1. Problem 3.1

Let $\mathcal{R}_g$ and $\mathcal{R}_r$ to be the reluctances of each air gap and the iron coil, respectively.

![Figure 1: Magnetic circuit for Problem 3.1](image)

Considering the fringing effects, the effective cross-section area $A_g$ for the air gap is $A_g = (d + g)(w + g) = 7.59 \text{ cm}^2$, where $A_r = 6 \text{ cm}^2$ is the cross-section area for the iron core. Assuming that $l_g$ and $l_r$ are the average length of each air gap and the coil, respectively, we can have

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{0.3 \text{ cm}}{(4\pi \times 10^{-7}) \times 7.59 \text{ cm}^2} = 3.145 \times 10^6 \frac{A \cdot t}{\text{Wb}}$$

$$\mathcal{R}_r = \frac{l_r}{\mu_r \mu_0 A_r (2000)(4\pi \times 10^{-7}) \times 6 \text{ cm}^2} = 2.653 \times 10^5 \frac{A \cdot t}{\text{Wb}}$$

As such, we can obtain the magnetic flux $\phi$ by:

$$\phi = \frac{N_i}{2\mathcal{R}_g + \mathcal{R}_r} = 0.153 \text{ mWb}$$

Correspondingly, the flux linkage is:

$$\lambda = N\phi = 30.65 \text{ mWb} \cdot t$$

with the inductance to be:

$$L = \frac{\lambda}{i} = 6.12 \text{ mH}$$

Then, the magnetic flux density in the iron core is given by:

$$B = \frac{\phi}{A_r} = 255 \text{ mT}$$
2. Problem 3.2

a. The dot marking is shown in Fig.2.

\[ \phi_3 = \phi_1 + \phi_2 \]  
\[ N_1 i_1 = \phi_1 R_1 + \phi_3 R_3 \]  
\[ N_2 i_2 = \phi_2 R_2 + \phi_3 R_3 \]

Solving the three equations (1)-(3), we have

\[ \lambda_1 = N_1 \phi_1 = 0.0003125(10i_1 - 3i_2) \]
\[ \lambda_2 = N_2 \phi_2 = 0.00015625(-6i_1 + 5i_2) \]

As such, \( L_1 = 0.003125 \) \( H \) and \( L_2 = 0.00078125 \) \( H \).

b. Given the equivalent magnetic circuit as shown in Fig.3, we can have:

\[ \phi_3 = \phi_1 + \phi_2 \]  
\[ N_1 i_1 = \phi_1 R_1 + \phi_3 R_3 \]  
\[ N_2 i_2 = \phi_2 R_2 + \phi_3 R_3 \]

As such, \( L_1 = 0.003125 \) \( H \) and \( L_2 = 0.00078125 \) \( H \).

c. After reversing the direction of the current in coil 2, we can reverse the direction of source \( N_2 i_2 \) in the equivalent magnetic circuit shown in Fig.4 to obtain:

\[ \phi_3 = \phi_1 - \phi_2 \]  
\[ N_1 i_1 = \phi_1 R_1 + \phi_3 R_3 \]  
\[ -N_2 i_2 = \phi_2 R_2 + \phi_3 R_3 \]
Solving the three equations above, we have

\[
\lambda_1 = N_1 \phi_1 = 0.0003125(10i_1 + 3i_2)
\]

\[
\lambda_2 = N_2 \phi_2 = 0.00015625(-6i_1 - 5i_2)
\]

As such, \( L_1 = 0.003125 \, H \) and \( L_2 = -0.00078125 \, H \).
3. Problem 3.3

The equivalent magnetic circuit is shown in Fig.5.

![Magnetic circuit for Problem 3.3](image)

Figure 5: Magnetic circuit for Problem 3.3

we can have:

\[ \phi_2 = \phi_1 + \phi_3 \]
\[ Ni = \phi_2 R_2 + \phi_3 R_1 \]
\[ Ni = \phi_2 R_2 + \phi_3 R_3 \]

with

\[ R_1 = R_3 = \frac{g}{\mu_0 A_1 \times 1.1} = \frac{0.25 \text{ cm}}{(4\pi \times 10^{-7}) \times 6 \text{ cm}^2 \times 1.1} = 3.016 \times 10^6 \frac{A \cdot t}{Wb} \]
\[ R_2 = \frac{g}{\mu_0 A_2 \times 1.1} = \frac{0.25 \text{ cm}}{(4\pi \times 10^{-7}) \times 12 \text{ cm}^2 \times 1.1} = 1.508 \times 10^6 \frac{A \cdot t}{Wb} \]

Therefore, we can have

\[ \phi_2 = \frac{Ni}{R_2 + \frac{R_3 R_1}{R_3 + R_1}} = 0.332 \text{ mWb} \]
\[ \phi_3 = \phi_1 = \frac{\phi_2}{2} = 0.166 \text{ mWb} \]

Correspondingly, the magnetic flux density in each leg (air gap) is given by:

\[ B_{2g} = \frac{\phi_2}{1.1A_2} = 0.251 \text{ T} \]
\[ B_{1g} = B_{3g} = \frac{\phi_1}{1.1A_1} = 0.251 \text{ T} \]
4. Special Problem 1

a. The equivalent magnetic circuit is shown in Fig.6. Given the equivalent circuit,

\[
\phi_3 = \phi_1 + \phi_2 \quad (4)
\]
\[
Ni = 2\phi_2 R_g + 2\phi_3 R_g \quad (5)
\]
\[
Ni = 2\phi_1 R_g + 2\phi_3 R_g \quad (6)
\]

with the reluctance of each air gap to be:

\[
R_g = \frac{g}{\mu_0 A} = \frac{1 \text{ mm}}{(4\pi \times 10^{-7}) \times 1 \text{ cm}^2} = 7.958 \times 10^6 \frac{A}{Wb}. \quad (7)
\]

b. Given the flux density \( B = 1 \ T \) in the bar \( x \), we can obtain:

\[
\phi_3 = BA = 10^{-4} \ Wb \quad (8)
\]

Solving (4)-(8), we can compute the current by:

\[
i = \frac{3R_g BA}{N} = 23.87 \ A
\]

c. Again, combining (4)-(8), the flux density of the left-hand coil is:

\[
\lambda_2 = N\phi_2 = N\frac{\phi_3}{2} = 5 \times 10^{-3} \ Wb \cdot t \quad (9)
\]