ECE 330 Exam #2, Spring 2012 Name: Solution
90 Minutes

Section (Check One) MWF 10am ______ TR 12:30pm ______

1. ________ / 25 2. ________ / 25

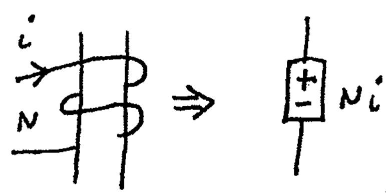
3. ________ / ~~25~~ 30 4. ________ / ~~25~~ 20 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



$W_m = \int_0^{\lambda} i d\hat{\lambda}$ $\quad W_m' = \int_0^{i} \lambda d\hat{i}$ $\quad W_m + W_m' = \lambda i$ $\quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\quad x \to \theta$

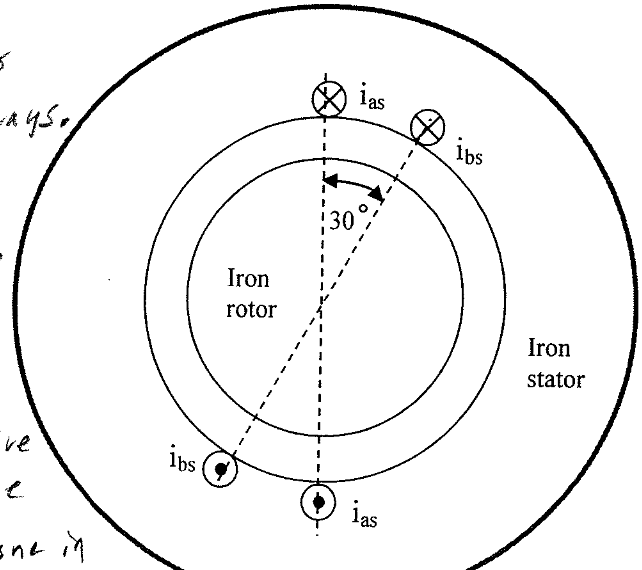
$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i d\lambda$ $\quad \underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\quad \underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\quad \lambda = \frac{\partial W_m'}{\partial i}$ $\quad i = \frac{\partial W_m}{\partial \lambda}$

Problem 1. (25 points)

The device below has two coils separated by 30 degrees as shown. Find an expression for the torque that must be tolerated by the coil installations (i.e. the torque created by the currents will try to move the coils) if the device radius is 0.01 meters, the depth into the paper is 0.02 meters, the air gap is 0.001 meters, and each current is 10 Amps DC. The coils are identical and each has 50 turns. The air gap permeability is μ_0 and the permeability of the iron is infinite.

Could solve this problem two ways. One way would solve for the H quantities in the four areas. This will take a long time and will give a triangle wave for L_{asbs} as done in the book and l_{asbs}



Radius $= .01m$

$l = .02\,m$

$g = .001\,m$

$i_{as} = 10A \quad i_{bs} = 10A$

$\theta = 30^0$

An easier way is to use a sinusoidal approximation for the flux linkage for two coils separated by an angle θ:

$$\left.\begin{aligned} \lambda_{as} &= L_s i_{as} + M\cos\theta\, i_{bs} \\ \lambda_{bs} &= M\cos\theta\, i_{as} + L_s i_{bs} \end{aligned}\right\} (\theta = 30^\circ \text{ in this problem})$$

$$W_m' = \left(\tfrac{1}{2} L_s i_{as}^2\right) + \left(M\cos\theta\, i_{as} i_{bs} + \tfrac{1}{2} L_s i_{bs}^2\right) \Rightarrow T^e = -M\sin\theta\, i_{as} i_{bs}$$

For $\theta = 30^\circ$, $i_{as} = 10A$, $i_{bs} = 10A$, $\boxed{T^e = -50M \quad N.m.}$

Maximum coupling is when $\theta = 0^\circ$. The flux linking the as coil due to i_{bs} would be $\phi = \mu_0 H_{out_{left}} \pi R l$ where H is due to i_{bs}. This would satisfy $\left(H_{out_{left}} - H_{out_{right}}\right) g = 50 \times 10$

By conservation of flux, $\mu_0 H_{out_{left}} \pi R l + \mu_0 H_{out_{right}} \pi R l = 0$

$H_{out_{left}} = \dfrac{500}{2g}$ so $50 \mu_0 \dfrac{500}{2g} \pi R l = 10M$ $\quad M = .001H \quad$ $\boxed{T^e = -.05\,NM}$

Problem 2. (25 points.)

An electromechanical system is described by the following flux-linkage vs current characteristic:

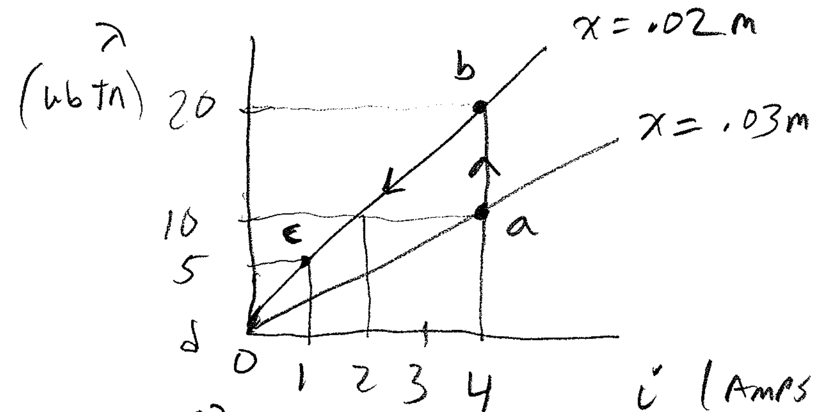
$$\lambda = \frac{.05}{x - .01}\, i$$

It is operated on the path a - b - c - d as indicated below, with x constant during b – c – d. The current is constant during a - b.

	a	b	c	d	
i (Amps)	i_a	i_b	i_c	0	
λ (Wb turns)	10	λ_b	5	0	
x (meters)	.03	.02	.02	.02	

Find the following things:

a) i_a, i_b, i_c, and λ_b
b) The energy stored in the coupling field at points a, b and c.
c) The force of electric origin at points a, b and c.
d) The Energy From the Electrical system into the coupling field as the system goes from a to b
e) The Energy From the Mechanical system into the coupling field as the system goes from a to b



A) $i_a = 4A = i_b$

$\lambda_b = 20$ Wb Tn

$i_c = 1$ Amp

b) $W_{m_a} = \int_0^{10} = 20J$

$W_{m_b} = \int_0^{20} = 40J$

$W_{m_c} = \int_0^{5} = 2.5J$

c) $w_m' = \dfrac{.025 i^2}{(x - .01)}$ $\qquad f^e = -\dfrac{.025 i^2}{(x - .01)^2}$

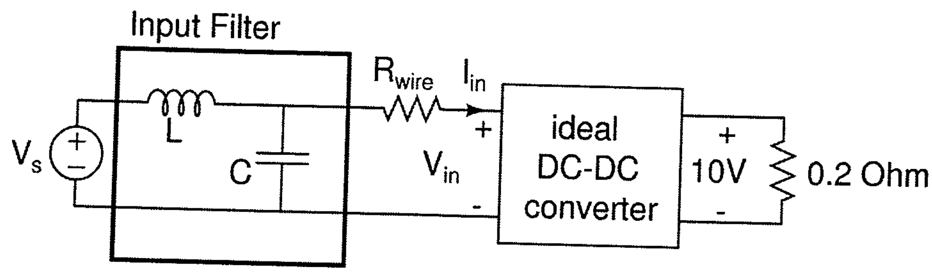
$f^e_a = -1000N$ $\qquad f^e_b = -4000N$ $\qquad f^e_c = -250N$

d) $\underset{a-b}{EFE} = \int_{10}^{20} 4\, d\lambda = 40J$

$\underset{a-b}{EFM} = W_{m_b} - W_{m_a} - \underset{a-b}{EFE} = 40 - 20 - 40 = -20J$

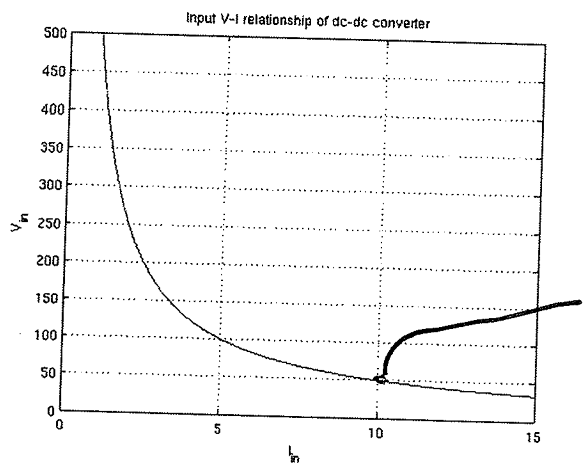
Check $\underset{a-b}{EFM} = -\int_{.03}^{.02} \left(-\dfrac{.025 \times 4^2}{(x - .01)^2}\right) dx = -\dfrac{.025 \times 16}{(x - 0.01)}\Bigg|_{.03}^{.02} = -.4(100 - 50) = -20J$ ✓

Problem 3. (30 points.)



A dc-dc converter with input filter

Shown above is an ideal (100% efficient) dc-dc converter with an L-C input filter and constant source voltage V_s. In this problem we will analyze the small-signal stability of this circuit. The following component values are used in the problem: $L = 1$ mH, $C = 1$ mF, $R_{wire} = 1$ Ohm. The ideal DC-DC converter provides a constant output power of $P_{out} = 500$ W to the load. The **input** voltage and current of the dc-dc converter are related by the equation: $P_{in} = P_{out} = V_{in} \times I_{in}$ (as shown below).



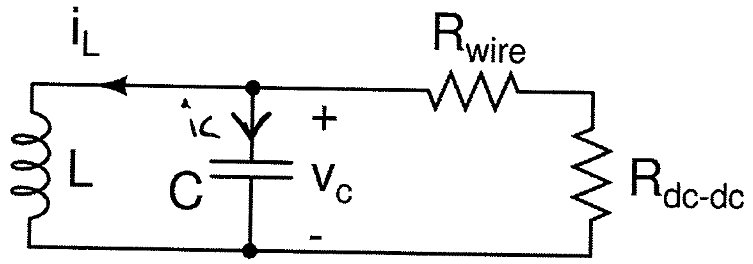
note negative slope $\frac{dV_{in}}{dI_{in}}$

a) Find an expression for the small signal input resistance ($R_{dc\text{-}dc} = dV_{in}/dI_{in}$) as a function of P_{in} and I_{in}, and evaluate it at the operating point ($I_{in} = 10$ A, $P_{in} = 500$ W).

$$P_{in} = V_{IN} I_{IN} \Rightarrow V_{IN} = \frac{P_{IN}}{I_{in}}$$

$$R_{DC\text{-}DC} = \frac{dV_{in}}{dI_{in}} = -\left.\frac{P_{IN}}{I_{in}^2}\right|_{\substack{P_{in}=500W \\ I_{in}=10A}} = -5\Omega$$

Shown below is the small-signal (linearized) circuit of the input filter and dc-dc converter, together with the wire resistance R_{wire}. Note that the component $R_{dc\text{-}dc}$ is a small-signal linearized resistance of the dc-dc converter and the load, and has the value that you found in (a).



Small signal circuit of input filter, wire resistance and the dc-dc converter with load.

$$i_C = C\frac{dv_C}{dt}$$

↑ note direction of current in figure

The circuit above can be expressed in a state-space representation as

$$\begin{bmatrix} \frac{dv_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = A \times \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

$$R_T = R_{wire} + R_{DC\text{-}DC}$$
$$= 1 - 5 = -4$$

b) Find the matrix A.

$$A = \begin{bmatrix} 250 & -1000 \\ 1000 & 0 \end{bmatrix}$$

$$\Uparrow$$

KVL: $L\frac{di_L}{dt} = v_C$

KCL: $i_L + C\frac{dv_C}{dt} + \frac{v_C}{R_T} = 0$

$$\frac{dv_C}{dt} = \frac{-v_C}{CR_T} - \frac{i_L}{C}$$

$$\frac{di_L}{dt} = \frac{v_C}{L}$$

$$\Rightarrow \begin{bmatrix} \frac{dv_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-1}{C(R_T)} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}}_{A} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

c) Find the eigenvalues of A.

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda + \frac{1}{R_T C} & \frac{1}{C} \\ -\frac{1}{L} & \lambda \end{vmatrix} = \lambda^2 + \frac{\lambda}{R_T C} + \frac{1}{LC} = 0$$

$$\lambda = -\frac{1}{2R_T C} \pm \frac{1}{2}\sqrt{\left(\frac{1}{R_T C}\right)^2 - \frac{4}{LC}}$$

$$= 125 \pm 992 \cdot j$$

d) Is the system stable? Explain your answer.

The system is unstable

$\mathrm{Re}\{\lambda\} > 0$

Problem 4. (20 points.)

Consider the following nonlinear equations:

$dx_1/dt = 3x_1 - x_1x_2$

$dx_2/dt = 2x_1x_2 + x_2$

The system has initial conditions $x_1(0) = 1$ and $x_2(0) = 2$

a) Find all equilibrium points of the system

$0 = 3x_1^e - x_1^e x_2^e \Rightarrow 0 = x_1^e(3 - x_2^e) \quad x_1^e = 0 \text{ or } x_2^e = 3$

$0 = 2x_1^e x_2^e + x_2^e \Rightarrow 0 = x_2^e(2x_1^e + 1) \quad x_2^e = 0 \text{ or } x_1^e = -\frac{1}{2}$

If $x_1^e = 0$, x_2^e must be zero $\Rightarrow$ $x_1^e = 0$, $x_2^e = 0$ is one soln.

other soln is $x_1^e = -\frac{1}{2}$, $x_2^e = 3$

b) Using Euler's method with a timestep $\Delta t = 0.01$ sec, determine the state values at $t = 0.01$ sec and $t = 0.02$ sec.

$x_1(0.1) = x_1(0) + \left.\frac{dx}{dt}\right|_{t=0} \Delta t$

$x_1(0.1) = 1 + (3 \cdot 1 - 1 \cdot 2) \cdot 0.01 = 1.01$

$x_2(0.1) = 2 + (2 \cdot 1 \cdot 2 + 2) \cdot 0.01 = 2.06$

$x_1(0.2) = 1.01 + (3 \cdot 1.01 - 1.01 \cdot 2.06)\, 0.01 = 1.0195$

$x_2(0.2) = 2.06 + (2 \cdot 1.01 \cdot 2.06 + 2.06) \cdot 0.01 = 2.1222$