

**ECE330: Power Circuits & Electromechanics**  
**Lecture 17. Energy conversion cycles**

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**Motor technology from Model 3 helps Tesla boost Model S range 10%**

New motor design helps Model S to drive 370 miles on a charge.

The Model 3 debuted with an alternative motor technology that Tesla calls a permanent magnet synchronous reluctance motor. A synchronous reluctance motor has a series of electromagnets around the stator, but the rotor doesn't have any windings or permanent magnets. Instead, the rotor contains veins of a magnetic material interspersed with non-magnetic material, arranged so that it has a preferred orientation in the magnetic field created by the stator.

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**Last time: Rotating reluctance machine**

$$L(\theta) = L_1 + L_2 \cos(2\theta)$$

$$P_e = vi$$

$$P_m = f \frac{dx}{dt}$$

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**Last time: Switched reluctance system**

Today: How much power?

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**Line integral definition of energy**

$$\text{Energy}(\lambda_1, x_1) - \text{Energy}(\lambda_0, x_0) = \int_C \underline{F(x)} \cdot \underline{dr} = \int_C i(\lambda, x) d\lambda + \int_C -f(\lambda, x) dx$$

Path independent (via conservation)

Energy From Electrical      Energy From Mechanical

Path dependent

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**Trajectories and force diagrams**

$$\text{Energy}(\lambda_1, x_1) - \text{Energy}(\lambda_0, x_0) = \int_C \underline{F(x)} \cdot \underline{dr} = \int_C i(\lambda, x) d\lambda + \int_C -f(\lambda, x) dx$$

**Monday:**

1. Draw trajectory from description
2. Draw force diagrams from trajectory

**Friday:** Analyze electrical machines as a sequence of trajectories

Function of start and end

Function of entire trajectory

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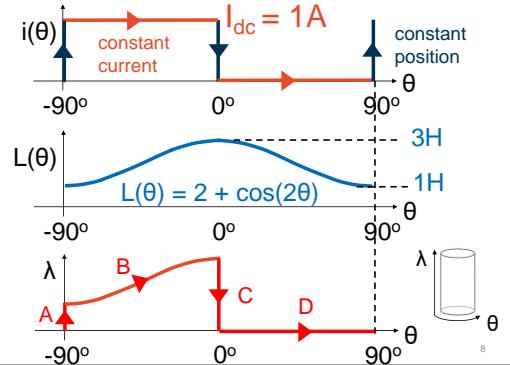
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## Today

- Example: Switched reluctance machine
- Example: Linear actuator

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## Reluctance machine conversion cycle

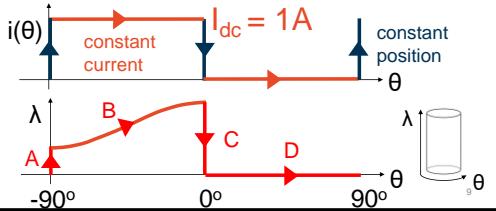


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## Reduction to EFE / EFM problem

$L(\theta) = 2 + \cos(2\theta)$ . Find the EFE / EFM:

- A. Constant  $\theta = -90^\circ$ , current 0 → 1A.
- B. Constant  $i = 1A$ , position  $-90^\circ \rightarrow 0^\circ$ .
- C. Constant  $\theta = 0^\circ$ , current 1A → 0A.
- D. Constant  $i = 0A$ , position  $0^\circ \rightarrow 90^\circ$ .



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## Reduction to EFE / EFM problem

$L(\theta) = 2 + \cos(2\theta)$ . Find the EFE / EFM:

- A. Constant  $\theta = -90^\circ$ , current 0 → 1A.

$$L'(\theta) = \frac{\partial L}{\partial \theta} = -2\sin(2\theta).$$

$$\tau_e = \frac{\partial}{\partial \theta} \left\{ \frac{1}{2} L(\theta) i^2 \right\} = -i^2 \sin(2\theta).$$

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## Reduction to EFE / EFM problem

$L(\theta) = 2 + \cos(2\theta)$ . Find the EFE / EFM:

- B. Constant  $i = 1A$ , position  $-90^\circ \rightarrow 0^\circ$ .

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## Reduction to EFE / EFM problem

$L(\theta) = 2 + \cos(2\theta)$ . Find the EFE / EFM:

- C. Constant  $\theta = 0^\circ$ , current 1A → 0A.

$$L'(\theta) = \frac{\partial L}{\partial \theta} = -2\sin(2\theta).$$

$$\tau_e = \frac{\partial}{\partial \theta} \left\{ \frac{1}{2} L(\theta) i^2 \right\} = -i^2 \sin(2\theta).$$

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### Reduction to EFE / EFM problem

$L(\theta) = 2 + \cos(2\theta)$ . Find the EFE / EFM:

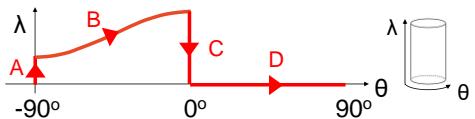
D. Constant  $i = 0A$ , position  $0^\circ \rightarrow 90^\circ$ .

$$L'(\theta) = \frac{\partial L}{\partial \theta} = -2\sin(2\theta).$$

$$\tau^e = \frac{\partial}{\partial \theta} \left( \frac{1}{2} L(\theta) i^2 \right) = -i^2 \sin(2\theta).$$

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### Reduction to EFE / EFM problem



$$i = L_{\min}^{-1} \Lambda \quad i = L_{\max}^{-1} \Lambda$$

$$i_{dc}$$

$$L(\theta) = 2 + \cos(2\theta)$$

$$H$$

$$I_{dc} = 1 \text{ A}$$

$$\text{EFE}_A = +0.5 \text{ J}$$

$$\text{EFE}_B = +2.0 \text{ J}$$

$$\text{EFE}_C = -1.5 \text{ J}$$

$$\text{EFE}_D = 0 \text{ J}$$

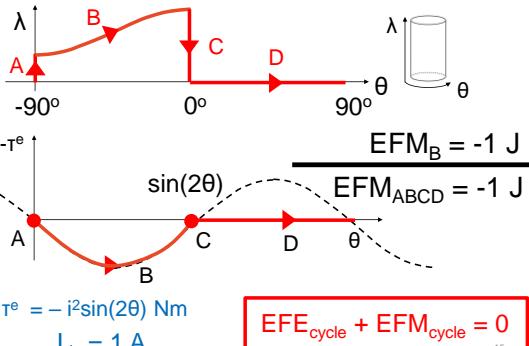
$$\text{EFE}_{ABCD} = +1 \text{ J}$$

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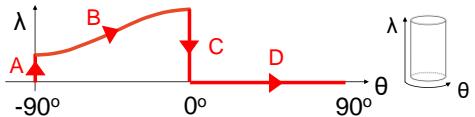
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### Reduction to EFE / EFM problem



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### High-level view: "Four-stroke engine"



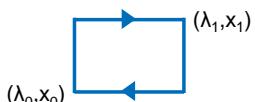
$$\text{Input} \quad P_e = vi \quad \text{Output} \quad P_m = f \frac{dx}{dt}$$

	A	B	C	D
EFE [J]	+1/2 J	+2 J	-3/2 J	0
EFM [J]	0	-1 J	0	0

$$\text{Av. power out} = 1 \text{ J / half-cycle} = 120 \text{ W @ 60 Hz}$$

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### Summary: Energy over a closed loop



$$\begin{aligned} & \text{Energy}(\lambda_1, x_1) - \text{Energy}(\lambda_1, x_1) \\ &= \oint_C \underline{F(r)} \cdot d\underline{r} = \oint_C i(\lambda, x) d\lambda + \oint_C -f(\lambda, x) dx \\ &= 0 \end{aligned}$$

Motor  
elec → mech

EFE|<sub>cycle</sub> > 0, EFM|<sub>cycle</sub> < 0

Generator  
mech → elec

EFE|<sub>cycle</sub> < 0, EFM|<sub>cycle</sub> > 0

⚠ Positive = power input, negative = power output

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### Today

- Example: Switched reluctance machine

- Example: Linear actuator

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**Linear actuator**

$$\lambda(i,x) = 5i/x.$$

Find  $EFE_{cycle}$ ,  $EFM_{cycle}$ . Motor or generator?

Game plan

- Get formula for energy
- Evaluate energy at each node
- For each path
  - $\Delta E = EFE + EFM$
  - $EFE = \int i d\lambda$

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**Linear actuator**

$$\lambda(i,x) = 5i/x.$$

Find  $EFE_{cycle}$ ,  $EFM_{cycle}$ . Motor or generator?

Game plan

- Get formula for energy
- Evaluate energy at each node
- For each path
  - $\Delta E = EFE + EFM$
  - $EFE = \int i d\lambda$

$E_a = 5J$ ,  $E_b = 2J$ ,  $E_c = 3J$ ,  $E_d = 6J$

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**Linear actuator**

$$\lambda(i,x) = 5i/x.$$

Find  $EFE_{cycle}$ ,  $EFM_{cycle}$ . Motor or generator?

Game plan

- Get formula for energy
- Evaluate energy at each node
- For each path
  - $\Delta E = EFE + EFM$
  - $EFE = \int i d\lambda$

$E_a = 5J$ ,  $E_b = 2J$ ,  $E_c = 3J$ ,  $E_d = 6J$

$EFE_{a \rightarrow b} = -1.5J$ ,  $EFM_{a \rightarrow b} = -1.5J$ ,

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**Linear actuator**

$$\lambda(i,x) = 5i/x.$$

Find  $EFE_{cycle}$ ,  $EFM_{cycle}$ . Motor or generator?

Game plan

- Get formula for energy
- Evaluate energy at each node
- For each path
  - $\Delta E = EFE + EFM$
  - $EFE = \int i d\lambda$

$E_a = 5J$ ,  $E_b = 2J$ ,  $E_c = 3J$ ,  $E_d = 6J$

$EFE_{a \rightarrow b} = -1.5J$ ,  $EFE_{b \rightarrow c} = -1.5J$ ,  $EFM_{a \rightarrow b} = -1.5J$ ,  $EFM_{b \rightarrow c} = +2.5J$ ,

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**Linear actuator**

$$\lambda(i,x) = 5i/x.$$

Find  $EFE_{cycle}$ ,  $EFM_{cycle}$ . Motor or generator?

Game plan

- Get formula for energy
- Evaluate energy at each node
- For each path
  - $\Delta E = EFE + EFM$
  - $EFE = \int i d\lambda$

$E_a = 5J$ ,  $E_b = 2J$ ,  $E_c = 3J$ ,  $E_d = 6J$

$EFE_{a \rightarrow b} = -1.5J$ ,  $EFE_{b \rightarrow c} = -1.5J$ ,  $EFE_{c \rightarrow d} = +2.5J$ ,  $EFM_{a \rightarrow b} = -1.5J$ ,  $EFM_{b \rightarrow c} = +2.5J$ ,  $EFM_{c \rightarrow d} = +0.5J$ ,

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**Linear actuator**

$$\lambda(i,x) = 5i/x.$$

Find  $EFE_{cycle}$ ,  $EFM_{cycle}$ . Motor or generator?

Game plan

- Get formula for energy
- Evaluate energy at each node
- For each path
  - $\Delta E = EFE + EFM$
  - $EFE = \int i d\lambda$

$EFE_{cycle} = -1.5 - 1.5 + 2.5 + 2.5 = +2 J$

$EFM_{cycle} = -1.5 + 2.5 + 0.5 - 3.5 = -2 J$

$EFE_{d \rightarrow a} = +2.5J$ ,  $EFM_{d \rightarrow a} = -3.5J$ ,

$EFE_{a \rightarrow b} = -1.5J$ ,  $EFE_{b \rightarrow c} = -1.5J$ ,  $EFE_{c \rightarrow d} = +2.5J$ ,  $EFM_{a \rightarrow b} = -1.5J$ ,  $EFM_{b \rightarrow c} = +2.5J$ ,  $EFM_{c \rightarrow d} = +0.5J$ ,

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