Name: KEY

ECE 430

Exam #2

Fall 2004

Wednesday, November 10, 2004
7 to 8:30 pm

Closed book, closed notes
Two 8.5" by 11" inch note sheets and calculators allowed

1. ________ / 25

2. ________ / 25

3. ________ / 25

4. ________ / 25

Total ________ / 100

1. **(25 points total)**

The system shown is intended to manipulate the vertical position of a work piece in air, shown here as a mass m. Values are as follows:

Gravity g ~ 10 m/s^2
Mass m = 1 kg
Damper B = 100 N/(m/s)
Spring constant k = 200 N/m
N = 200 turns, R = 1 ohm

With no current, the work piece stabilizes at a vertical position such that the air gap is 2 cm.

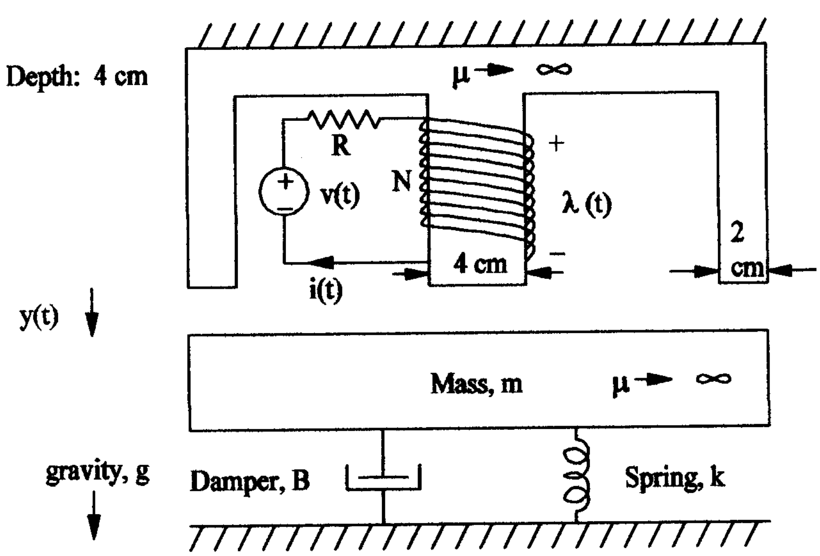
(3 pts) a) Draw a magnetic circuit that represents the magnetic field system. Show the values of the circuit components.

(5 pts) b) Find the flux linkage for the coil in terms of i, y, and other parameters as given.

(5 pts) c) Find either the co-energy or energy in this system, then find the force of electrical origin acting on the work piece.

(5 pts) d) Write the state equations that describe the system.

(7 pts) e) What current will be needed to produce an equilibrium position with a 5 mm gap? Is this position stable?



No current

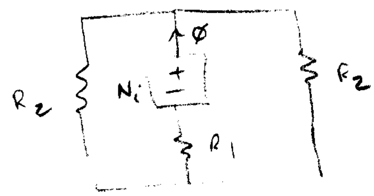
$mg = k(y - \ell)$

$10 = 200(0.02 - \ell)$

$\ell = -0.03$

Extra Blank Sheet (please indicate which problem)

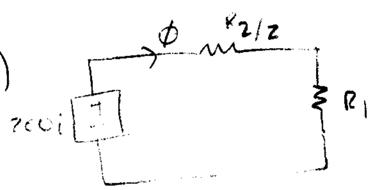
a)



$R_2 = \dfrac{Y}{\mu(0.02)(0.04)}$ $\quad R_1 = \dfrac{Y}{\mu(0.04)^2}$ $\quad R_2 = 2R_1$

$R_2 = 9.95e^8 Y$ $\quad R_1 = 4.97e^8 Y$

b)



$\phi = \dfrac{Ni}{(R_2/2 + R_1)} = \dfrac{Ni}{2R_1} = 2.01e^{-7}\dfrac{i}{Y}$

$\lambda = N\phi = \boxed{4.02\times10^{-5}\dfrac{i}{Y}}$

c) $W_m' = \frac{1}{2}(4.02\times10^{-5})\dfrac{i^2}{Y} = \boxed{2.01\times10^{-5}\dfrac{i^2}{Y}}$ $\quad f^e = \dfrac{\partial W_m'}{\partial Y} = \boxed{-2.01\times10^{-5}\dfrac{i^2}{Y^2}}$ (up)

d) KVL: $v = iR + \dfrac{d\lambda}{dt}$ $\quad$ Newton: $M\ddot{y} = Mg - K(y-\ell) - B\dot{y} + f^e$

$\ddot{y} = \frac{1}{M}\left[Mg - K(y-\ell) - B\dot{y} + f^e\right]$

State Space:

$\dot{y} = v$

$\dot{v} = g - 200(y-\ell) - 100v - 2.01\times10^{-5}\dfrac{i^2}{y^2}$

$\dot{i} = \dfrac{(v - iR)Y}{4.02\times10^{-5}} + \dfrac{v}{Y}$

e) Equilibrium:

$v = 0$,

$0 = (1)10 - 200(y^e + 0.03) - \cancel{100v}^{0} - 2.01\times10^{-5}\dfrac{i^2}{y^{e2}} \Rightarrow i^2 = \dfrac{3}{.804} \Rightarrow \boxed{i^e = 1.93\text{ A}}$

Stability

$A = \left.\begin{bmatrix} 0 & 1 \\ \dfrac{2.01\times10^{-5}i^2}{y^3} - K_y & -B \end{bmatrix}\right|_{eq} = \begin{bmatrix} 0 & 1 \\ 598 & -100 \end{bmatrix}$ $\quad |\lambda I - A| = 0$

$\begin{vmatrix} \lambda & 1 \\ 598 & \lambda + 100 \end{vmatrix} = 0$ $\quad \lambda^2 + 100\lambda - 598 \Rightarrow \boxed{\text{Unstable}}$

2. (25 points total)

A rotating electric machine has the following electric terminal relationships for the stator and rotor flux linkages:

$$\lambda_s = (L_s \cos 2\theta)\, i_s + (M \sin\theta)\, i_r$$

$$\lambda_r = (M \sin\theta)\, i_s + (L_r \cos 2\theta)\, i_r$$

where θ is the angle of the rotor, i_s is the current in the stator winding, and i_r is the current in the rotor winding. L_s, L_r, and M are constant inductance terms.

(15 pts) a. Find the torque of electric origin acting on the rotor in terms of θ, i_s, i_r, L_s, L_r, and M.

(10 pts) b. If the machine is operated $i_r = I_{dc}$ (i.e., a constant dc rotor current) and spun at a speed $\omega = d\theta/dt$, what is the stator terminal voltage, $v_s(t)$, in terms of the various parameters and currents?

a. $$W_m' = L_s \cos 2\theta \frac{i_s^2}{2} + M \sin\theta\, i_s i_R + L_R \cos 2\theta \frac{i_R^2}{2}$$

$$T^e = \frac{\partial W_m'}{\partial \theta} = -2L_s \sin 2\theta \frac{i_s^2}{2} + M \cos\theta\, i_s i_R - 2L_R \sin 2\theta \frac{i_R^2}{2}$$

b. LET $\theta = \omega t + \gamma$

$i_R = I_{DC}$

$$v_s = \frac{d\lambda_s}{dt} + i_s R = -2L_s \sin 2\theta \frac{d\theta}{dt} i_s + L_s \cos 2\theta \frac{di_s}{dt} + M \cos\theta \frac{d\theta}{dt} i_R + \cancel{M \sin\theta \frac{di_R}{dt}}$$

$$v_s = -2L_s \sin[2(\omega t+\gamma)]\, \omega i_s + L_s \cos[2(\omega t+\gamma)] \frac{di_s}{dt} + M \cos(\omega t+\gamma)\, \omega I_{DC}$$

3. **(25 points total)**

Assume the state space equations for system are

$$\dot{x}_1 = 2x_1 - 0.02\,x_1x_2$$
$$\dot{x}_2 = 0.01\,x_1x_2 - x_2 + u(t)$$

(13 pts) a) Assume the initial conditions for the system are $x_1(0) = 100$, and $x_2(0) = 50$, and $u(t) = 10\cos 5t$. Using Euler's method with a time step of $\Delta t = 0.05$ second, determine $x_1(0.1)$ and $x_2(0.1)$. That is, solve for the values after the first two time steps.

(12 pts) b) Now assume that $u(t) = 0$. The system of equations has an equilibrium point at the origin ($x_1 = 0$, $x_2 = 0$). Linearize the system about this equilibrium point, determine the eigenvalues of the linearized system, and then tell whether this equilibrium point is stable.

$$x_1(t+\Delta t) \approx x(t) + \Delta t\, \dot{x}_1\big|_t$$
$$x_2(t+\Delta t) \approx x(t) + \Delta t\, \dot{x}_2\big|_t$$

$x_1(0) = 100$
$x_2(0) = 50$
$u(t) = 10\cos 5t$

$$x_1(0.05) \approx 100 + (0.05)(200 - 100) = 105$$
$$x_2(0.05) \approx 50 + \cancel{50}\ 0.05(50 - 50 + 10\cos 0) = 50.5$$

$$x_1(0.1) \approx 105 + 0.05(210 - 106.05) = 110.1975$$
$$x_2(0.1) \approx 50.5 + 0.05(53.025 - 50.5 + 10\cos 0.25) = 51.1107$$

rad

b)
$$\begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 2 & 0 \\ 0 & \lambda + 1 \end{vmatrix}$$
$$= (\lambda - 2)(\lambda + 1)$$
$$= \lambda^2 - \lambda - 2$$

EIGENVALUES: 2, −1

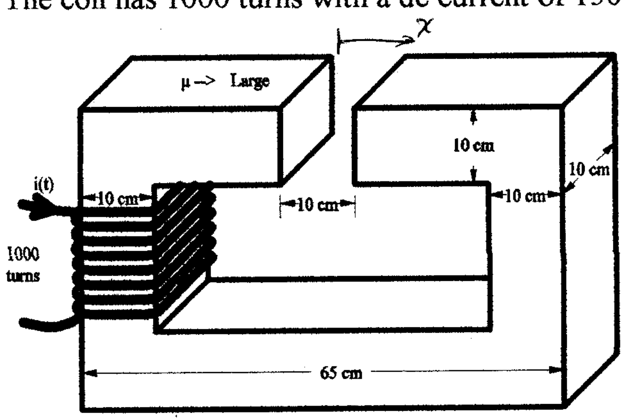
UNSTABLE

4. **(Short answer - 25 points total)**

(8 pts) a) Explain briefly how a synchronous electric motor works.

In a poly phase machine, a rotating field is developed on the stator. This field will attract a permanent magnet or electromagnet on the rotor, and produces torque if it moves at the same speed. In a single phase machine, non-zero average torque is developed if the rotor PM or EM moves at a speed that matches the electrical input frequency.

(9 pts) b) Calculate the force of electric origin exerted on the piece on the right side of the air gap. The height and depth of the material is 10 cm. Even though the air gap is rather long you may neglect fringing. Assume the core permeability is infinite. The coil has 1000 turns with a dc current of 150 amps.



$$\mathcal{R} = \frac{l}{\mu A} = \frac{x}{\mu_0(0.01m^2)}$$

$$\lambda = N\phi = \frac{N^2 i}{\mathcal{R}} = \frac{iN^2\mu_0(0.01m^2)}{x}$$

$$W_m' = \frac{N^2 i^2 \mu_0(0.01m^2)}{2x}$$

$$\frac{dW_m'}{dx} = f^e = \frac{-N^2 i^2 \mu_0(0.01m^2)}{2x^2}$$

$$f^e = 14.1\ kN$$

(8 pts) c) The co-energy for a magnetic device is $W_m' = \frac{i^3}{4x}$, find the energy stored in the coupling field W_m.

$$W_m = \lambda i - W_m' \qquad \lambda = \frac{dW_m'}{di} = \frac{3i^2}{4x}$$

$$\lambda i = \frac{3i^3}{4x}$$

$$\text{or } \frac{4}{3}\left(\frac{x}{3}\right)^{1/2}\lambda^{3/2} = W_m = \frac{1}{2}\frac{i^3}{x}$$