ECE 430 Exam #2, Spring 2010 Name: Solution
90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. ________ / 25 2. ________ / 25

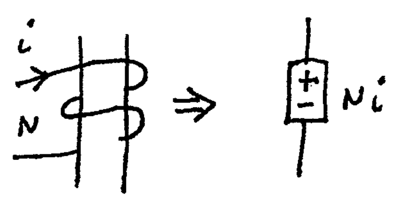
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^*$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\Re = \frac{l}{\mu A}$ $MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $B = \mu H$ $\phi = BA$ $\lambda = N\phi$ $\lambda = Li$ (if linear)

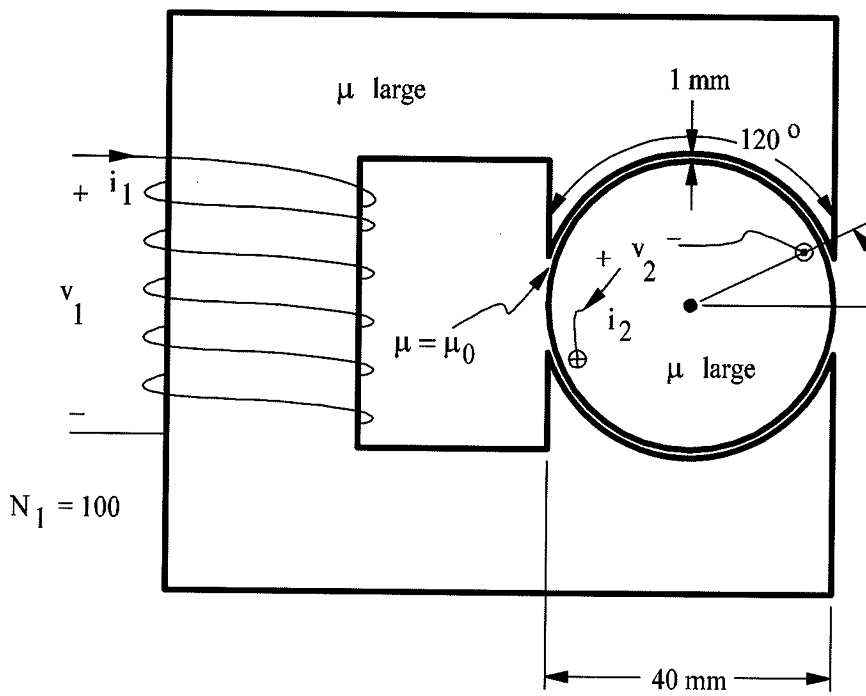


$W_m = \int_0^{\lambda} i \, d\hat{\lambda}$ $W_m' = \int_0^{i} \lambda \, d\hat{i}$ $W_m + W_m' = \lambda i$ $f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $x \to \theta$

$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i \, d\lambda$ $\underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\lambda = \frac{\partial W_m'}{\partial i}$ $i = \frac{\partial W_m}{\partial \lambda}$

Problem 1. (25 points)



This structure is 100 mm long, into the paper.

$g = .001\,m$

$r = 20\,mm = .02\,m$

$l = .1\,m$

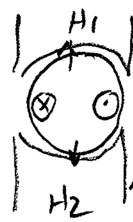
a) For the structure above, find $\lambda_1(i_1, i_2, \theta)$ and $\lambda_2(i_1, i_2, \theta)$. The winding resistances are small. You may use sinusoidal approximations for position effects as appropriate. (15 points)

b) Let $i_1 = 10$ A. If $d\theta/dt = 120\pi$ rad/s, what is the voltage $v_2(t)$ when $i_2 = 0$? (10 points)

a) $\lambda_1 = L_1 i_1 + M\cos\theta\, i_2$

$\lambda_2 = M\cos\theta\, i_1 + L_2 i_2$

$\theta = 0$



$H_1 g - H_2 g = -N_1 i_1 - N_2 i_2$

$\mu_0 H_1 \left(\frac{2\pi}{3} r l\right) + \mu_0 H_2 \left(\frac{2\pi}{3} r l\right) = 0$

$H_1 = -H_2 = -\frac{N_1 i_1}{2g} - \frac{N_2 i_2}{2g}$

$\phi_{cw} = \mu_0 H_2 \frac{2\pi}{3} r l$

$\lambda_1 = N_1 \phi_{cw} = \frac{\mu_0 \frac{2\pi}{3} r l N_1^2}{2g} i_1 + \frac{\mu_0 N_1 N_2 \frac{2\pi}{3} r l}{2g} i_2$

$= .0263\, i_1 + .0263\, i_2$

$\lambda_1 = .0263\, i_1 + .0263 \cos\theta\, i_2$

$\lambda_2 = .0263 \cos\theta\, i_1 + .0263\, i_2$

b) $v_{2_{oc}} = -.0263 \sin\theta \times 120\pi \times 10$

$= -99.15 \sin\theta$ volts

For $\theta = 120\pi t + \theta(0)$

$v_{2_{oc}} = 99.15 \cos(120\pi t + \theta(0) + 90^\circ)$

Problem 2. (25 points.)

A mathematical model of an electromechanical system is:

$\lambda_1 = (a/x)\, i_1 + (b/x)\, i_3$

$\lambda_2 = (c/x)\, i_2 + (d/x)\, i_3$

$\lambda_3 = (e/x)\, i_1 + (f/x)\, i_2 + (g/x)\, i_3$

$$L = \frac{1}{x}\begin{bmatrix} a & 0 & b \\ 0 & c & d \\ e & f & g \end{bmatrix}$$

a) If this relationship came from a conservative coupling field, what can you say about the constants a, b, c, d, e, f, g? (7 points)
b) Find an expression for the force of electrical origin in the positive x direction (10 points)
c) Find an expression for the energy stored in the coupling field in terms of the currents plus x and the given parameters (8 points)

a) Symmetry of L gives $b=e$ and $d=f$

b) $W_m' = \frac{1}{2}\frac{a}{x} i_1^2 + \frac{1}{2}\frac{c}{x} i_2^2 + \frac{e}{x} i_1 i_3 + \frac{f}{x} i_2 i_3 + \frac{g}{x}\frac{i_3^2}{2}$

$$f^e = -\frac{1}{x^2}\left[\frac{1}{2} a i_1^2 + \frac{1}{2} c i_2^2 + e i_1 i_3 + f i_2 i_3 + g \frac{i_3^2}{2}\right]$$

c) $W_m = W_m'$ because it is linear

$=$ see above

Problem 3. (25 points)

The flux linkage-current relationship for a rotational electromechanical system with one electrical input and one mechanical input is given as:

$$\lambda = (1 + \cos 2\theta)i$$

a) Find the maximum possible torque when i = 1 Ampere (3 points).
b) Find the energy stored in the coupling field (W_m) when the angle is 90 degrees and i =1 Ampere. (3 points)
c) Find the energy transferred from the electrical system into the coupling field when the current stays at i = 1 Ampere while the angle changes from zero to 90 degrees. (9 points)
d) Find the energy transferred from the mechanical system into the coupling field when the current stays at i=1 Ampere while the angle changes from zero to 90 degrees. (10 points)

a) $W_m' = \frac{1}{2}(1+\cos 2\theta)i^2 \qquad T^e = -\sin 2\theta\, i^2$

$$T^e_{max}\Big|_{i=1} = +1 \quad N.m$$

b) $W_m\Big|_{\substack{\theta=90^\circ \\ i=1}} = W_m' = \frac{1}{2}(1-1)1^2 = 0\,J$

c) $EFE_{a-b} = \int_{\lambda_a}^{\lambda_b} i\Big|_{\substack{path \\ i=1}} d\lambda = \int_{(1+1)1^2}^{(1-1)1^2} 1\, d\lambda = \int_2^0 1\, d\lambda = \lambda\Big|_2^0 = -2\,J$

d) $EFm_{a-b} = \int_0^{90^\circ} -T^e\Big|_{i=1} d\theta = \int_0^{90^\circ} \sin 2\theta\, d\theta = -\frac{1}{2}\cos 2\theta\Big|_0^{90^\circ} = \frac{1}{2}+\frac{1}{2} = 1\,J$

Check $(-2)+(1) = W_{m_b} - W_{m_a} = 0 - 1 = -1$ ✓

Problem 4. (25 points)

A machine with one coil on the stator and one coil on the rotor has the two flux linkage vs current relationships as:

$$\lambda_r(i_r, i_s, \theta) = 3.6 i_r + 3 i_s cos\theta$$
$$\lambda_s(i_r, i_s, \theta) = 3 i_r cos\theta + 1.8 i_s$$

The machine is mounted horizontally and a pendulum is attached to the shaft. The pendulum exerts a torque on the shaft equal to $T_L = +6cos\theta$ N-m. Newton's second law for this machine is $J\frac{d^2\theta}{dt^2} = T^e + T_L$. The motor is excited with dc currents on both the rotor and stator. The rotor current, i_r, is 2 A.

a) A point where the sum of the two torques is zero is found at $\theta = 45°$. What is the stator current, i_s ? (10 points)
b) With the stator and rotor currents as in part a), are there any other points where the sum of the two torques is zero? If so, what are the values of θ for these other points? (15 points)

a) $w_m' = 1.8 i_r^2 + 3 i_r i_s \cos\theta + 0.9 i_s^2$

$T^e = -3 i_r i_s \sin\theta$

$-3 \times 2 \times i_s \sin 45° + 6 \cos 45° = 0$

$-6 i_s (.707) + 6(.707) = 0$

$\underline{i_s = 1\ A}$

b) $-6\sin\theta + 6\cos\theta = 0$ or $\sin\theta = \cos\theta$