ECE 330 Exam #1, Fall 2013 Name: Solution
90 Minutes

Section (Check One) MWF 10am _______ MWF 2:00pm _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^*$ $\bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

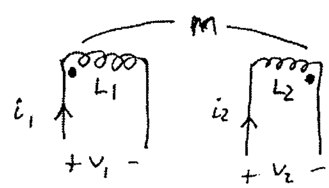
$0 < \theta < 180^\circ$ (lag)
$-180^\circ < \theta < 0$ (lead)

$I_L = \sqrt{3}I_\phi$ (delta)
$V_L = \sqrt{3}V_\phi$ (wye)

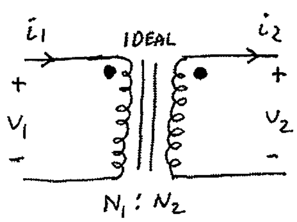
$\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n} da$ $\Re = \frac{l}{\mu A}$ $MMF = Ni = \phi\Re$

$\phi = BA$ $\lambda = N\phi$ $k = \frac{M}{\sqrt{L_1 L_2}}$ 1 hp = 746 Watts

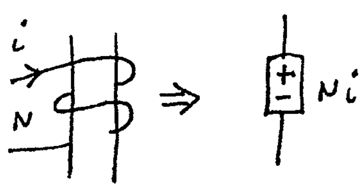


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$



Problem 1. (25 points)

A single phase, 120 Volt (RMS) source serves a passive load. A variable single-phase capacitor bank is connected in parallel with the load. Measurements of the source current for various values of capacitor Vars give:

Capacitor Vars:	0	100	200	300	400	500	600	700
Source current (Amps):	6.93	6.56	6.28	6.10	6.03	6.07	6.23	6.49

(a) By just looking at the numbers in the table above, about how many Vars would you say the original load (without the capacitors) consumes?

Put answer here: 400 Vars (minimum current = unity pf)

(b) What is the exact value of the original load P + jQ (without the capacitors)?

$$P^2+Q^2 = (120\times 6.93)^2 = 691{,}559 = P^2+Q^2$$

$$P^2+(Q-300)^2 = (120\times 6.10)^2 = 535{,}824 = P^2+Q^2-600Q+90{,}000$$

$$691{,}559 - 535{,}824 = 600Q - 90{,}000 \qquad \boxed{Q = 410 \text{ vars}}$$

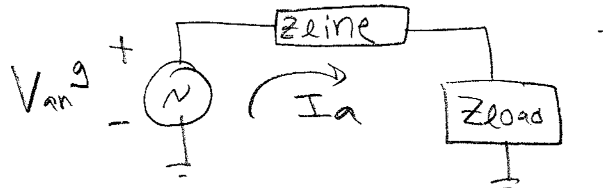
$$\boxed{P = 724 \text{ watts}}$$

Problem 2. (25 pts)

Assume a balanced, three-phase, 60 Hz sequence power system supplies a delta-connected load such that the line-to-line voltage at the load is 2.4 kV. A three-phase grounded-wye-connected generator supplies the load through a line impedance of 0.1 + j1 Ω per phase. The total power consumed by the load is 240 kVA at a lagging power factor of 0.8.

$S_{1\phi} = \frac{240}{3} \angle \cos^{-1}(0.8)$ kVA

(a). Draw the per-phase equivalent.



$\frac{2400}{\sqrt{3}} \angle 0 = V_{an}^{\lambda}$

$S_{1\phi} = V_{1\phi} I_{1\phi}^{*}$

$\left(\frac{\frac{240000}{3} \angle \cos^{-1}(0.8)}{\frac{2400}{\sqrt{3}} \angle 0} \right)^{*} = I_a$

$57.735 \angle 36.87°$

(b). Find the total (three-phase) real and reactive power supplied by the source. Make sure to include units.

$-V_{an}^{g} + Z_{line} I_a + \frac{2400}{\sqrt{3}} \angle 0 = 0$

$V_{an}^{g} = (0.1 + j1)(46.188 - j34.61) + \frac{2400}{\sqrt{3}} \angle 0°$

$= 1424.9 + j42.72 = 1425.54 \angle 1.7174°$

$S_{3\phi} = 3 \cdot S_{1\phi} = 3 \cdot (1425.54 \angle 1.7)(57.735 \angle 36.87)$

$= \overset{kW}{193} + j\underset{kVAR}{154}$ kVA

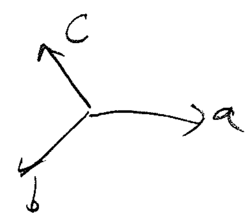
(c). Using the line-to-neutral A-phase voltage at the load as your angle reference, find all three line-to-line voltages at the source (assume ABC sequence). Include magnitude and angle.

$V_{ab}^{g} = \sqrt{3} \times 1425.54 \angle 31.717°$ V

$2469.11 \angle 31.717°$

$V_{bc}^{g} = 2469 \angle -88.28$ V

$V_{ca}^{g} = 2469 \angle 151.717$



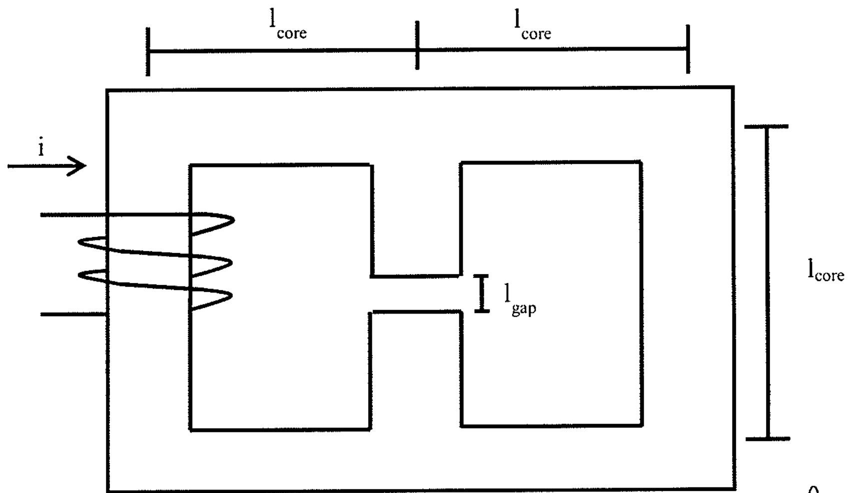
(d). Assume the power consumed by the load increases uniformly per phase to become 300 kVA with unity power factor. The system remains balanced. What is the magnitude of the neutral current in the generator?

b/c balanced

0

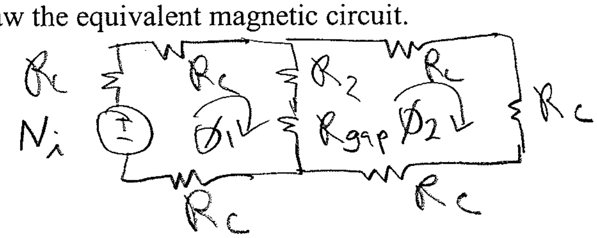
Problem 3. (25 points)

Consider the iron geometry given in the figure below. Assume fringing in the air gap such that $A_{gap} = 1.1 * A_{core}$, and assume the following values: lcore = 10 cm, lgap = 0.1 cm, $A_{core} = 2\ cm^2$, N = 100, and $\mu_r = 1000$.



$$R_C = \frac{l}{\mu A} = \frac{0.1}{1000 \times 4\pi \times 10^{-7} \times 0.0002}$$
$$= 397887\ AT/W$$

(a) Draw the equivalent magnetic circuit.

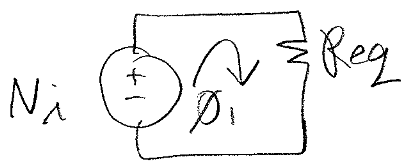


$$R_g = \frac{0.001}{4\pi \times 10^{-7} \times 0.00022} = 3.61716 \times 10^6\ AT/W$$

$$R_2 = \frac{(0.1 - 0.001)}{1000 \times 4\pi \times 10^{-7} \times 0.0002}$$
$$= 393908\ AT/W$$

(b) Find the inductance of the coil.

$$R_{eq} = 3R_c + [(R_2 + R_g) // 3R_c]$$
$$R_B = 919906\ AT/W$$
$$R_2 + R_g = 4.01107 \times 10^6\ AT/W$$
$$R_{eq} = 2.11 \times 10^6\ AT/W$$



$$\emptyset_1 = \frac{100 i}{R_{eq}} \quad L = \frac{N\emptyset}{i} = \frac{N^2}{R_{eq}}$$
$$= 0.000473 H$$

Continued on the next page

4.73 mH

(c) Find the current needed to generate a flux in the middle leg of 5x10^{-6} Wb.

$(\phi_1 - \phi_2) = 5\times10^{-6} \Rightarrow \phi_1 = 5\times10^{-6} + \phi_2$ $\quad \phi_2 = \dfrac{MMF}{3R_c}$

$5\times10^{-6} \times (R_2 + R_g) = MMF = 20.0553$

$\phi_2 = 0.000017$

$\phi_1 = 5\times10^{-6} + 0.000017 = 0.000022$

Loop 1: $-Ni + 3R_c\phi_1 + MMF = 0$

$\underline{i} = (3R_c\phi_1 + 20.0553)/100 = \underline{0.463\,A}$

(d) Find the flux density (Wb/m^2) in the right leg corresponding to the values given in part c.

$\phi_2 = 0.000017$

$\underline{B_2} = \dfrac{\phi_2}{A_2} = \dfrac{0.000017}{0.0002} = \underline{0.085\ Wb/m^2}$

Problem 4. (25 points)

Two identical coils (each with zero resistance) are located near each other. Coil #2 is open circuited.

When a 60Hz sinusoidal voltage of 120 Volts (RMS) is applied to coil #1, the coil #1 current is 6 Amps (RMS) and the voltage measured on the open-circuited coil #2 is 70 Volts (RMS).

(a) What are the self inductances of coil #1 and #2 in Henries?

$120\angle 0 = j2\pi 60 L_1 \times 6\angle\theta \qquad 120 = 2\pi 60 \times 6 \times L_1$

$$\boxed{L_1 = L_2 = 0.053 H}$$

(b) What is the magnitude of the mutual inductance between coil #1 and coil #2 in Henries?

$70\angle\theta_2 = j2\pi 60 m \times 6\angle\theta \qquad 70 = 2\pi 60 \times 6 \times M$

$$\boxed{m = 0.0309\ H}$$

(c) What are the current magnitudes in coil #1 and #2 if a short circuit is placed across coil #2 while the given voltage is applied across coil #1?

$$120\angle 0 = j20\,\bar{I}_1 + j11.65\,\bar{I}_2$$

$$0 = j11.65\,\bar{I}_1 + j20\,\bar{I}_2 \Rightarrow \bar{I}_2 = -0.583\,\bar{I}_1$$

$$120\angle 0 = (j20 - j11.65 \times 0.583)\,\bar{I}_1$$

$$\bar{I}_1 = \frac{120\angle 0}{j13.2} = -j9.09$$

$$\boxed{|\bar{I}_2| = 5.3\ Amps} \qquad \boxed{|\bar{I}_1| = 9.09\ Amps}$$