ECE 330 Exam #2, Fall 2016 Name: Solutions
90 Minutes

Section (Check One) MWF 9am _______ MWF 10am _______

1. ________ / 25 2. ________ / 25

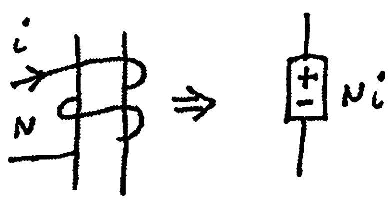
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\qquad \overline{V} = \overline{Z}\overline{I}$ $\qquad \overline{S} = \overline{V}\overline{I}^*$ $\qquad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n}da$ $\qquad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}da$ $\qquad MMF = Ni = \phi\Re$

$\Re = \frac{l}{\mu A}$ $\qquad B = \mu H$ $\qquad \phi = BA$ $\qquad \lambda = N\phi$ $\qquad \lambda = Li$ (if linear)



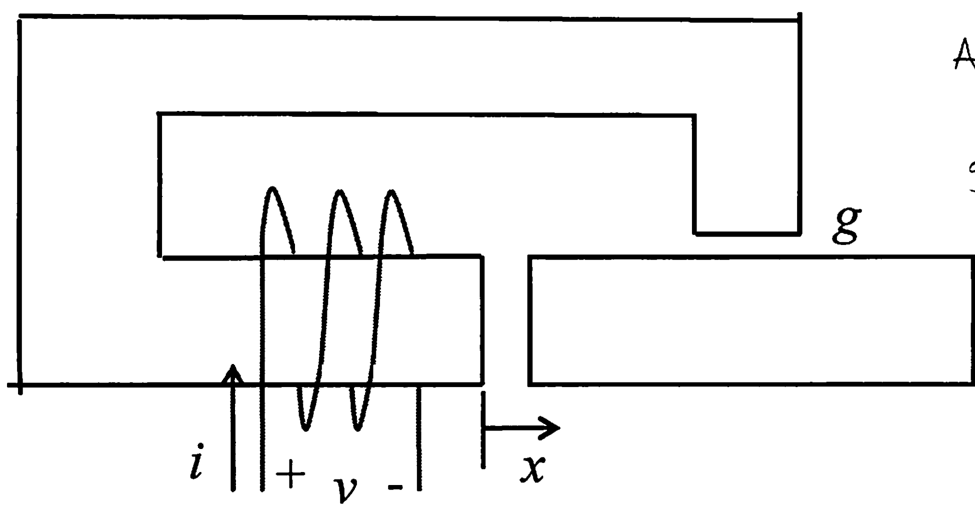
$W_m = \int_0^{\lambda} i\,d\hat{\lambda}$ $\qquad W_m' = \int_0^{i} \lambda\,d\hat{i}$ $\qquad W_m + W_m' = \lambda i$ $\qquad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\qquad x \to \theta$

$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i\,d\lambda$ $\qquad \underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\qquad \underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\qquad \lambda = \frac{\partial W_m'}{\partial i}$ $\qquad i = \frac{\partial W_m}{\partial \lambda}$

Problem 1. (25 points)

For the structure drawn in the figure below, the movable member is constrained to move left and right only as indicated in the figure where "x" is the distance to the left edge of the movable member. Assume the cross sectional area of all air gaps is 4 cm^2, and that the core is infinitely permeable. The member with the coil is fixed. The gap, g, is 1mm, and the number of turns of the coil is 100.

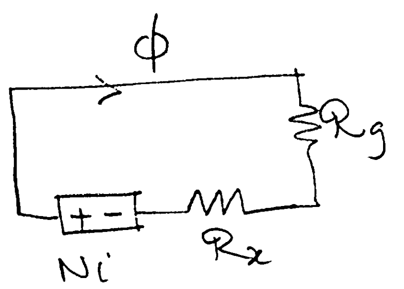


$A_g = 4\,cm^2 = 4\times10^{-4}\ m^2$

$g = 1\ mm = 10^{-3}\ m$

For the questions below, express all of these as functions of current (i) and/or position (x) and/or velocity and/or time as appropriate. You may neglect fringing in the gap, and you may assume the iron is infinitely permeable.

a) Find the flux linkage, λ. (defined for the voltage polarity shown)



$\phi = \dfrac{N i \mu_0 A}{x+g}$

$\lambda = \dfrac{N^2 i \mu_0 A}{x+g}$

$= \dfrac{16\pi\times10^{-7}}{x+0.001}\, i$

$\mathcal{R}_x = \dfrac{x}{\mu_0 A} = \dfrac{x}{16\pi\times10^{-11}}\ H^{-1}$

$\mathcal{R}_g = \dfrac{g}{\mu_0 A} = \dfrac{1}{16\pi\times10^{-8}}\ H^{-1}$

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$\mathcal{R}_{total} = \dfrac{x+0.001}{16\pi\times10^{-11}}$

b) Determine an expression for the voltage, v.

$$V = \frac{d\lambda}{dt} = \frac{\partial \lambda}{\partial i} \cdot \frac{di}{dt} + \frac{\partial \lambda}{\partial x} \cdot \frac{dx}{dt}$$

$$= \frac{N^2 \mu_0 A}{x+g} \frac{di}{dt} - \frac{N^2 \mu_0 A i}{(x+g)^2} \cdot \frac{dx}{dt}$$

$$= \frac{16\pi \times 10^{-7}}{x+0.001} \frac{di}{dt} - \frac{16\pi \times 10^{-7}}{(x+0.001)^2} \cdot \nu$$

c) Find an expression for the co-energy.

$$W_m' = \int_0^i \lambda(\hat{i}, x) \cdot d\hat{i} = \int_0^i \frac{N^2 \mu_0 A}{x+g} \hat{i} \, d\hat{i}$$

$$= \frac{\mu_0 A N^2 i^2}{2(x+g)} = \frac{8\pi \times 10^{-7}}{x+0.001} i^2$$

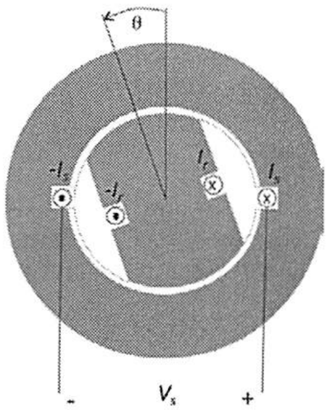
d) Find an expression for the force of electrical origin, f^e, acting on the movable piece.

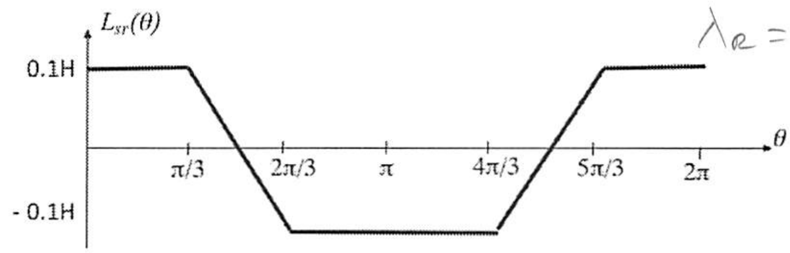
$$f^e = \frac{\partial W_m'}{\partial x} = \frac{-N^2 i^2 \mu_0 A}{2(x+g)^2}$$

$$= -\frac{8\pi \times 10^{-7} i^2}{(x+0.001)^2}$$

Problem 2. **(25 points)**

A single phase generator consists of a coil in the stator and a coil on the rotor with a mutual inductance, L_{sr}, between them. The variation L_{sr} with θ is shown in the figure below. The rotor is driven at a constant speed of 377 radians per second. The stator and rotor coils have 10 turns each. The rotor is fed with a constant dc current of 1A. Even though it may not be obvious, assume the rotor and stator self inductances are approximately constant.

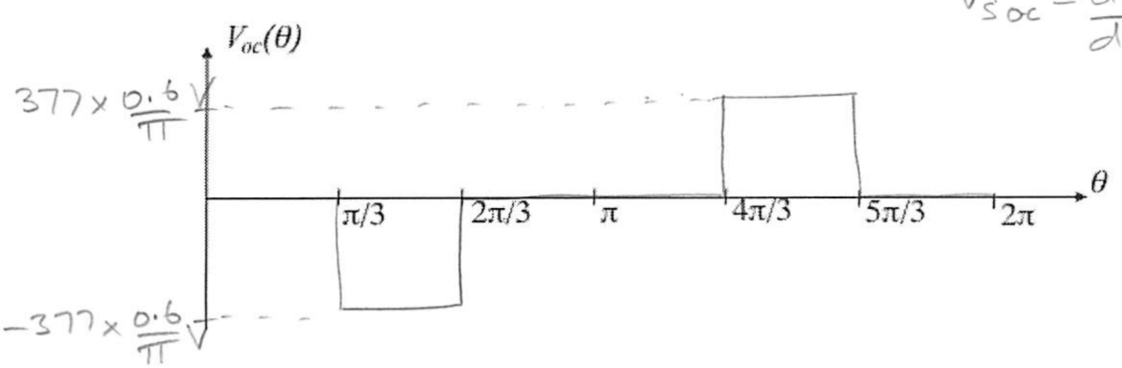




$$\lambda_s = L_{ss} I_s + L_{SR}(\theta) I_R$$
$$\lambda_R = L_{SR}(\theta) I_s + L_{RR} I_R$$

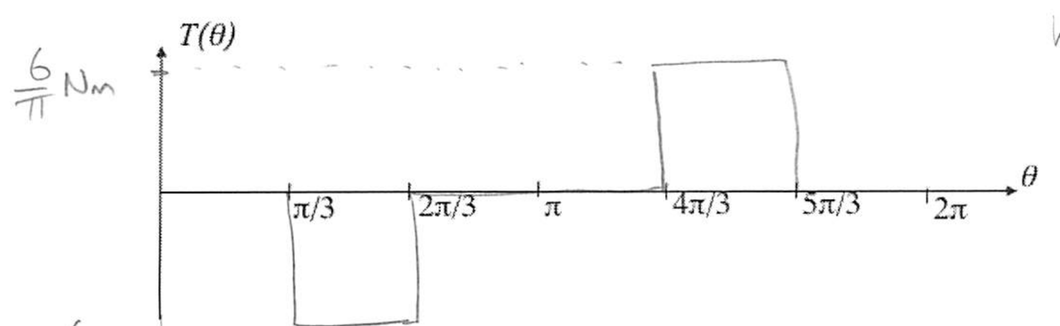
a) Plot the open circuit voltage (the case with stator current = 0) as a function of theta using the axes below. Label all points (compute all peaks). Do not use sinusoidal approximations.

$$V_{s\,oc} = \frac{d\lambda_s}{dt} = \frac{\partial \lambda_s}{\partial \theta}\cdot\frac{d\theta}{dt} + \frac{\partial \lambda_s}{\partial I_R}\cdot\cancel{\frac{dI_R}{dt}}^{\,0}$$
$$= \frac{dL_{SR}(\theta)}{d\theta}\cdot I_R \cdot \omega$$
$$= \text{Slope of } L_{SR}(\theta) \times \omega$$



b) The stator coil is ramped up to 10A dc fed through the positive terminal. Plot the torque variation under this condition. Label all points and do not use sinusoidal approximations.

$$W_m' = \frac{1}{2} L_{SS} I_S^2 + \frac{1}{2} L_{RR} I_R^2$$
$$+ L_{SR}(\theta) I_S I_R$$



$$T^e(\theta) = \frac{\partial W_m'}{\partial \theta}$$
$$= \frac{dL_{SR}(\theta)}{d\theta} I_S I_R$$
$$= \text{slope} \times 1 \times 10$$

Problem 3. (25 points.)

A device has the following linear flux linkage vs current characteristic:

$$\lambda = (50/(1000x+1))\ i$$

a) How much energy is given to the coupling field by the mechanical system if x is changed from 0.21m to 0.25m while the current remains constant at 10 Amps?
b) How much energy is given to the coupling field by the electrical system during that same path from x equals 0.21m to 0.25m while the current remains constant at 10 Amps?
c) Draw a magnetic device that would give this type of flux linkage vs current characteristic

a) $$W_m' = \frac{25 i^2}{1000x+1} \qquad f^e = -\frac{25{,}000 i^2}{(1000x+1)^2}$$

$$EFM_{a-b} = \int_{.21}^{.25} \frac{25000\,(10)^2}{(1000x+1)^2}\,dx = -\frac{2500}{1000x+1}\Bigg|_{.21}^{.25} = -\frac{2500}{251} + \frac{2500}{211}$$

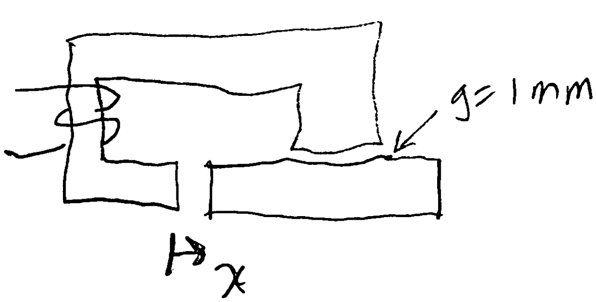
$$= -9.96 + 11.85 = \boxed{1.89\ J}$$

b) $$EFE_{a-b} = \int_{500/211}^{500/251} 10\, d\lambda = 10(1.992 - 2.37) = \boxed{-3.78\ J}$$

Note: $W_{m_b} = W'_{m_b} = \frac{2500}{251} = 9.96\ J \qquad W_{m_a} = W'_{m_a} = \frac{2500}{211} = 11.85\ J$

$$W_{m_b} - W_{m_a} = -1.89\ J \stackrel{?}{=} EFE_{a-b} + EFM_{a-b} = -3.78 + 1.89 = -1.89\ J$$ ✓

c) Same as that of Problem 1



Problem 4. (25 points.)

The dynamics of a system are described by the following non-linear differential equations:

+ $M\, d^2\delta/dt^2 + B\, d\delta/dt = P - KE \sin\delta$ and, $T dE/dt = -E + V_{in}$

where M=0.03, B=0.01, P=0.5, K=1.0, T=1.5, and Vin =1.0 are all positive constants.

a) If you were to write the above differential equations in state-space form, which variables would you take as dynamic states?

$$\delta, \quad \omega = \frac{d\delta}{dt}, \quad E$$

b) For the states you specified in a), write the state-space dynamic model equations in terms of the states you have selected.

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{1}{M}(P - KE\sin\delta - B\omega) \qquad \frac{dE}{dt} = \frac{1}{T}(-E + V_{in})$$

c) Find all the equilbrium points of the state-space model in b) that correspond to values of δ between 0 and π.

$$\omega_e = 0$$

$$E_e = 1 \qquad \delta_e = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$0.5 - \sin\delta = 0$$

d) If you start from $\delta(0)=0$, $\omega(0)=d\delta/dt$ (at $t=0$)$=0$, E(0)=1.0 use Euler's method with a time step of 0.01 seconds to find δ, ω and E at time equal to 0.01 and 0.02 using a time step of 0.01 sec.

$$\delta(.01) = 0 + (0).01 = 0 \qquad \delta(.02) = 0 + (0.167).01 = .00167$$

$$\omega(.01) = 0 + \frac{1}{.03}(0.5 - 0).01 = 0.167 \qquad \omega(.02) = 0.167 + \frac{1}{.03}(0.5 - 0).01 = 0.333$$

$$E(.01) = 1 + \frac{1}{1.5}(-1+1).01 = 1 \qquad E(.02) = 1 + \frac{1}{1.5}(-1+1).01 = 1$$

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