ECE 430 Exam #2, Fall 2010 Name: Solution

90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. ________ / 25 2. ________ / 25

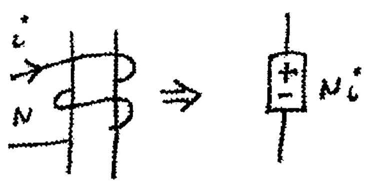
3. ________ / 25 4. ________ / 25 Total ________ / 100

## Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\qquad \bar{V} = \bar{Z}\bar{I}$ $\qquad \bar{S} = \bar{V}\bar{I}^*$ $\qquad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\qquad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\qquad \Re = \frac{l}{\mu A}$ $\qquad MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $\qquad B = \mu H$ $\qquad \phi = BA$ $\qquad \lambda = N\phi$ $\qquad \lambda = Li$ (if linear)



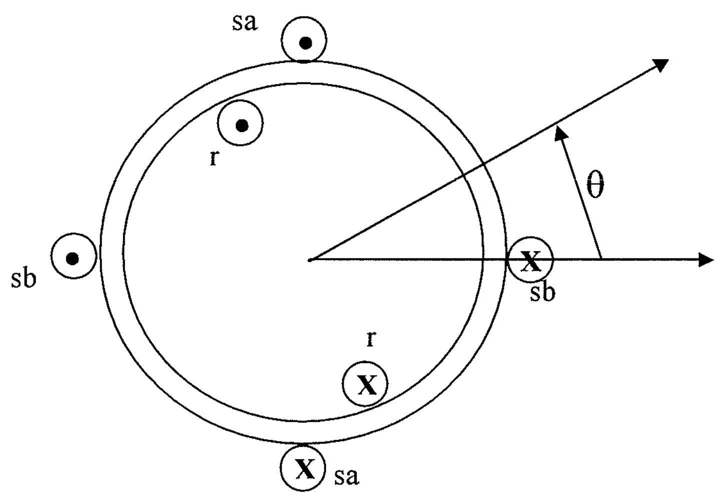

$W_m = \int_0^\lambda i \, d\hat{\lambda}$ $\qquad W_m' = \int_0^i \lambda \, d\hat{i}$ $\qquad W_m + W_m' = \lambda i$ $\qquad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\qquad x \rightarrow \theta$

$f^e \rightarrow T^e$

$\underset{a \rightarrow b}{EFE} = \int_a^b i \, d\lambda$ $\qquad \underset{a \rightarrow b}{EFM} = -\int_a^b f^e dx$ $\qquad \underset{a \rightarrow b}{EFE} + \underset{a \rightarrow b}{EFM} = W_{mb} - W_{ma}$ $\qquad \lambda = \frac{\partial W_m'}{\partial i}$ $\qquad i = \frac{\partial W_m}{\partial \lambda}$

**Problem 1. (25 points)**

Assume the two stator coils (sa and sb) shown below are identical and the rotor coil has a resistance of 1.5 Ohms.




a) Write down $\lambda_{sa}(i_{sa},i_{sb},i_r,\theta)$, $\lambda_{sb}(i_{sa},i_{sb},i_r,\theta)$, and $\lambda_r(i_{sa},i_{sb},i_r,\theta)$ by inspection using inductance parameters $L_s$, $L_r$, and M. You may use sinusoidal approximations for position effects as appropriate and you should assume the positive polarity for the flux linkages and voltages is on the "X" mark of each coil. (9 points)

b) Find an expression for the torque of electrical origin in the positive θ direction in terms of the currents, θ, and the mutual inductance between stator and rotor (M). (8 points)

c) ~~b)~~ Let $i_r$ = 8 A (DC). If $d\theta/dt = 120\pi$ rad/s, what are the voltages $v_{sa}(t)$ and $v_{sb}(t)$ when $i_{sa}=i_{sb}=$ 0 in terms of the mutual inductance M and the angle θ? (8 points)

a) $\lambda_{sa} = L_s i_{sa} + M\cos\theta\, i_r$

$\lambda_{sb} = L_s i_{sb} + M\sin\theta\, i_r$

$\lambda_r = M\cos\theta\, i_{sa} + M\sin\theta\, i_{sb} + L_r i_r$

b) $W_m' = \left(\frac{1}{2}L_s i_{sa}^2\right) + \left(\frac{1}{2}L_s i_{sb}^2\right) + \left(M\cos\theta\, i_{sa} i_r + M\sin\theta\, i_{sb} i_r + \frac{1}{2}L_r i_r^2\right)$

(extra space on next page)

$T^e = -M\sin\theta\, i_{sa} i_r + M\cos\theta\, i_{sb} i_r$

c) $v_{sa} = \frac{d\lambda_{sa}}{dt} = -8M \times 120\pi \sin\theta$ $\qquad$ $v_{sb} = \frac{d\lambda_{sb}}{dt} = 8M \times 120\pi \cos\theta$

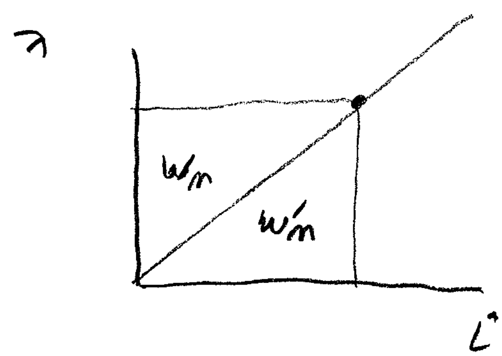
**Problem 2. (25 points.)**

a) Explain in your own words why it is usually easier to use coenergy (Wm') rather than the energy stored in the coupling field (Wm) to compute the force or torque of electrical origin.

Because you compute $\lambda$ in terms of $i$ when you do the magnetic analysis. It comes directly from laws.

If you use $W_m$, then you must solve for $i$ in terms of $\lambda$.

b) Show graphically why the energy stored in the coupling field and the coenergy are equal for magnetically linear systems.




c) Why is the integral for energy stored in the coupling field done along a constant x (or theta) path while the EFE integral might not be?

when computing $W_m$, you need to integrate the force over x while the current and flux linkage are zero. Therefore when you integrate current over flux linkage, you must keep x at its final value.

EFE is the integral of $i$ over $\lambda$ along a specific path which could involve changing x.

**Problem 3. (25 points)**

An electromechanical system has the following flux linkage-current relationship:

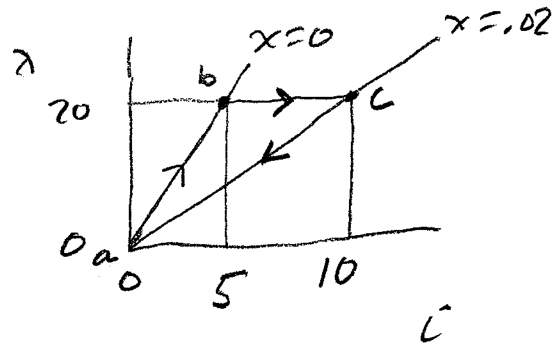
$$\lambda = \frac{0.08}{(0.02+x)} i$$

In the following questions, EFE stands for "Energy From the Electrical system into the coupling field", and EFM stands for "Energy From the Mechanical system into the coupling field".

Consider the following to be point a: x = 0, i = 0, λ = 0.

a) Find the EFE and EFM as the system is moved along constant x from point a to point b which has i = 5 Amps.
b) Find the EFE and EFM as the system is moved along constant λ from point b to point c which has x = 0.02 meters.
c) Find $W_m$ at point c
d) Find the EFE and EFM as the system is moved along constant x from point c to point d which has λ = i = 0.
e) For this cycle, is this a motor or a generator? Explain why.

a)




$$w_m' = \frac{0.04 i^2}{.02+x} \qquad f^e = -\frac{.04 i^2}{(.02+x)^2}$$

$$\underset{a-b}{EFE} = \int_0^{20} \frac{\lambda}{4} d\lambda = \frac{\lambda^2}{8}\Big|_0^{20} = 50 \text{ J}$$

$$w_{m_a} = 0 \qquad w_{m_b} = \int_0^{20} \frac{\lambda}{4} d\lambda = 50 J \qquad \underset{a-b}{EFM} = 0 \text{ J}$$

b) $$\underset{b-c}{EFE} = \int_{20}^{20} \frac{(.02+x)\lambda}{.08} d\lambda = 0 \qquad \underset{b-c}{EFM} = -\int_0^{.02} \frac{-.04}{(.02+x)^2}\left(\frac{(.02+x)20}{.08}\right)^2 dx = 50 J$$

c) $w_{m_c} = \frac{1}{2} 20 \times 10 = 100 J$

d) $$\underset{c-d}{EFE} = \int_{20}^{0} \frac{\lambda}{2} d\lambda = -100 J \qquad \underset{c-d}{EFM} = 0 \qquad w_{m_d} - w_{m_c} = -100 J$$

e) $$\underset{cycle}{EFE} = 50 + 0 - 100 = -50 J$$ so Generator $\left(\text{or } \underset{cycle}{EFM} = 50J\right)$

From problem 3, $\lambda = \frac{.08}{.02+x} i$

**Problem 4. (25 points)**

The movable mass of the electromechanical system of problem 3 has a mass of 0.5 kg. The electrical system is producing a current of 4 Amps (DC) and the wires have a total resistance of 3 Ohms. There is an external force equal to 200 Newtons being applied in the positive x direction and the mass is stationary under these conditions.

a) What is the equilibrium value of x for the condition described above (constrained to be positive)?
b) Describe as best you can what happens if the applied voltage is suddenly changed from 12 Volts to 24 Volts (DC).
c) Solve the electrical and mechanical dynamic equations of motion using Euler's method for one time step of 0.1 seconds to see what happens after the moment of suddenly changing the voltage.

a) $f^e \Big| = -\frac{.04 i^2}{(.02+x)^2}$ from problem 3

$$0 = 200 - \frac{.04 \times 4^2}{(.02+x_e)^2} \qquad 200 = \frac{.64}{(.02+x_e)^2}$$

$$(.02+x_e)^2 = \frac{.64}{200} \qquad .02+x_e = \pm .0566$$

$$x_e > 0 \qquad \boxed{x_e = .0366 \text{ m}}$$

b) The current will increase to 8 Amps
The $f^e$ will increase which will start the mass moving in the negative x direction, It will not stop until $x = 0$ (slam shut).

c) $v = iR + \frac{d\lambda}{dt}$ $\qquad 24 = 3i + \frac{.08}{(.02+x)}\frac{di}{dt} - \frac{.08 i v}{(.02+x)^2}$ $\qquad i(0) = 4$

$\frac{dx}{dt} = v$ $\qquad 0.5\frac{dv}{dt} = 200 - \frac{.04 i^2}{(.02+x)^2}$ $\qquad v(0) = 0$

$x(0) = .0366$ m

$$i(0.1) = 4 + \frac{.02+.0366}{.08}(24 - 3\times 4 + 0)\times 0.1 = 4.849 \text{ A}$$

$$x(0.1) = .0366 + 0 \times 0.1 = .0366 \text{ m}$$

$$v(0.1) = 0 + \frac{1}{0.5}\left(200 - \frac{.04 \times 4^2}{(.02+.0366)^2}\right)\times .1 = 0$$