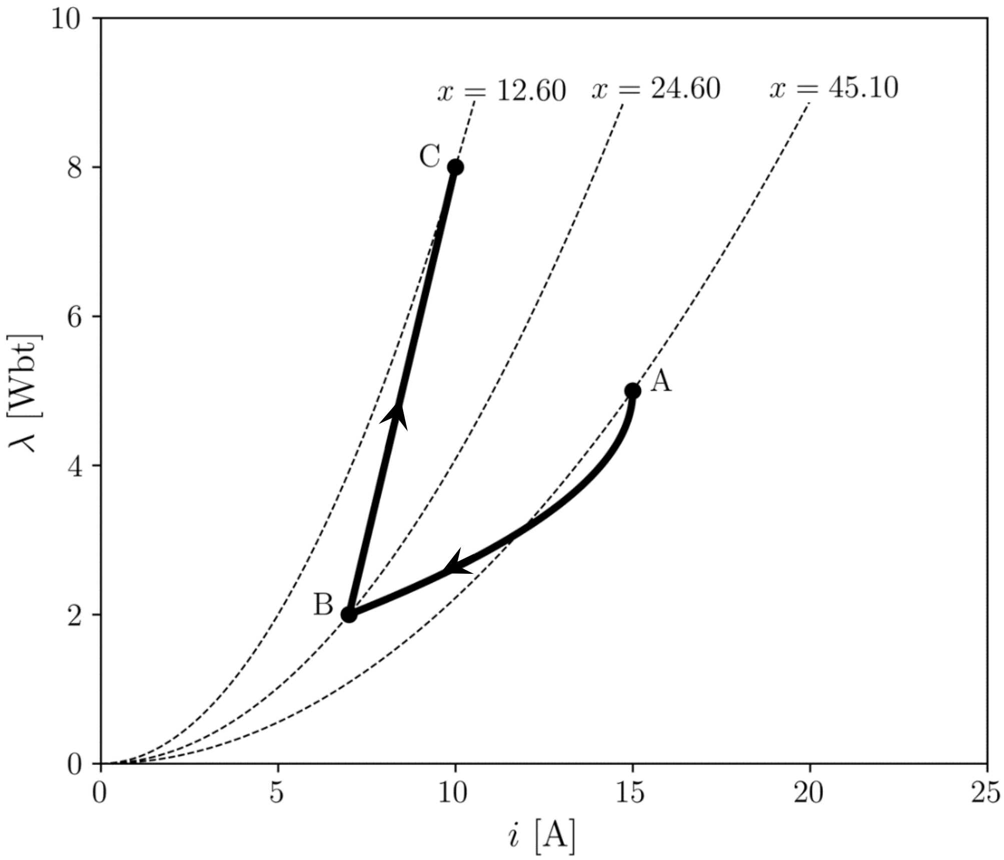
b) $f^e = \dfrac{\partial W_m'}{\partial x}$

$$f^e = -\frac{1}{2}\left(\frac{1}{(x-0.5)^2}\right) i_1^2 - \frac{1}{2}\left(\frac{0.5}{(x-0.5)^2}\right) i_2^2 + \frac{0.25}{(x-0.5)^2} i_1 i_2$$

$$\boxed{f^e = -43.75\,N}$$

c) f^e acts to close the air gap, so it acts in the negative x direction.

Problem 3 (25 Points)



An electro-mechanical system that has the flux linkage relationship $\lambda = \frac{1}{x-0.1} i^2$ moves from point A ($i_a = 15$ A, $\lambda_a = 5$ Wbt) to point C ($i_c = 10$ A and $\lambda_c = 8$ Wbt) through the intermediate point B ($i_b = 7$ A, $\lambda_c = 2$ Wbt). The equation for the curve between points A and B is given as

$$i = -\frac{8}{9}(\lambda - 5)^2 + 15$$

and the curve between points B and C is a straight line. The values for x at each given point are also given.

For the movement between these points, find:

a) The energy at point A and C.
b) The EFE when moving from point A to C.
c) The EFM when moving from point A to C.
d) If we move back from point C to point A along a straight line, what is the EFM over this cycle? Is the system behaving like a motor or a generator? Explain.

3. a) $\lambda = \frac{1}{x-0.1} i^2$ Non-Linear.

$W_m \neq W_m'$

$W_m + W_m' = i\lambda$

$W_m' = \int_0^i \lambda \, d\hat{i} \Rightarrow W_m' = \int_0^i \frac{1}{x-0.1} \hat{i}^2 \, d\hat{i} = \frac{1}{3}\left(\frac{1}{x-0.1}\right) i^3$

$W_m = i\lambda - \frac{1}{3}\left(\frac{1}{x-0.1}\right) i^3 \Rightarrow W_m = \frac{1}{x-0.1} i^3 - \frac{1}{3}\left(\frac{1}{x-0.1}\right) i^3$

$W_m = \frac{2}{3}\left(\frac{1}{x-0.1}\right) i^3$

A: $i = 15$, $x = 45.10$

$W_{ma} = \frac{2}{3}\left(\frac{1}{45.10-0.1}\right)(15)^3$

$\boxed{W_{ma} = 50\,\text{J}}$

C: $i = 10$, $x = 12.60$

$W_{mc} = \frac{2}{3}\left(\frac{1}{12.60-0.1}\right)(10)^3$

$\boxed{W_{mc} = 53.33\,\text{J}}$

b) $\underset{a\to c}{EFE} = \underset{a\to b}{EFE} + \underset{b\to c}{EFE}$

$\underset{a\to b}{EFE} = \int_5^2 \left(-\frac{8}{9}(\lambda-5)^2 + 15\right) d\lambda = -\frac{8}{27}(\lambda-5)^3 + 15\lambda \Big|_5^2$

$= -\frac{8}{27}(2-5)^3 + 15(2) - \left(-\frac{8}{27}[5-5]^3 + 15(5)\right) \Rightarrow \underset{a\to b}{EFE} = -37\,\text{J}$

$\underset{b\to c}{EFE} = \frac{8-2}{2}(7+10) = \frac{6}{2}(17) = 3(17) = 51\,\text{J}$

$\underset{a\to c}{EFE} = -37 + 51 \Rightarrow \boxed{\underset{a\to c}{EFE} = 14\,\text{J}}$

c) $\underset{a\to c}{EFM} = W_{mc} - W_{ma} - \underset{a\to c}{EFE} = 53.33 - 50 - 14 \Rightarrow \boxed{\underset{a\to c}{EFM} = -10.67\,\text{J}}$

d) $\underset{c\to a}{EFE} = \frac{5-8}{2}(10+15) \Rightarrow -\frac{3}{2}(25) \Rightarrow \underset{c\to a}{EFE} = -37.5\,\text{J}$

$\underset{cycle}{EFE} = 14 - 37.5 \Rightarrow \underset{cycle}{EFE} = -23.5\,\text{J}$

$\boxed{\underset{cycle}{EFM} = 23.5\,\text{J}}$

Generator. EFM>0, which means putting in mechanical energy to get out electrical energy.

Problem 4 (25 Points)

You are working with an Australian consulting firm to bring electricity to an isolated rural community in Papua New Guinea. The firm is currently installing a micro hydro plant, which will use the natural river flow across a small waterfall to generate electricity.
As usual, a three-phase Wye-connected synchronous machine will be used for the actual generation. This machine has **6 poles** and is rated for **30 kW** at **50 Hz**, **440 V** rms line-to-line. Its synchronous reactance is **5 Ω**.

a) How much mechanical torque must the hydro turbine apply to the synchronous machine in order for it to generate rated power and frequency? (In order to get a back-of-the-envelop figure, assume zero losses in the entire system. That is, zero armature resistance, zero windage loss, zero friction.)

The micro hydro plant will be located in an isolated, rural part of Papua New Guinea. Therefore, it is crucial that the synchronous machine will also able to supply reactive power.

b) What is the excitation voltage and torque angle of the machine when it is supplying rated power and also delivering **3 kVARs** of reactive power to the rural electric grid? (We want the generator to also act as a capacitor.)

a)

conversion to rad

$$\omega_{spd} = \left(\frac{2}{6}\right)(50\,\text{Hz})(2\pi) = 104.7198 \text{ rad/s}$$

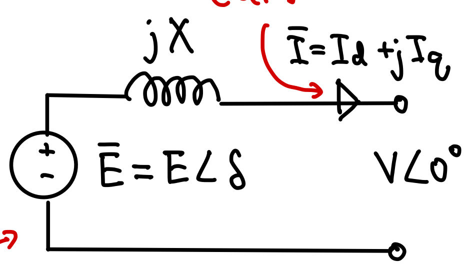
6-pole is factor 3X slower than 2-pole

torque · spd = power

$$\tau = \frac{P}{\omega} = \frac{30\,\text{kW}}{104.72\,\text{rad/s}} = \underline{286.4789\,\text{Nm}}$$

b)

Careful: orientation



generator model

$X = 5\Omega$

$V = 440/\sqrt{3}\ \text{V}$

$P_{3\phi} = 30\ \text{kW}$

$Q_{3\phi} = 3\ \text{kVAR}$

convert line to line to phase to neutral

$P_{1\phi} = VI_d = 10\ \text{kW}$ (per-phase)

$\Rightarrow I_d = P_{1\phi}/V = 39.3648\ \text{A}$

$Q_{1\phi} = -VI_q = 1\ \text{kVAR}$ (per-phase)

$\Rightarrow I_q = -Q_{1\phi}/V = -3.9365\ \text{A}$

$$\begin{aligned}\bar{E} &= V\angle 0° + jX(I_d + jI_q) \\ &= (V - I_q X) + jI_d X \\ &= 273.72 + j196.82\ \text{V} \\ &= \underline{377.1356}\,\angle\,\underline{35.7191°}\ \text{V}\end{aligned}$$

excitation voltage

torque angle