ECE330: Power Circuits & Electromechanics

Lecture 14. Co-Energy via line integrals

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Schedule

- ~~Wed 3/4: Voltage of mechanical origin~~
- ~~Fri 3/6: Practical transformers (Banerjee)~~
- ~~Mon 3/9: Force of electrical origin~~
- ~~**Wed 3/11: Quiz 5 + Review**~~
- ~~Fri 3/13: Energy via line integrals~~
- **Mon 3/16: Spring break**
- **Wed 3/18: Spring break**
- **Fri 3/20: Spring break**
- Mon 3/23: Co-energy via line integrals
- **Wed 3/11: Homework 6 + Review**

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Today

- (Review) Line integral definition of energy
- Line integral definition of co-energy
- Examples

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$$\int_C \mathbf{F}(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = \int_0^1 \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, \mathrm{d}t$$

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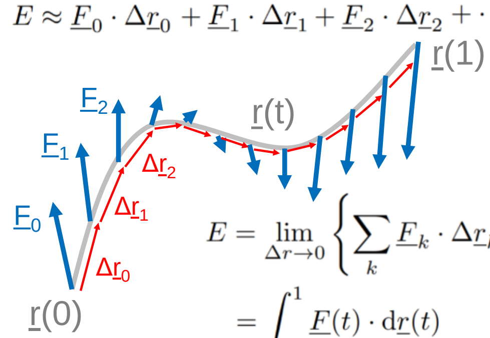
$$E \approx \underline{F}_0 \cdot \Delta\underline{r}_0 + \underline{F}_1 \cdot \Delta\underline{r}_1 + \underline{F}_2 \cdot \Delta\underline{r}_2 + \cdots$$



$$E = \lim_{\Delta r \to 0} \left\{ \sum_k \underline{F}_k \cdot \Delta\underline{r}_k \right\}$$

$$= \int_0^1 \underline{F}(t) \cdot \mathrm{d}\underline{r}(t)$$

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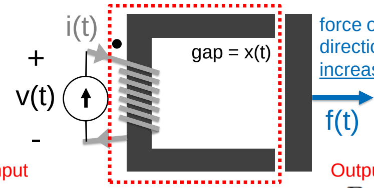
Conservative electromechanical system



Input $P_e = vi$ → Output $P_m = f\dfrac{dx}{dt}$

Storage $\dfrac{dE}{dt} = i\dfrac{d\lambda}{dt} - f\dfrac{dx}{dt}$

Energy over Δt

$$\Delta E = i\Delta\lambda - f\Delta x$$

Closed system = Conservative

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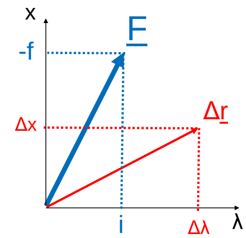
Line integral view of stored energy

$$\Delta E = i\Delta\lambda - f\Delta x$$
$$= (i, -f)\cdot(\Delta\lambda, \Delta x)$$
$$= \underline{F}\cdot\Delta\underline{r}$$



Position r = (λ, x)

Force F = (i, -f)

Flux as spatial dim. Current as force.

etymology for "mmf"

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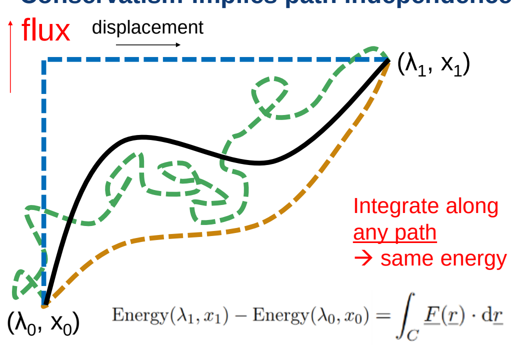
Conservatism implies path independence



Integrate along any path → same energy

$$\text{Energy}(\lambda_1, x_1) - \text{Energy}(\lambda_0, x_0) = \int_C \underline{F}(\underline{r})\cdot d\underline{r}$$

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Line integral definition of energy

Important

$$\text{Energy}(\lambda_1, x_1) - \text{Energy}(\lambda_0, x_0)$$
$$= \int_C \underline{F}(\underline{r})\cdot d\underline{r} = \underbrace{\int_C i(\lambda, x)\, d\lambda}_{\text{Energy From Electrical}} + \underbrace{\int_C -f(\lambda, x)\, dx}_{\text{Energy From Mechanical}}$$

Path independent (via conservation)

Path dependent

$$\text{Energy}(0, x) = 0$$ (Can be any constant)

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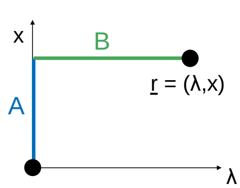
Derivation of the energy integral

Important

$$\text{Energy}(\lambda, x) - \text{Energy}(0, 0)$$
$$= \int_C i(\lambda', x')\, d\lambda' + \int_C -f(\lambda', x')\, dx'$$



$$= \left(\int_A i(\lambda', x')\, d\lambda' + \int_A -f(\lambda', x')\, dx'\right)$$

$$+ \left(\int_B i(\lambda', x')\, d\lambda' + \int_B -f(\lambda', x')\, dx'\right)$$

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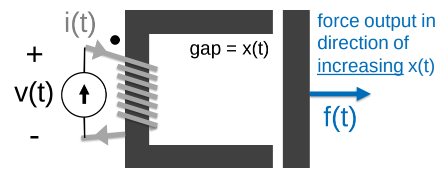
Definition of co-energy



$$\text{Co-energy}(i,x) = \lambda i - \text{Energy}(\lambda, x)$$

$$\frac{dE_{co}}{dt} = \lambda\frac{di}{dt} + i\frac{d\lambda}{dt} - \frac{dE}{dt}$$

Swaps the roles of current and flux

positive force

$$= \lambda\frac{di}{dt} + \cancel{i\frac{d\lambda}{dt}}^{\,0} - \left(\cancel{i\frac{d\lambda}{dt}}^{\,0} - f\frac{dx}{dt}\right) = \lambda\frac{di}{dt} + f\frac{dx}{dt}$$

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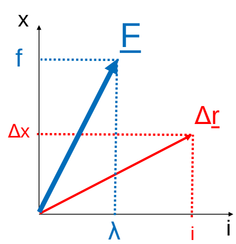
Line integral view of co-energy

$$\frac{dE_{co}}{dt} = \lambda\frac{di}{dt} + f\frac{dx}{dt}$$

$$\Delta E_{co} = \lambda\Delta i + f\Delta x$$
$$= (\lambda, f)\cdot(\Delta i, \Delta x)$$
$$= \underline{F}\cdot\Delta\underline{r}$$



Position $\underline{r}$ = (i, x)

Force $\underline{F}$ = (λ, f)

Current as spatial dim. Flux as force.

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Line integral definition of energy

$$\text{Co-energy}(i_1, x_1) - \text{Co-energy}(i_0, x_0)$$
$$= \int_C \underline{F}\cdot d\underline{r} = \int_C \lambda(i,x)\,di + \int_C f(i,x)\,dx$$

Path independent (via conservation)

Path dependent

No physical interpretation!!

$$\text{Co-energy}(0,x) = 0$$ (Can be any constant)

$$\text{Co-energy}(i,x) = \lambda i - \text{Energy}(\lambda, x)$$

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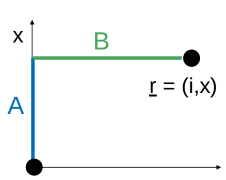
Derivation of the co-energy integral

Important

$$\text{Energy}(\lambda, x) - \text{Energy}(0,0)$$
$$= \int_C i(\lambda', x')\,d\lambda' + \int_C -f(\lambda', x')\,dx'$$



$$= \left(\int_A \lambda(i', x')\,di' + \int_A f(i', x')\,dx'\right)$$

$$+ \left(\int_B \lambda(i', x')\,di' + \int_B f(i', x')\,dx'\right)$$

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- ~~Line integral definition of co-energy~~
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Example 1. $\lambda(i,x) = L(x)i, \quad L(x) = \dfrac{\mu_0 A N^2}{x}$

Compute force via energy, co-energy

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Example 2. $\lambda(i,x) = L(x)i^2, \quad L(x) = \dfrac{\mu_0 A N^2}{x}$

Compute force via energy, co-energy

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Example 3. $\lambda(i,\theta) = L(\theta)i^2, \qquad L(x) = L_0 \cos\theta$

Compute force via energy, co-energy

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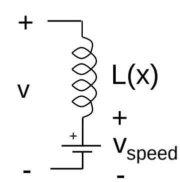
Conservative electromagnetic systems

Flux & current relations

$$\lambda = \lambda(x,i), \quad i = i(\lambda, x)$$



Voltage of mechanical origin

$$v = \underbrace{\frac{\partial \lambda}{\partial i}}_{L(x)} \frac{\mathrm{d}i}{\mathrm{d}t} + \underbrace{\frac{\partial \lambda}{\partial x} \frac{\mathrm{d}x}{\mathrm{d}t}}_{v_{\text{speed}}}$$

Force of electric origin
(direction of increasing x)

$$f^e = +\frac{\partial}{\partial x}[\text{Co-energy}]$$
$$= -\frac{\partial}{\partial x}[\text{Energy}]$$

Energy & Co-energy

$$\text{Co-energy} = \int_0^i \lambda(i', x)\,\mathrm{d}i'$$

$$\text{Energy} = \int_0^\lambda i(\lambda', x)\,\mathrm{d}\lambda'$$