ECE 330 Exam #1, Fall 2011 Name: Solution

90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

## Useful information

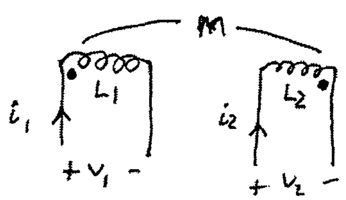
$\sin(x) = \cos(x - 90^\circ)$ $\quad \overline{V} = \overline{Z}\overline{I}$ $\quad \overline{S} = \overline{V}\overline{I}^*$ $\quad \overline{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3} I_\phi$ (delta) $\quad \overline{Z}_Y = \overline{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

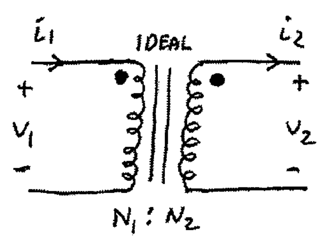
$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3} V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\phi = BA$ $\quad \lambda = N\phi$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

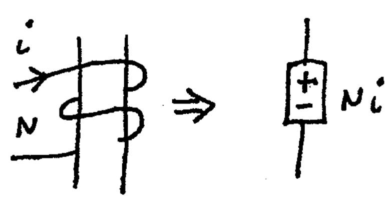



$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$




$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



**Problem 1.** **(25 points)**

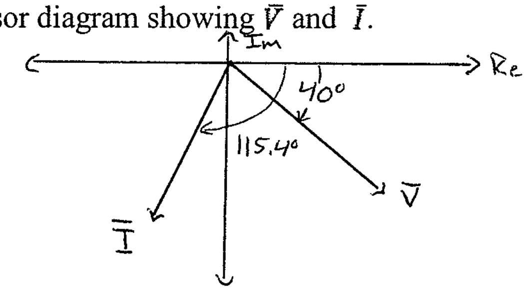
(a) Write the phasor form of the voltage $v(t) = 100\sqrt{2}\cos(377t - 40°)$ V

$$\bar{V} = 100\angle{-40°}\ V$$

(b) Write the time domain form of the complex current $\bar{I} = 25.6\angle - 115.4°$ A.

$$i(t) = 25.6\sqrt{2}\cos(\omega t - 115.4°)\ A$$

(c) Draw the phasor diagram showing $\bar{V}$ and $\bar{I}$.



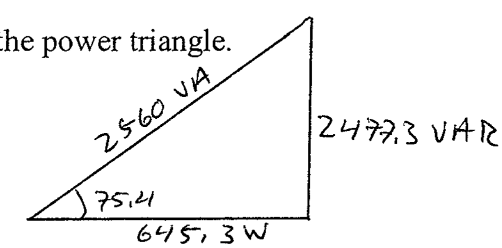

(d) The source voltage given by $\bar{V}$ is applied across an impedance. Find the complex impedance that results in current $\bar{I}$ in the circuit.

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{100\angle{-40°}\ V}{25.6\angle{-115.4°}\ A} = 3.9063\angle{75.4°}\ \Omega$$

(e) Find the complex power $\bar{S}$ consumed by the load and draw the power triangle.

$$\bar{S} = \bar{V}\bar{I}^* = (100\angle{-40°}\ V)(25.6\angle{115.4°}\ A)$$

$$\bar{S} = 2560\angle{75.4°}\ VA$$




(f) Find the power factor. Indicate whether leading or lagging. If the power factor is lagging, find how much capacitive VARs needs to be added in parallel to the impedance to raise the power factor to unity.

$\theta_V - \theta_I > 0°$

$$pf = \cos(75.4°) = 0.252\ \text{lag}$$

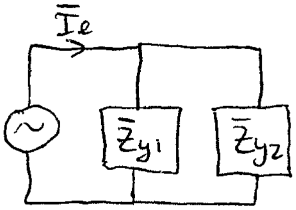
In order to reach unity, $\boxed{2477.3\ VAR}$ need to be added using shunt capacitance.

**Problem 2. (25 pts)**

A three-phase wye-connected balanced load of 12 kW at 0.8 power factor lag is connected in parallel with another three-phase wye-connected balanced load of 24 kVA at 0.6 power factor lead. The line-line voltage is 440V.

(a) Draw the per-phase equivalent of the circuit.

$\bar{V}_{an} = \frac{440}{\sqrt{3}} \angle \theta^\circ$ V




Load1

$\bar{S}_{1\phi} = \frac{4kW}{.8}(.8 + j\sin(\cos^{-1}(0.8))) = \bar{V}_{en}\bar{I}_\phi^* = \frac{|\bar{V}_{en}|^2}{\bar{Z}^*}$

$\bar{Z}_{y1} = \left(\frac{440^2/3}{\bar{S}_{1\phi}}\right)^* = 12.91 \angle 36.87^\circ\ \Omega$

Load2

$\bar{Z}_{y2} = \left(\frac{440^2/3}{8kVA(.6 - j\sin(\cos^{-1}(.6)))}\right)^* = 8.07 \angle -53.13^\circ\ \Omega$

(b) Find the total complex power consumed by the two loads and the power factor (mention lead/lag).

$\bar{S}_{1,3\phi} = 12kW + j\left(\frac{12kW}{.8}\right)\sin(\cos^{-1}(.8)) = (12 + j9)$ kVA

$\bar{S}_{2,3\phi} = (24)(.6)kW - 24j\sin(\cos^{-1}(.6)) = (14.4 - j19.2)$ kVA

$\therefore \bar{S}_t = \bar{S}_{1,3\phi} + \bar{S}_{2,3\phi} = (26.4 - j10.2)$ kVA $= 28.302 \angle -21.12$ kVA

$pf = \cos(-21.12) = 0.933$ lead

(c) Find the magnitude of the line current and phase currents in the two loads.

$\bar{S}_t = 3\bar{V}_\phi \bar{I}_\phi^*$

$(26.4 - j10.2)\text{kVA} = 3\bar{V}_{en}\bar{I}_\ell^*$

$\bar{I}_\ell^* = \frac{(26.4 - j10.2)\text{kVA}}{3\ 440/\sqrt{3}\ \angle\theta}$

$\bar{I}_a = 37.14 \angle \theta + 21.12^\circ$ A

$\bar{S}_1 = 3\bar{V}_{en}\bar{I}_{\phi 1}^*$

$\bar{I}_{\phi 1} = \left(\frac{(12 + j9)\text{kVA}}{3\ \frac{440}{\sqrt{3}} \angle\theta}\right)$

$\bar{I}_{\phi 1} = 19.68 \angle \theta - 36.87^\circ$ A

$\bar{S}_2 = 3\bar{V}_{en}\bar{I}_{\phi 2}^*$

$\bar{I}_{\phi 2} = \left(\frac{(14.4 - j19.2)\text{kVA}}{3\ \frac{440}{\sqrt{3}} \angle\theta}\right)^*$

$\bar{I}_{\phi 2} = 31.49 \angle \theta + 53.13^\circ$ A

(d) Assuming a-b-c phase sequence, and that angle on $\bar{V}_{AB} = 0$, write the phasor form of the line voltages $\bar{V}_{AB}$, $\bar{V}_{BC}$, $\bar{V}_{CA}$ and line currents $\bar{I}_A$, $\bar{I}_B$, $\bar{I}_C$.

$V_{ab} = 440 \angle 0^\circ$ V

$V_{bc} = 440 \angle -120^\circ$ V

$V_{ca} = 440 \angle +120^\circ$ V

if $\angle \bar{V}_{ab} = 0$

$\Downarrow$

$\angle \bar{V}_{an} = -30$

$\therefore \theta = -30^\circ$

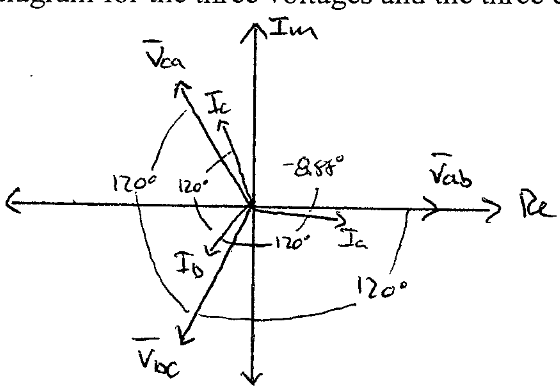
$\bar{I}_a = 37.14 \angle -30 + 21.12^\circ$ A

$= 37.14 \angle -8.88^\circ$ A

$\bar{I}_b = 37.14 \angle -128.88^\circ$ A

$\bar{I}_c = 37.14 \angle 111.12^\circ$ A

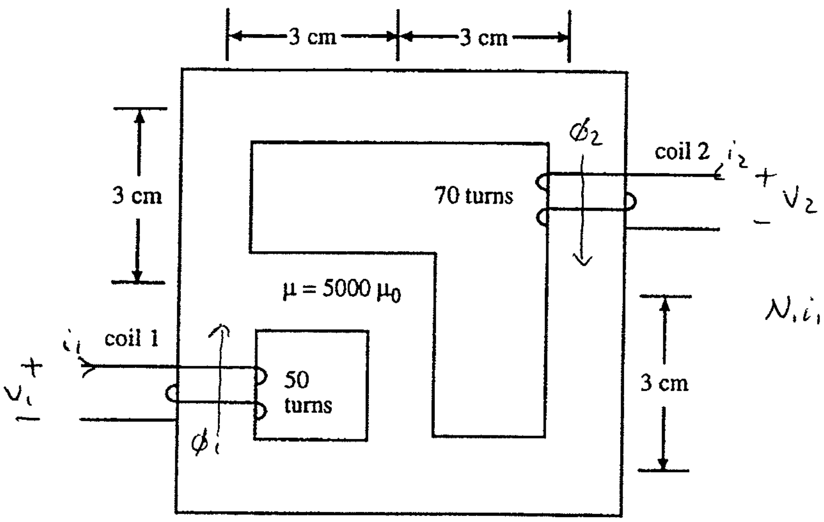
(e) Draw the phasor diagram for the three voltages and the three currents found in part (d).



$$V_1 = 50\frac{d\phi_1}{dt}, \quad \lambda_1 = 50\phi_1$$

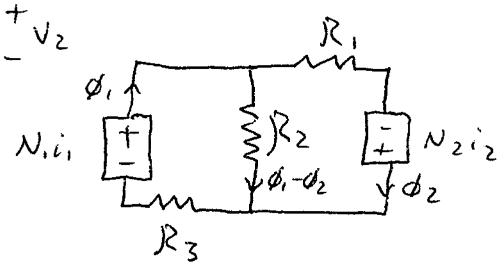
$$V_2 = 70\frac{d\phi_2}{dt}, \quad \lambda_2 = 70\phi_2$$

**Problem 3. (25 points)**




All legs have cross sections 1 cm by 1 cm

Equiv. Magnetic Circuit:




Determine the self inductances of the two coils and the mutual inductance between them.

$$R_1 = \frac{(18\times10^{-2}\,m)}{(5000)(4\pi\times10^{-7}\,H/m)(1\times10^{-4}\,m^2)} = 286{,}479\ \frac{At}{Wb}$$

$$R_2 = R_3 = \frac{(6\times10^{-2}\,m)}{(5000)(4\pi\times10^{-7}\,H/m)(1\times10^{-4}\,m^2)} = 95493\ \frac{At}{Wb}$$

KVL

$$50i_1 = R_2(\phi_1-\phi_2) + R_3\phi_1$$

$$70i_2 = R_2(\phi_2-\phi_1) + R_1\phi_2$$

$$\Downarrow$$

$$(R_2+R_3)\phi_1 + (-R_2)\phi_2 = 50i_1$$

$$(-R_2)\phi_1 + (R_1+R_2)\phi_2 = 70i_1$$

$$\begin{bmatrix}(R_2+R_3) & -R_2\\ -R_2 & (R_1+R_2)\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \begin{bmatrix}50i_1\\ 70i_2\end{bmatrix}$$

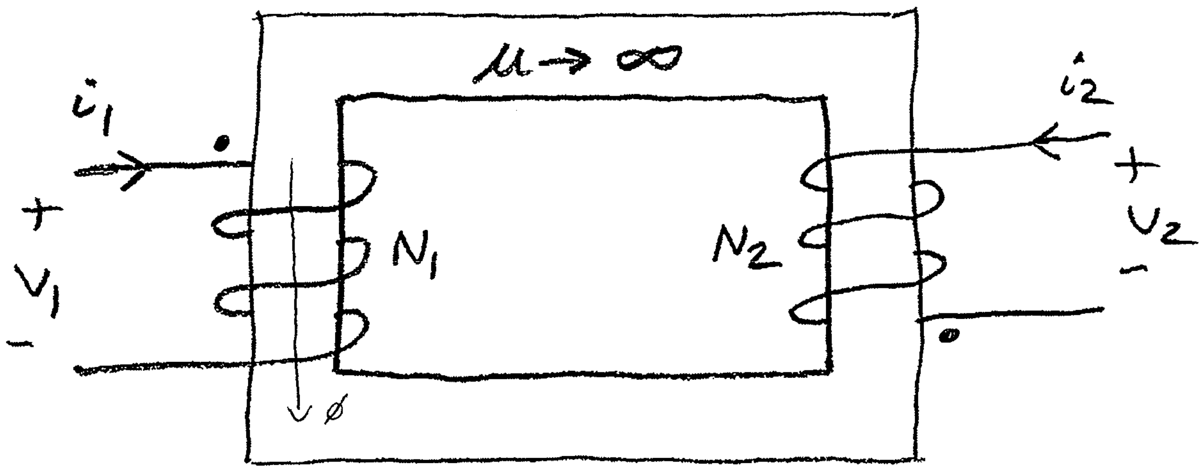
$$\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \frac{\begin{bmatrix}(R_1+R_2) & R_2\\ R_2 & (R_2+R_3)\end{bmatrix}}{(R_2+R_3)(R_1+R_2)-R_2^2}\begin{bmatrix}50i_1\\ 70i_2\end{bmatrix}$$

$$\lambda_1 = 50\phi_1 = \underbrace{\frac{50(R_1+R_2)50}{(R_2+R_3)(R_1+R_2)-R_2^2}}_{L_{11}}i_1 + \underbrace{\frac{50(R_2)50}{(R_2+R_3)(R_1+R_2)-R_2^2}}_{M}i_2$$

$$\lambda_2 = 70\phi_2 = \underbrace{\frac{70(R_2)50}{(R_2+R_3)(R_1+R_2)-R_2^2}}_{M}i_1 + \underbrace{\frac{70(R_2+R_3)70}{(R_2+R_3)(R_1+R_2)-R_2^2}}_{L_{22}}i_2$$

$$\boxed{\begin{aligned}L_{11} &= 0.01496\,H\\ L_{22} &= 0.01466\,H\\ M &= 0.00523\,H\end{aligned}}$$

**Problem 4. (25 points)**




(a) Put the dots on the two coils

(b) Write the ideal transformer equations for this device

$$0 = N_1 i_1 - N_2 i_2 \qquad \boxed{N_1 i_1 = N_2 i_2}$$

$$v_1 = N_1 \frac{d\phi}{dt}$$

$$\boxed{\frac{v_1}{v_2} = -\frac{N_1}{N_2}}$$

$$v_2 = -N_2 \frac{d\phi}{dt}$$

(c) If $v_1(t) = 170 \cos(377t)$ volts, and $N_1 = 200$ turns, what cross-sectional area is needed to make the peak value of the flux density in the iron exactly 1.0 Tesla?

Applying Faraday's Law:

$$\oint_C E \cdot dl = \frac{-d}{dt} \int_S \bar{B} \cdot \hat{n}\, ds$$

$$-170 \cos(377t) = -\frac{d}{dt} N\phi(t)$$

$$\int \frac{170}{N} \cos(377t)\, dt = \phi(t)$$

$$\phi(t) = \frac{170}{200 \cdot 377} \sin(377t)$$

$$|\phi| = |B| A$$

$$|B| = 1.0\,T$$

$$A = \frac{170}{200 \cdot 377} = \boxed{0.00225\, m^2}$$