ECE 330 Exam #2, Spring 2013

90 Minutes

Name: Solutions

Section (Check One) MWF 10am ______ TR 12:30pm ______

1. ________ / 25 2. ________ / 25

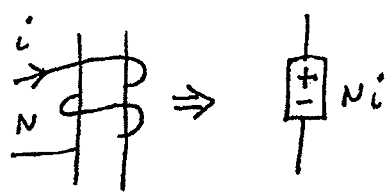
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



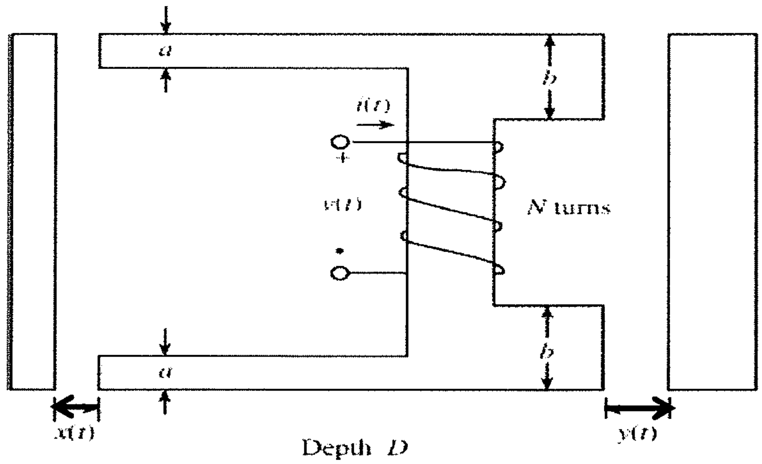
$W_m = \int_0^\lambda i\, d\hat{\lambda}$ $\quad W_m' = \int_0^i \lambda\, d\hat{i}$ $\quad W_m + W_m' = \lambda i$ $\quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\quad x \to \theta$

$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i\, d\lambda$ $\quad \underset{a\to b}{EFM} = -\int_a^b f^e dx$ $\quad \underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma}$ $\quad \lambda = \frac{\partial W_m'}{\partial i}$ $\quad i = \frac{\partial W_m}{\partial \lambda}$

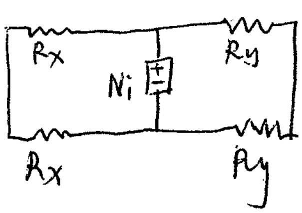
Problem 1. (25 points)

The device shown here has a fixed center core and two movable side pieces. The device is made from infinitely permeable material, and all three pieces have depth D into the paper. A current source $i(t)$ drives the terminals of an N-turn coil wound on the fixed core.



a) Find the flux $\lambda(i,x,y)$ linked by the electrical terminal pairs (15 points);
b) Compute the co-energy $W_m'(i,x,y)$ (5 points);
c) Compute the force~~s~~ of electric origin f_x^e acting on the left moving part (5 points).

a) Equivalent Magnetic Circuit



$$R_{eq} = (2R_x) // (2R_y) = \frac{\frac{2x}{\mu_0 aD} \times \frac{2y}{\mu_0 bD}}{\frac{2x}{\mu_0 aD} + \frac{2y}{\mu_0 bD}}$$

$$= \frac{\frac{4xy}{\mu_0^2 ab D^2}}{\frac{2xb+2ya}{\mu_0 abD}}$$

$$= \frac{4xy}{\mu_0 D(2xb+2ya)} = \frac{2xy}{\mu_0 D(xb+ya)}$$

$$\therefore \underset{(i,x,y)}{\lambda} = N\phi = N\frac{Ni}{R_{eq}} = \frac{N^2 i}{R_{eq}} = \underline{\frac{\mu_0 D N^2(xb+ya)}{2xy}\, i}$$

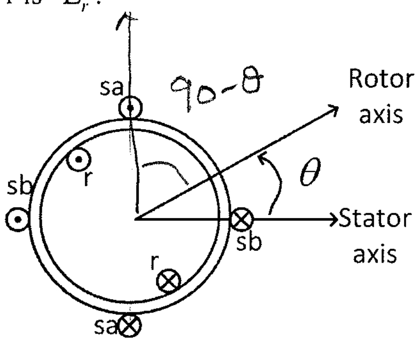
b) $$W_m' = \int_0^i \lambda(\hat{i},x,y)\,d\hat{i} = \underline{\frac{\mu_0 D N^2 (xb+ya)}{2xy}\left(\frac{i^2}{2}\right)}$$

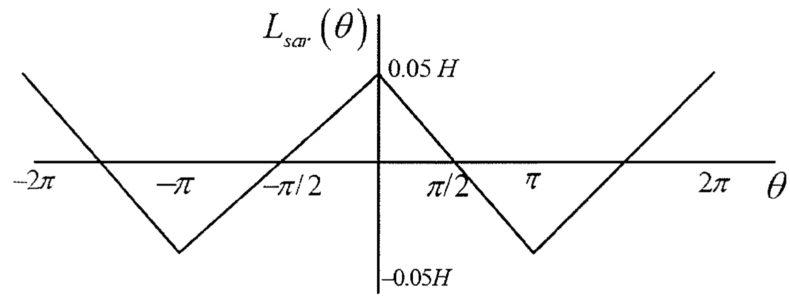
c) $$f_x^e = \frac{\partial W_m'}{\partial x} = \frac{\partial}{\partial x}\left[\frac{\mu_0 D N^2 b i^2}{4y} + \frac{\mu_0 D N^2 a i^2}{4x}\right]$$

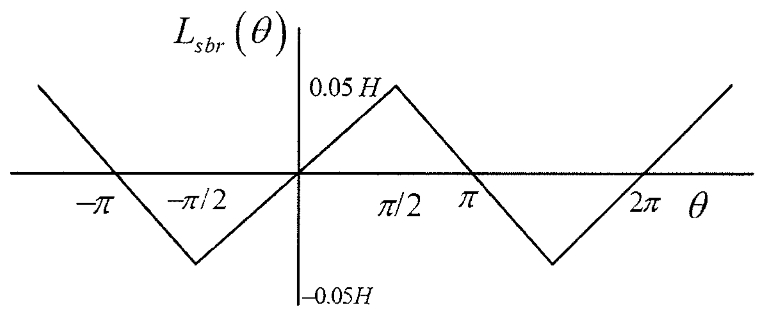
$$= \underline{-\frac{\mu_0 D N^2 a i^2}{4x^2}}$$

Problem 2. (25 points.)

The rotating system shown below has two coils on the stator (sa and sb) plus one coil on the rotor (r). The magnetic axes are shown for sa and r. The mutual inductance variations between the stator coils and the rotor coil are also shown. The self-inductance for sa and sb is L_s for each coil, and the self inductance for r is L_r.







a) Write the flux linkage equations $\lambda_{sa}, \lambda_{sb}, \lambda_r$ in terms of $i_{sa}, i_{sb}, i_r, L_s, L_r, \theta$ when $\theta \in (0, \pi/2)$. (15 Points). Do not use Sinusoidal approximations

b) Compute the Torque of electric origin in terms of the three currents i_{sar}, i_{sbr}, i_r when $\theta \in (0, \pi/2)$. (10 Points)

a)

$$L_{sar}(\theta) = 0.05 - \frac{0.1\theta}{\pi} \qquad \theta \in (0, \pi/2).$$

$$L_{sbr}(\theta) = \frac{0.1\theta}{\pi} \qquad \theta \in (0, \pi/2)$$

$$\lambda_{sa} = L_s i_{sa} + L_{sar}(\theta)\, i_r$$

$$\lambda_{sb} = L_s i_{sb} + L_{sbr}(\theta)\, i_r$$

$$\lambda_r = L_{sar}(\theta)\, i_{sa} + L_{sbr}(\theta)\, i_{sb} + L_r i_r$$

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$$\lambda_{sa} = L_s i_{sa} + \left[0.05 - \frac{0.1\theta}{\pi}\right] i_r$$

$$\lambda_{sb} = L_s i_{sb} + \frac{0.1\theta}{\pi} i_r$$

$$\lambda_r = \left[0.05 - \frac{0.1\theta}{\pi}\right] i_{sa} + \frac{0.1\theta}{\pi} i_{sb} + L_r i_r$$

(b)

$$W_m' = \int_0^{i_{sa}} L_s \hat{i}_{sa}\, d\hat{i}_{sa} + \int_0^{i_{sb}} L_s \hat{i}_{sb}\, d\hat{i}_{sb} + \int_0^{i_r} \left[L_{sar}(\theta) i_{sa} + L_{sbr}(\theta) i_{sb} + L_r \hat{i}_r \right] d\hat{i}_r$$

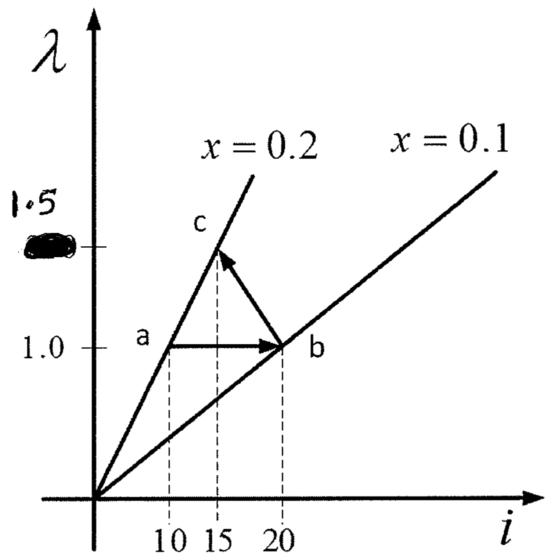
$$W_m' = \frac{1}{2} L_s i_{sa}^2 + \frac{1}{2} L_s i_{sb}^2 + L_{sar}(\theta) i_{sa} i_r + L_{sbr}(\theta) i_{sb} i_r + \frac{1}{2} L_r i_r^2$$

$$W_m' = \frac{1}{2} L_s i_{sa}^2 + \frac{1}{2} L_s i_{sb}^2 + \left[0.05 - \frac{0.1}{\pi}\theta\right] i_{sa} i_r + \frac{0.1\theta}{\pi} i_{sb} i_r + \frac{1}{2} L_r i_r^2$$

$$T^e = \frac{\partial W_m'}{\partial \theta} = \frac{-0.1}{\pi} i_{sa} i_r + \frac{0.1}{\pi} i_{sb} i_r$$

Problem 3. (25 points.)

An electromechanical device is operated over the path $a \to b \to c$ shown in the figure below. The system is known to be electrically linear, i.e., $\lambda = L(x)i$.



(in Joules)

a) Calculate the energy stored in the coupling field (W_m) at points 'a', 'b' and 'c'. (10 Points)

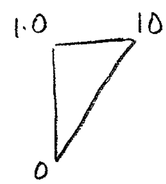
b) Calculate $EFE|_{a\to b\to c}$ in Joules. (5 points)

c) Calculate $EFM|_{a\to b\to c}$ in Joules. (5 Points)

d) Suppose the system takes the path $a \to c$ directly. Is $EFM|_{a\to c} = EFM|_{a\to b\to c}$? Explain your answer. If $EFM|_{a\to c} \neq EFM|_{a\to b\to c}$ then state the value of $EFM|_{a\to c}$ (5 Points).

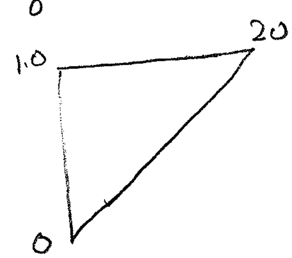
(Note : You must clearly show the steps for parts a) - c))

a) $W_{ma} =$



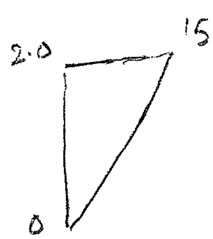
$= \frac{1}{2} \times 1.0 \times 10 = 5J$

$W_{Mb} =$



$= \frac{1}{2} \times 20 \times 1 = 10J$

$W_{Mc} =$



$= \frac{1}{2} \times 15 \times 1.5 = 11.25J$

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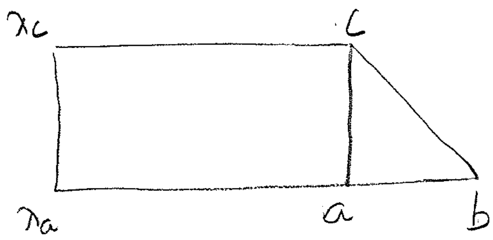
b) EFE = 

$= 0.5 \times 15 + \frac{1}{2} \times 0.5 \times 5$

$\Rightarrow$ EFE = 8.75 J

c) $W_{Mc} - W_{Ma} =$ EFE + EFM

$6.25 = 8.75 +$ EFM

$\Rightarrow$ EFM $= 10 - 8.75 = -2.5$ J

d) EFM is path dependent. Along the direct path $A \to C$ x is constant. Hence $EFM|_{a \to c} = 0$.

Problem 4. (25 points.)

An electromechanical system has the following equation of motion:

$$0.1\,\frac{d^2x}{dt^2} + x = x^2$$

a) Write this in the standard state-space form; 10

b) Find all the equilibrium points; 7

c) With initial conditions $x(0) = 0.1$ and $\frac{dx}{dt}$ (at $t = 0$) = 0, use Euler's method to find $x(0.1)$ and $x(0.2)$ with a time step of 0.1 second. 8

a)

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \frac{dx}{dt} \end{aligned}\right\} \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 10(x_1^2 - x_1) \end{aligned}$$

b)

$$\begin{aligned} 0 &= x_2 \\ 0 &= 10(x_1^2 - x_1) \end{aligned} \quad \Rightarrow \quad \begin{cases} x_1^e = 0 \text{ or } 1 \\ x_2^e = 0 \end{cases}$$

c)

$$\begin{bmatrix} x_1(0.1) \\ x_2(0.1) \end{bmatrix} = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} + 0.1 \times \begin{bmatrix} x_2(0) \\ 10[x_1^2(0) - x_1(0)] \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} + 0.1 \begin{bmatrix} 0 \\ 10(0.1^2 - 0.1) \end{bmatrix} = \begin{bmatrix} 0.1 \\ -0.09 \end{bmatrix}$$

$$\begin{bmatrix} x_1(0.2) \\ x_2(0.2) \end{bmatrix} = \begin{bmatrix} x_1(0.1) \\ x_2(0.1) \end{bmatrix} + 0.1 \times \begin{bmatrix} x_2(0.1) \\ 10[x_1^2(0.1) - x_1(0.1)] \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ -0.09 \end{bmatrix} * 0.1 \begin{bmatrix} -0.09 \\ 10(0.1^2 - 0.1) \end{bmatrix} = \begin{bmatrix} 0.091 \\ -0.18 \end{bmatrix}$$

$$\therefore x(0.1) = x_1(0.1) = 0.1$$

$$x(0.2) = x_1(0.2) = 0.091$$