Problem 1 (40 pts.)

10pts a) Given a two port electrical and one port mechanical system

$$\lambda_1(i_1,i_2,\theta) = 2i_1 + 3i_2\cos\theta$$
$$\lambda_2(i_1,i_2,\theta) = 3i_1\cos\theta + i_2 + 5$$

find W_m' and T^e

$$W_m' = i_1^2 + 3i_1i_2\cos\theta + \frac{1}{2}i_2^2 + 5i_2$$

$$T^e = \frac{\partial W_m'}{\partial\theta} = -3i_1i_2\sin\theta$$

15pts b) Given the following system

$$\dot{x}_1 = 2x_1 + x_2^2 + 1 \qquad x_1(0) = 0$$
$$\dot{x}_2 = x_1x_2 - x_1^2 + 3x_2 \qquad x_2(0) = 1$$

Use the Euler's method with step size $\Delta t = 0.1$ calculate $x_1(t)$ and $x_2(t)$ at $t = 0.1$ and $t = 0.2$

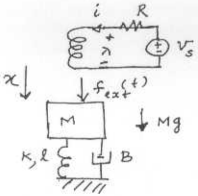
$x_1(.1) = 0 + (0 + 1 + 1)0.1 = 0.2$

$x_2(.1) = 1 + (0 - 0 + 3)0.1 = 1.3$

$x_1(.2) = 0.2 + (2\times0.2 + 1.3^2 + 1)0.1 = 0.509$

$x_2(.2) = 1.3 + (0.2\times1.3 - 0.2^2 + 3\times1.3)\,0.1 = 1.712$

15pts c)



ℓ is the value of x for which $f_{spring} = 0$

include gravity

$\lambda = \frac{10i}{x}$

If $f_{ext}(t) = 10\sin t$, write the electromechanical model in state space form.

$$v_s = iR + \frac{d\lambda}{dt} = iR - \frac{10i}{x^2}\frac{dx}{dt} + \frac{10}{x}\frac{di}{dt}$$

$$w_m' = \frac{5i^2}{x} \qquad f^e = -\frac{5i^2}{x^2}$$

$$m\frac{d^2x}{dt^2} = -k(x-\ell) - B\frac{dx}{dt} + 10\sin t + mg - \frac{5i^2}{x^2}$$

$x = x$

$\dot{x} = v$

$i = i$

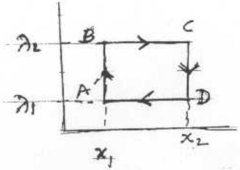
$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = \frac{1}{m}\left[-k(x-\ell) - Bv + 10\sin t + mg - \frac{5i^2}{x^2}\right]$$

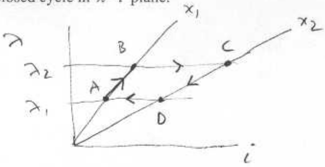
$$\frac{di}{dt} = \frac{x}{10}\left[-iR + \frac{10iv}{x^2} + v_s\right]$$

Problem 2 (30 pts)

Given the $\lambda - i$ relationship of a device as $\lambda = \dfrac{i}{(1+x)}$. The system traverses in the closed cycle as the following

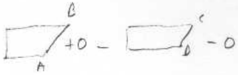


a) Sketch the closed cycle in $\lambda - i$ plane.

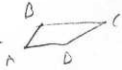


b) For the problem above, find $\mathrm{EFE}|_{\mathrm{cycle}}$. Is the device acting as a generator or a motor?

$$\mathrm{EFE}_{\text{cycle}} = \int_{A-B} i\,d\lambda + \int_{B-C} i\,d\lambda + \int_{C-D} i\,d\lambda + \int_{D-A} i\,d\lambda =$$



$=$ −



Generator

c) For the problem above, what is the value of $\mathrm{EFM}|_{\mathrm{cycle}}$?

$\mathrm{EFM}_{\text{cycle}} =$ +

<u>OR</u>

b) $$EFE_{cycle} = \int_{\lambda_1}^{\lambda_2} (1+x_1)\lambda \, d\lambda + 0 + \int_{\lambda_2}^{\lambda_1} (1+x_2)\lambda \, d\lambda$$

$$= (1+x_1)\frac{(\lambda_2^2-\lambda_1^2)}{2} + (1+x_2)(\lambda_1^2-\lambda_2^2)$$

$$= \frac{\lambda_2^2-\lambda_1^2}{2}(1+x_1-x_2-1) = \frac{\lambda_2^2-\lambda_1^2}{2}(x_1-x_2) \quad < 0$$

gen

c) $$EFM_{cycle} = \frac{\lambda_2^2-\lambda_1^2}{2}(x_2-x_1)$$

Problem 3 (30 pts)

a) Find the equilibrium of the following system in the interval $0 < x_1 < \pi$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -5\sin x_1 - x_2 + 2.5$$

$x_2 = c$

$5\sin x_1 = 2.5$ $\quad\quad x_1^e = \frac{\pi}{6}, \frac{5\pi}{6}$

b) Linearize and find eigenvalues of the system around <u>one</u> of them.

$\Delta\dot{x}_1 = \Delta x_2$

$\Delta\dot{x}_2 = -5\cos x_1^e \,\Delta x_1 - \Delta x_2$

$$A = \begin{bmatrix} 0 & 1 \\ -5\cos x_1^e & -1 \end{bmatrix} \quad A_{e_1} = \begin{bmatrix} 0 & 1 \\ -4.33 & -1 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 4.33 & \lambda+1 \end{vmatrix} = \lambda^2 + \lambda + 4.33$$

$$\lambda = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 17.32}$$

$$= -0.5 \pm j\,2.02$$