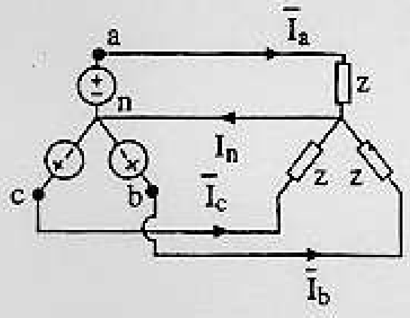
Problem 1 (40 pts.)

(a) Given $\bar{V}_{an} = 100\angle 10^\circ$ V, $Z = 5\angle 25^\circ\,\Omega$, and the following circuit in a-b-c sequence. Find $\bar{V}_{ab}, \bar{I}_a, \bar{I}_n$ and total complex power delivered by this three-phase circuit.



$$\bar{V}_{ab} = \bar{V}_{an}(\sqrt{3}\angle 30^\circ) = \boxed{100\sqrt{3}\angle 40^\circ \text{ V}}$$

$$\bar{I}_a = \frac{\bar{V}_{an}}{Z} = \frac{100\angle 10^\circ}{5\angle 25^\circ} = \boxed{20\angle -15^\circ \text{ A}}$$

$$\bar{I}_n = 0$$

$$\bar{S}_T = 3\,\bar{V}_{an}\,\bar{I}_a^*$$

$$= 3(100\angle 10^\circ)(20\angle 15^\circ) = \boxed{6000\angle 25^\circ \text{ VA}}$$

(b) A single phase, 60 Hz, 1000 V voltage source supplies a load of 10 kVA at 0.8 PF lagging. Find the value of capacitance in Farads of a capacitor connected in parallel with the load to improve the overall PF to 0.9 lagging.

Before adding capacitor:

$$\bar{S} = 10(0.8 + j0.6) = 8 + j6 \text{ kVA}$$

$$\Rightarrow P = 8 \text{ kW}, \quad Q^{old} = 6 \text{ kVAr}$$

After adding capacitor:

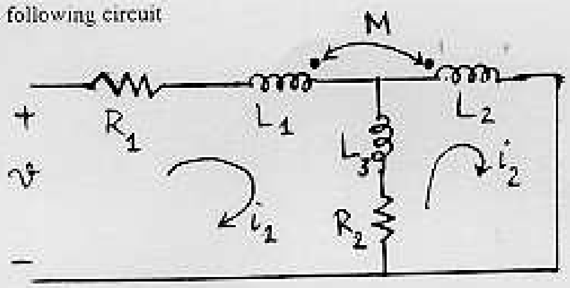
$$Q^{new} = P\tan(\cos^{-1}(0.9)) = 3.875 \text{ kVAr}$$

$$Q^{cap} = Q^{new} - Q^{old} = -2.125 \text{ kVAr}$$

$$|Q^{cap}| = V^2 \omega C$$

$$C = \frac{|Q^{cap}|}{V^2\omega} = \frac{2125}{1000^2 \times 2\pi 60} = \boxed{5.64 \times 10^{-6} \text{ F}}$$

(c) Write the loop equations for the following circuit



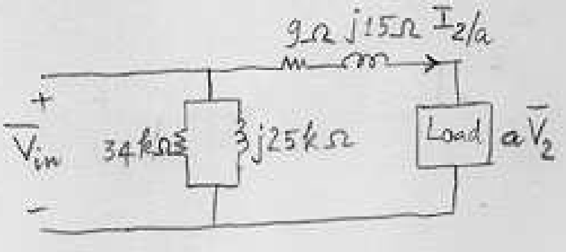
$$v = R_1 i_1 + L_1 \frac{di_1}{dt} + L_3 \frac{d}{dt}(i_1 - i_2) + R_2(i_1 - i_2) - M\frac{di_2}{dt}$$

$$0 = R_2(i_2 - i_1) + L_3 \frac{d}{dt}(i_2 - i_1) + L_2 \frac{di_2}{dt} - M\frac{di_1}{dt}$$

(d) A 10 kVA, 2400/240 V transformer has $R_C = 34\ k\Omega$, $X_m = 25\ k\Omega$, $R_{eq} = 9\Omega$, $X_{eq} = 15\ \Omega$ (all quantities referred to the HV side). The transformer is supplying a load of 8 kVA at 0.9 PF leading at rated voltage of 240 V. Find voltage regulation and efficiency of this transformer at the given load.

$$a = \frac{2400}{240} = 10$$

$$\bar{V}_2 = 240\angle 0^\circ\ V$$



$$\bar{I}_2/a = \left(\frac{\bar{S}_L}{a\bar{V}_2}\right)^* = \left(\frac{8000\angle -\cos^{-1}0.9}{2400\angle 0^\circ}\right)^* = 3.333\angle 25.84^\circ\ A$$

$$\bar{V}_{in} = a\bar{V}_2 + (9+j15)\bar{I}_2/a = 2400 + (9+j15)(3.333\angle 25.84^\circ)$$

$$= 2405.9\angle 1.38^\circ\ V$$

$$VR = \frac{V_{no\ load} - V_{load}}{V_{load}} = \frac{2405.9 - 2400}{2400} = \boxed{0.25\%}$$

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_i} = \frac{8000 \times 0.9}{8000 \times 0.9 + 9 \times 3.333^2 + 2405.9^2/34000} = \boxed{96.4\%}$$

(e)

(i) Put the following equation in state space form by defining $x_1 = x$, $x_2 = \dot{x}$, and $x_3 = \ddot{x}$.

$\dddot{x} - 2\ddot{x} + 5\dot{x} - x + x^3 = 0$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = 2x_3 - 5x_2 + x_1 - x_1^3$$

(ii) For the system in (i) with $x_1(0) = 1$, $x_2(0) = 0.5$, and $x_3(0) = 0$, using Euler's method with step size $\Delta t = 0.1$ to compute $x_1(0.1)$, $x_2(0.1)$, and $x_3(0.1)$.

$$x_1(0.1) = x_1(0) + \Delta t\,(x_2)\big|_{t=0} = 1 + 0.1 \times 0.5 = \boxed{1.05}$$
$$x_2(0.1) = x_2(0) + \Delta t\,(x_3)\big|_{t=0} = 0.5 + 0.1 \times 0 = \boxed{0.5}$$
$$x_3(0.1) = x_3(0) + \Delta t\,(2x_3 - 5x_2 + x_1 - x_1^3)\big|_{t=0}$$
$$= 0 + 0.1\,(2 \times 0 - 5 \times 0.5 + 1 - 1^3) = \boxed{-0.25}$$

(iii) Find the equilibrium point(s) of this system for $x_1 > 0$.

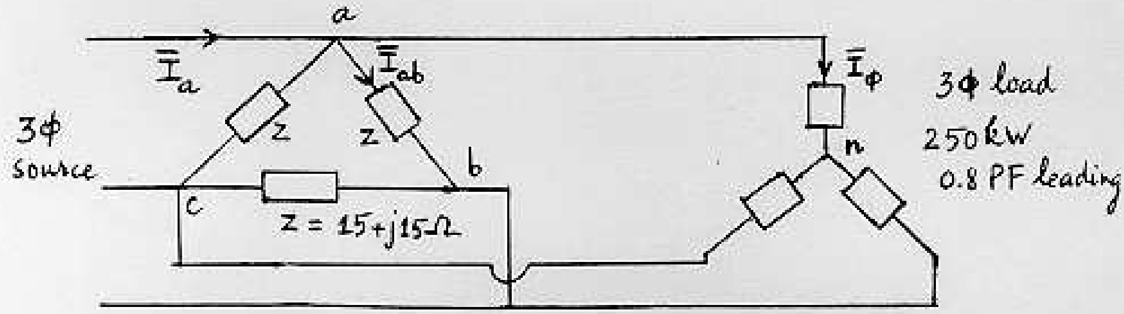
$$\begin{cases} 0 = x_2 \\ 0 = x_3 \\ 0 = 2x_3 - 5x_2 + x_1 - x_1^3 \end{cases} \Rightarrow \boxed{\begin{array}{l} x_1^e = 1 \\ x_2^e = 0 \\ x_3^e = 0 \end{array}}$$

(iv) Linearize the system around the equilibrium point(s) found in (iii).

$$\Delta\dot{x}_1 = \Delta x_2$$
$$\Delta\dot{x}_2 = \Delta x_3$$
$$\Delta\dot{x}_3 = (1 - 3x_1^2)\big|_e \Delta x_1 - 5\,\Delta x_2 + 2\,\Delta x_3$$

$$\boxed{\begin{array}{l} \Delta\dot{x}_1 = \Delta x_2 \\ \Delta\dot{x}_2 = \Delta x_3 \\ \Delta\dot{x}_3 = -2\Delta x_1 - 5\Delta x_2 + 2\Delta x_3 \end{array}}$$

Problem 2 (15 pts.)



(a) (10 pts.) The line-line voltage is 460 V at 60 Hz. Phase sequence is a-b-c. Find $\overline{I}_\phi, \overline{I}_{ab}, \overline{I}_a$. Take $\overline{V}_{an}$ as reference.

$\overline{V}_{ab} = 460\angle 30^\circ\ V \quad , \quad \overline{I}_{ab} = \dfrac{\overline{V}_{ab}}{Z} = \dfrac{460\angle 30^\circ}{15+j15} = \boxed{21.68\angle -15^\circ\ A}$

$\Rightarrow \overline{I}_{ca} = 21.68\angle 105^\circ\ A$

$\overline{I}_\phi = \dfrac{250\times 10^3}{\sqrt{3}\times 460\times 0.8}\angle \cos^{-1} 0.8 = \boxed{392.22\angle 36.87^\circ\ A}$

$\overline{I}_a = \overline{I}_{ab} - \overline{I}_{ca} + \overline{I}_\phi = 21.68\angle -15^\circ - 21.68\angle 105^\circ + 392.22\angle 36.87^\circ$

$= \boxed{399.26\angle 31.53^\circ\ A}$

(b) (5 pts.) Find total complex power supplied by the source.

$\overline{S}_{3\phi,\Delta} = 3\dfrac{V_{ab}^2}{Z^*} = 3\,\dfrac{460^2}{15-j15} = 21.16 + j21.16\ kVA$

$\overline{S}_{3\phi,Y} = \dfrac{250}{0.8}(0.8 - j0.6) = 250 - j187.5\ kVA$

$\overline{S}_T = \overline{S}_{3\phi,\Delta} + \overline{S}_{3\phi,Y} = \boxed{271.16 - j166.34\ kVA}$

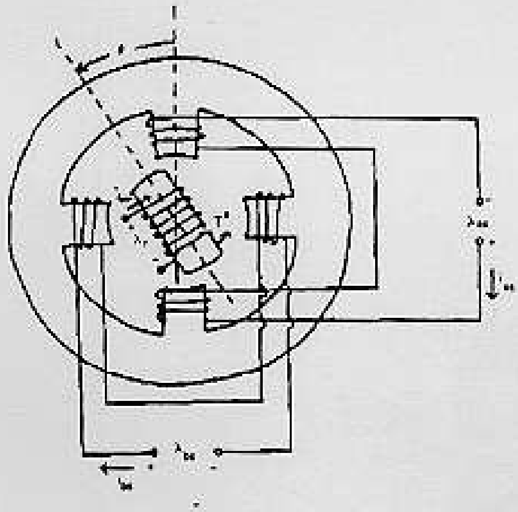
Problem 3 (15 pts.)

(a) The smooth air gap machine shown below has the following electrical terminal relations:

$\lambda_{as} = (L_0 + L_1 \cos 2\theta) i_{as} + (M \cos \theta) i_r$

$\lambda_{bs} = (L_0 - L_1 \cos 2\theta) i_{bs} + (M \sin \theta) i_r$

$\lambda_r = (M \cos \theta) i_{as} + (M \sin \theta) i_{bs} + (L_2 \cos 4\theta) i_r$



(i) (3 pts.) Find the co-energy W'_m.

(ii) (3 pts.) Find the torque of electric origin T^e.

(b) The energy of an electromechanical system is given by $W_m(x,\lambda) = 5x\lambda^2$.

(i) (3 pts.) Find the energy transferred from the mechanical system to the coupling field (i.e. the EFM) as x moves from 1m to 2m and current i follows the path $i = 8x^2$ amps.

(ii) (3 pts.) Find the energy transferred from the electrical system (i.e. the EFE) over the same path.

(iii) (3 pts.) Use the formula given for $W_m(x,\lambda)$ directly to confirm that the total change in energy $\Delta W_{m,(1\to2)}$ over the prescribed path is given by

$$\Delta W_{m,(1\to2)} = EFM_{(1\to2)} + EFE_{(1\to2)}$$

Problem 4 (15 pts.)

A 1 MVA, 6600 V_{LL}, 10 pole, wye-connected, 60 Hz, 3-phase synchronous motor has $x_s = 15\,\Omega$. I_f and the load are such that the stator current is 95 A at a power factor of 0.8 leading with rated voltage at the terminals. Neglect all losses.

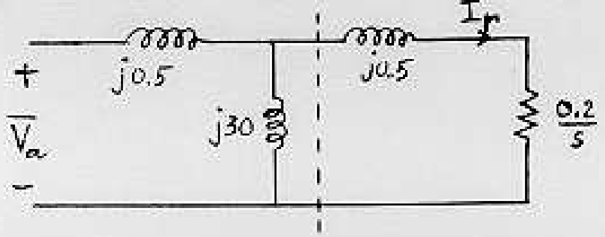
(a) (4 pts.) Compute δ and T^e.

(b) (3 pts.) Compute the speed of rotation of the motor in revolution per minute (rpm).

(c) (4 pts.) Compute the reactive power drawn from the power supply by the motor.

(d) (4 pts.) Draw the phasor diagram that identifies $\bar{I}_a$, $\bar{V}_a$, $\bar{E}_{ar}$, δ, and the power factor angle θ.

Problem 5 (15 pts.)
The per phase equivalent circuit of a 3 phase, 4 pole, 60 Hz, 460 V (line-line) of a wound rotor induction motor is given below.



(a) (3 pts.) Find the Thevenin equivalent circuit to the left of the dotted line.

$$jX_T = (j0.5) /\!/ (j30) = \frac{j0.5 \times j30}{j30.5} = \boxed{j0.492\ \Omega}$$

$$V_T = V_a \frac{j30}{j30 + j0.5} = \frac{460}{\sqrt{3}} \cdot \frac{30}{30.5} = \boxed{261.23\ V}$$

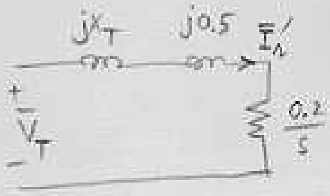
(b) (3pts.) If the motor runs at 1740 RPM, find the value of slip s.

$$N_s = \frac{120 f}{P} = \frac{120 \times 60}{4} = 1800\ RPM$$

$$s = \frac{N_s - N_{act}}{N_s} = \frac{1800 - 1740}{1800} = \boxed{0.0333} = \frac{1}{30}$$

(c) (3 pts.) Find $\bar{I}_r'$ and the power P_{ag} across the air gap.

$$\bar{I}_r' = \frac{261.23 \angle 0^\circ}{\frac{0.2}{1/30} + j(0.492 + 0.5)} = \boxed{42.955 \angle -9.39^\circ\ A}$$



$$P_{ag} = 3 \times 42.955^2 \cdot \frac{0.2}{1/30} = \boxed{33.212\ kW}$$

(d) (3 pts.) Find mechanical power developed P_m.

$$P_m = P_{ag}(1 - s) = 33.212 \times (1 - 1/30) = \boxed{32.105\ kW}$$

(e) (3 pts.) Find the torque of electric origin.

$\omega_s = 120\pi$,

$$T^e = \frac{P_m}{\omega_m} = \frac{32105}{\omega_s(1-s)/P/2} = 2 \frac{32105}{120\pi(1 - 1/30)} = \boxed{176.2\ N\text{-}m}$$