ECE 430 Exam #1, Spring 2009 Name: _____Solution_________________
90 Minutes

Section (Check One) MWF _______ TTH _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. _________ / 25 Total ________ / 100

Useful information

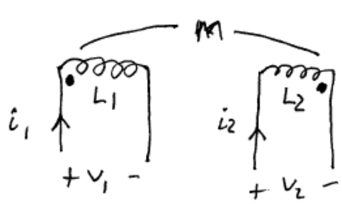
$\sin(x) = \cos(x - 90^\circ)$ $\qquad \overline{V} = \overline{Z}\overline{I}$ $\qquad \overline{S} = \overline{V}\overline{I}^*$ $\qquad \overline{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $\qquad I_L = \sqrt{3} I_\phi$ (delta) $\qquad \overline{Z}_Y = \overline{Z}_\Delta / 3$ $\qquad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

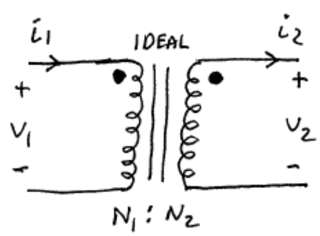
$-180^\circ < \theta < 0$ (lead) $\qquad V_L = \sqrt{3} V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\qquad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\qquad \Re = \frac{l}{\mu A}$ $\qquad MMF = Ni = \phi\Re$

$\phi = BA$ $\qquad \lambda = Li = N\phi$ $\qquad k = \frac{M}{\sqrt{L_1 L_2}}$ $\qquad$ 1 hp = 746 Watts



$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\qquad N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

1. (25 points)

Three single-phase loads are connected in parallel. The load voltage is $v_{load} = 283\sin(120\pi t)$.

The load characteristics are:

Load #1: 100 Amps at 0.8 power factor lag

Load #2: 8 KW at 0.7 power factor lag

Load #3: $(5 + j2)\Omega$

The impedance of each of the two wires that serves this combination of loads is $(0.1 + j0.1)\Omega$.

a) Find the total real and reactive powers provided by the source (10 points)

b) Find the source voltage and current time domain quantities (10 points)

c) Find the overall system power factor (5 points)

$$v_{load} = 283\sin(120\pi t) \Rightarrow \overline{V}_{load} = 200\angle -90$$

Load 1 characteristics:

$$\bar{I}_{L_1} = 100\angle(-90 - 36.87) = 100\angle -126.87$$

$$S_{L_1} = \overline{V}_{load}\bar{I}_{L_1}^{\;*} = 20000\angle 36.87$$

Load 2 characteristics:

$$S_{L_2} = \frac{P}{\cos\theta}\angle\cos^{-1}0.7 = 11420\angle 45$$

$$\bar{I}_{L_2} = \left(\frac{S_{L_2}}{\overline{V}_{load}}\right)^* = 57.1\angle -135$$

Load 3 characteristics:

$$S_{L_3} = \overline{V}_{load}\frac{\overline{V}_{load}^*}{Z_3^*} = \frac{\left|\overline{V}_{load}\right|^2}{Z_3^*} = 7420\angle 21.8$$

$$\bar{I}_{L_2} = \frac{\overline{V}_{load}}{Z_3} = 37.14\angle -111$$

Total current:

$$\bar{I}_{L_{total}} = \bar{I}_{L_1} + \bar{I}_{L_2} + \bar{I}_{L_3} = 192.21\angle -126.25$$

Source voltage:

$$\overline{V}_s = \overline{V}_{load} + 2Z_{line}\bar{I}_{L_{total}} = 253.85\angle -88$$

$$S_{total} = S_{L_1} + S_{L_2} + S_{L_3} + 2\times Z_{line}\times I_{total}^2 = 48790\angle 38.25\text{ ; } P_{total} = 38310\text{ ; } Q_{total} = 30200$$

$$v_s(t) = 253.85\sqrt{2}(\cos(120\pi t - 88))\text{ ; } i_s(t) = 192.21\sqrt{2}(\cos(120\pi t - 126.85))$$

$$PF = \cos(38.25) = 0.78\ lag$$

2. (25 points)

The following three-phase, balanced loads are connected across a three-phase, wye-connected source ($v_{load} = 679\cos(120\pi t)$ – line to line):

Load #1: Wye-connected load with 60 kVA (3-phase) at 0.9 PF lag;
Load #2: Delta-connected load with 100 kW (3-phase) at 0.8 PF lag;

Calculate the following:

a) The total complex power (3-phase) consumed by both loads (5 points)
b) Total source line current RMS magnitude (5 points)
c) The phase current RMS magnitude for each load. (10 points)
d) A delta-connected capacitor bank is added in parallel to make the overall power factor equal to unity. Determine the required VARS per phase. (5 points)

$$v_{load} = 679\cos(120\pi t) \Rightarrow \overline{V}_{load} = 480\angle 0$$

Load 1 characteristic:

$$S_{L_1} = 60\angle\cos^{-1} 0.9 = 54000 + 26150j$$

Load 2 characteristic:

$$S_{L_2} = \frac{100}{0.8}\angle\cos^{-1} 0.8 = 100{,}000 + 75{,}000j$$

A) Total complex power:

$$S_{total} = S_{L_1} + S_{L_2} = 184250\angle 33.29$$

B) Total line current:

$$S_{total} = \sqrt{3}V_{load}I_{Line} \Rightarrow I_{Line} = 221.26$$

C) Phase currents:

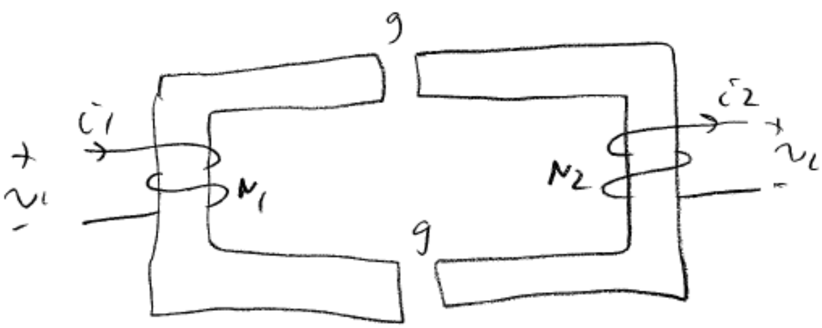
$$S_1 = \sqrt{3}V_{load}I_{Line} = \sqrt{3}V_{load}I_{phase} \Rightarrow I_{phase} = 72.16$$
$$S_2 = \sqrt{3}V_{load}I_{Line} = \sqrt{3}V_{load}\sqrt{3}I_{phase} \Rightarrow I_{phase} = 86.8$$

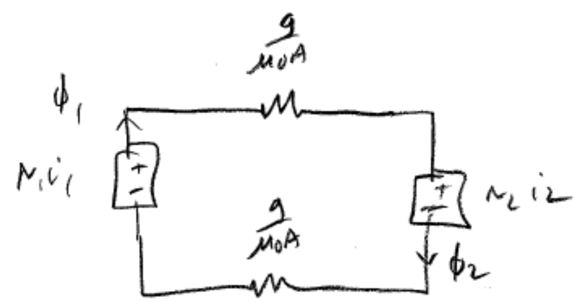
D) Power factor correction reactive power:

$$Q_{3\phi} = -101150 \Rightarrow Q_{1\phi} = \frac{Q_{3\phi}}{3} = -33720$$

3. (25 points)

An electric tooth brush has two parts - the base which has a coil and a handset which also has a coil. This could be modeled as two "C" sections of steel (each with a coil) separated by an air gap of distance "g" when the handset is on the base. Draw your own magnetic circuit of this and compute an expression for the mutual inductance between the handset coil and the base coil. Assume all of your own polarities and current directions. Express your answer in terms of the two coil turns, the gap g, the permeability of free space, and the cross sectional area of the steel. Neglect fringing and assume infinite permeability in the steel.





$\lambda_1 = N_1 \phi_1$

$\lambda_2 = N_2 \phi_2$

$$-N_1 i_1 + \phi_1 \frac{2g}{\mu_0 A} + N_2 i_2 = 0$$

$$\phi_1 = \frac{N_1 i_1 - N_2 i_2}{\frac{2g}{\mu_0 A}}$$

$$\lambda_1 = \frac{N_1^2 i_1}{\frac{2g}{\mu_0 A}} - \frac{N_1 N_2 i_2}{\frac{2g}{\mu_0 A}}$$

$$\boxed{m = \frac{N_1 N_2 \mu_0 A}{2g}}$$

4. (25 points)

Two coils (each with zero resistance) are located near each other. Coil #2 is open circuited. When a 60Hz sinusoidal voltage of 120 Volts (RMS) is applied to coil #1, the coil #1 current is 5 Amps (RMS) and the voltage measured on the open-circuited coil #2 is 70 Volts (RMS).

a) What is the self inductance of coil #1?

b) What is the magnitude of the mutual inductance between coil #1 and coil #2?

$$120\sqrt{2}\cos 2\pi 60t = L_1\frac{di_1}{dt} \pm \cancel{m\frac{di_2}{dt}} \rightarrow 0$$

Open Circuit

$$\frac{di_1}{dt} = \frac{120\sqrt{2}}{L_1}\cos 2\pi 60t \qquad i_1 = \frac{120\sqrt{2}}{2\pi 60 L_1}\sin 2\pi 60t$$

$$\frac{120}{2\pi 60 L_1} = 5A \qquad L_1 = \frac{120}{2\pi 60\times 5} H$$

$$= 0.0637 H$$

$$v_2 = \pm m\frac{di_1}{dt} = \pm m\times\frac{120\sqrt{2}}{L_1}\cos 2\pi 60t$$

$$70 = \frac{m\times 120}{L_1} \qquad m = \frac{70 L_1}{120}$$

$$= 0.0371 H$$