Name (Print) SOLUTION.

Section (circle one) 9 am 10 am

ECE330 B&C EXAM #2 Fall 98

Problem 1 (30)

An electromechanical system with three electrical ports and one rotational mechanical port (θ) has the following equations for flux linkages

$$\lambda_1 = L_{11}i_1 + (M\cos\theta)i_3$$
$$\lambda_2 = L_{22}i_2 + (M\sin\theta)i_3$$
$$\lambda_3 = (M\cos\theta)i_1 + (M\sin\theta)i_2 + L_{33}i_3$$

where M, L_{11}, L_{22} and L_{33} are constants.

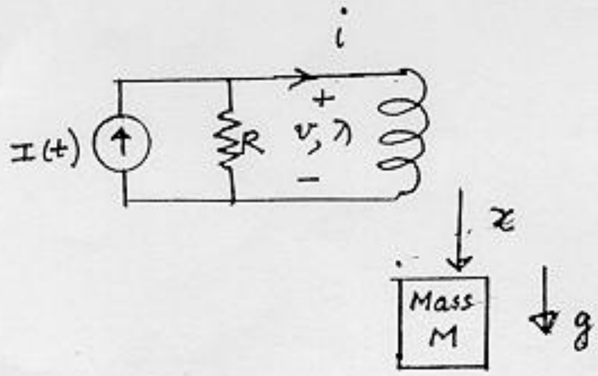
(15) a) Compute the co-energy W'_m.

(15) b) Compute the torque of electric origin $T^e_\theta(i_1, i_2, i_3, \theta)$.

$$W_m' = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + \frac{1}{2}L_{33}i_3^2 + M\cos\theta\, i_1 i_3 + M\sin\theta\, i_2 i_3$$

$$T^e_\theta = -M\sin\theta\, i_1 i_3 + M\cos\theta\, i_2 i_3$$
$$= \frac{\partial W_m'}{\partial \theta}$$

Problem 2 (35)



Distance x of the mass M is measured from a fixed reference. $\lambda(i,x)$ is given as $\lambda(i,x) = \frac{Ci^2}{x}$.

(10) a) Find the force of electric origin.

(15) b) Write the equations of the system in state space form. $I(t)$ is a specified function of time.

(10) c) What is $W_m(\lambda, x)$?

a) $W_m' = \int_0^i \lambda \, di = \frac{Ci^3}{3x}$; $f^e = \frac{\partial W_m'}{\partial x} = -\frac{C}{3}\frac{i^3}{x^2}$

b) $Mg + f^e = M\frac{d^2x}{dt^2}$

$i + \frac{1}{R}\frac{d\lambda}{dt} = I(t)$

$i + \frac{1}{R}\left(-\frac{Ci^2}{x^2}\frac{dx}{dt} + \frac{2Ci}{x}\frac{di}{dt}\right) = I(t)$

$\frac{dx}{dt} = v$

$\frac{dv}{dt} = \frac{1}{M}\left[Mg - \frac{C}{3}\frac{i^3}{x^2}\right]$

$\frac{di}{dt} = \frac{xR}{2Ci}\left[I(t) + \frac{Ci^2}{Rx^2}v - i\right]$

c) $W_m + W_m' = \lambda i$

$W_m = \lambda i - W_m' = \frac{Ci^3}{x} - \frac{Ci^3}{3x} = \frac{2}{3}\frac{Ci^3}{x} = \frac{2}{3}\frac{C}{x}i^3 = \frac{2}{3}\frac{C}{x}\left(i^2\right)^{3/2}$

$= \frac{2}{3}\frac{C}{x}\left(\frac{\lambda x}{C}\right)^{3/2}$

Problem 3 (35)

In the following state space model

$$\dot{x}_1 = x_2 \qquad = f_1(x_1, x_2\, x_3)$$
$$\dot{x}_2 = -0.1x_2x_3 - x_1^3 + x_1 \qquad = f_2(x_1, x_2\, x_3)$$
$$\dot{x}_3 = -2x_2x_3 - x_3 + 5 \qquad = f_3(x_1, x_2\, x_3)$$

Initial conditions are $x_1(0) = 1,\ x_2(0) = 0.5,\ x_3(0) = 10$ at $t = 0$.

(10) a) With a time step of 0.1 sec, find using Euler's method x_1, x_2, x_3 at $t = 0.1$ sec.

(10) b) Find all the possible static equilibrium points.

(15) c) Find the linearized model at any <u>one</u> of the equilibrium points.

$$x_1(0.1) = x_1(0) + \Delta t\, f_1(x_1(0), x_2(0), x_3(0))$$
$$= 1 + 0.1\,(0.5) = \boxed{1.05}$$

$$x_2(0.1) = x_2(0) + \Delta t\, f_2(x_1(0), x_2(0), x_3(0))$$
$$= 0.5 + 0.1\left[-(0.1)(0.5)(10) - 1^3 + 1\right]$$
$$= 0.5 + 0.1\,(-0.5) = \boxed{0.45}$$

$$x_3(0.1) = x_3(0) + 0.1\left[-2(0.5)(10) - 10 + 5\right]$$
$$= 10 + 0.1\,[-15] = \boxed{8.5}$$

Eq. pt $\quad \dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$

$x_3^e = 5, \quad x_2^e = 0 \quad x_1(1 - x_1^2) = 0 \Rightarrow x_1 = (0, +1, -1)$

Eq. pts are.
$$\#1 \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \#2, \quad \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} \#3$$

Linearized Model

$$A = \left.\begin{bmatrix} 0 & 1 & 0 \\ -3x_1^2 + 1 & -0.1x_3 & -0.1x_2 \\ 0 & -2x_3 & -1-2x_2 \end{bmatrix}\right|_0$$

$$\#1 \begin{bmatrix} 0 & 1 & 0 \\ 1 & -0.5 & 0 \\ 0 & -10 & -1 \end{bmatrix} \quad \#2 \begin{bmatrix} 0 & 1 & 0 \\ -2 & -0.5 & 0 \\ 0 & -10 & -1 \end{bmatrix} \quad \#3 \begin{bmatrix} 0 & 1 & 0 \\ -2 & -0.5 & 0 \\ 0 & -10 & -1 \end{bmatrix}.$$