ECE 430 Exam #2, Spring 2009
90 Minutes

Name: Solution

Section (Check One) MWF _______ TTH _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m $\qquad \int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da \qquad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$

$\Re = \frac{l}{\mu A} \qquad MMF = Ni = \phi \Re \qquad B = \mu H \qquad \phi = BA \qquad \lambda = N\phi \qquad \lambda = Li$ (if linear)

$W_m = \int_0^{\lambda} i d\hat{\lambda} \qquad W_m' = \int_0^{i} \lambda d\hat{i} \qquad W_m + W_m' = \lambda i \qquad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x} \qquad x \to \theta$

$f^e \to T^e$

$\underset{a \to b}{EFE} = \int_a^b i d\lambda \qquad \underset{a \to b}{EFM} = -\int_a^b f^e dx \qquad \underset{a \to b}{EFE} + \underset{a \to b}{EFM} = W_{mb} - W_{ma} \qquad \lambda = \frac{\partial W_m'}{\partial i} \qquad i = \frac{\partial W_m}{\partial \lambda}$

1. **(25 points)**

A 2-phase machine, with two stator coils and one rotor coil, has following flux linkage-current relationship

$$\begin{bmatrix}\lambda_{sa}\\ \lambda_{sb}\\ \lambda_r\end{bmatrix} = \begin{bmatrix}L_s & 0 & M\cos\theta\\ 0 & L_s & M\sin\theta\\ M\cos\theta & M\sin\theta & L_r\end{bmatrix} \times \begin{bmatrix}i_{sa}\\ i_{sb}\\ i_r\end{bmatrix}$$

a) Compute the co-energy W'_m (10 points)

b) Compute the energy W_m (5 points)

c) Find the torque of electrical origin T^e (10 points)

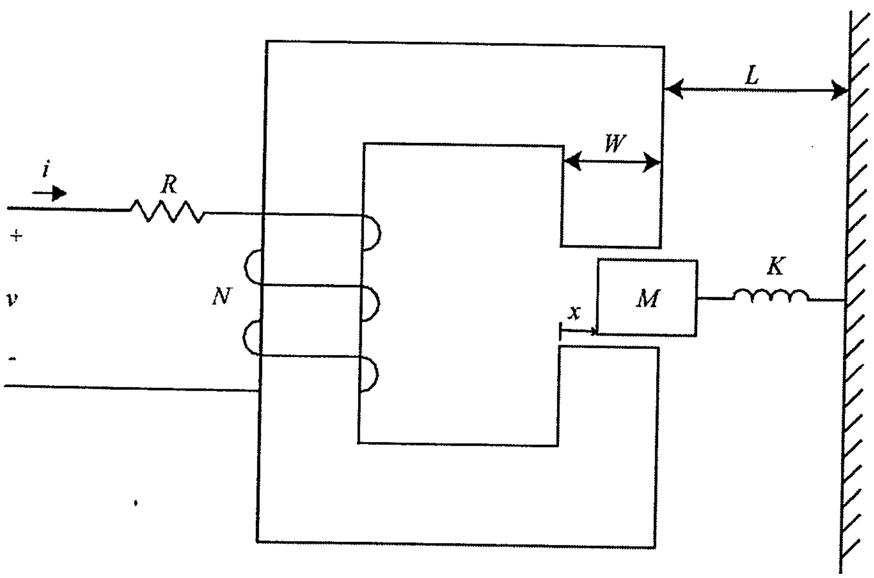
a) $W'_m = \frac{1}{2} L_s i_{sa}^2 + \frac{1}{2} L_s i_{sb}^2 + M\cos\theta\, i_{sa} i_r + M\sin\theta\, i_{sb} i_r + \frac{1}{2} L_r i_r^2$

b) $W_m = W'_m =$ " (linear system)

c) $T^e = \frac{\partial W'_m}{\partial \theta} = -M\sin\theta\, i_{sa} i_r + M\cos\theta\, i_{sb} i_r$ NM

2. (25 points)

The figure below shows a magnetic strucuture composed of an iron core and a moveable iron plunger. The plunger is connected to a solid structure through a spring with stiffness coefficient K. The spring exerts no force when the plunger is located at x = C. The plunger is separated from the core by a constant gap of length *g* above and below the plunger. The horizontal length of the plunger is *W*. The plunger and the core both have a depth of *D*. The iron core and plunger both have infinite permeability. You should neglect fringing in the air gaps. You may neglect the force of gravity and all friction.



Depth: D

$\mu \to \infty$

$v = iR + \frac{d\lambda}{dt}$

$\lambda = L(x)\, i$

$\nu = \frac{dx}{dt}$

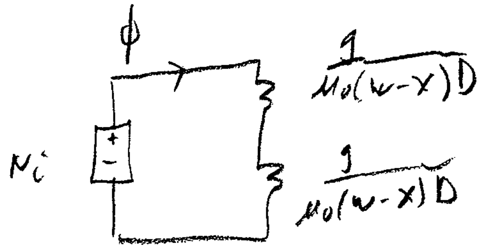
$m\frac{d\nu}{dt} = -k(x-c) + f^e$

For v = dλ/dt,

a) Find the flux linkage λ in terms of current i and position x plus μ_0, N, W, g, and D.

b) Find the force of electric origin either in terms of flux linkage λ and position x, or in terms of current i and position x (whichever you prefer).

c) Select either flux linkage λ or current i as an independent state and write the three differential equations that describe all of the dynamics of this electromechanical system with voltage v as the only input.



$\lambda = N\phi = \frac{N^2 i}{\frac{2g}{\mu_0(w-x)D}} = \frac{N^2\mu_0(w-x)D}{2g}\, i$

$W_m' = \frac{N^2\mu_0(w-x)D}{4g}\, i^2$

$f^e = -\frac{N^2\mu_0 D}{4g}\, i^2$

(this page is extra space for showing work)

$$v = iR + \frac{N^2 \mu_0 (w-x) D}{2g}\frac{di}{dt} - \frac{N^2 \mu_0 D i v}{2g}$$

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = -K(x-c) - \frac{N^2 \mu_0 i^2 D}{4g}$$

OR

$$v = \frac{2g\lambda R}{N^2 \mu_0 (w-x) D} + \frac{d\lambda}{dt}$$

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = -K(x-c) - \frac{N^2 \mu_0 D}{4g}\left(\frac{2g\lambda}{D N^2 \mu_0 (w-x)}\right)^2$$

$$= -K(x-c) - \frac{g\lambda^2}{D N^2 \mu_0 (w-x)^2}$$

3. (25 points)

The co-energy of a device is given by

$$Wm'(i,x) = \frac{i^5}{24x} + \frac{i^3}{6x} + \frac{i}{x}$$

$$\lambda = \frac{5i^4}{24x} + \frac{3i^2}{6x} + \frac{1}{x}$$

Find:

a) The force of electrical origin $f^e(i,x)$

b) The energy stored in the coupling field Wm as a function of i and x.

a) $$f^e = -\frac{i^5}{24x^2} - \frac{i^3}{6x^2} - \frac{i}{x^2}$$

b) $$W_m = \lambda i - W_m' = \frac{5i^5}{24x} + \frac{3i^3}{6x} + \frac{i}{x} - \frac{i^5}{24x} - \frac{i^3}{6x} - \frac{i}{x}$$

$$= \frac{4i^5}{24x} + \frac{2i^3}{6x} = \frac{i^5}{6x} + \frac{i^3}{3x}$$

4. (25 points)

An electromechanical system has a nonlinear flux linkage-current relationship:

$$i = \frac{\lambda^2}{x}$$

a) Find the energy stored in the coupling field when i = 2.0 Amps, x = 0.02 m, λ = 0.2 WbTn

b) Find the force of electric origin when i = 2.0 Amps, x = 0.02 m, λ = 0.2 WbTn

c) Find the energy transferred from the mechanical system into the coupling field (EFM) as the position x changes from 0.02 m to 0.01 m along a constant flux linkage (0.2 WbTn) path.

a) $W_m = \int_0^{\lambda} i \, d\hat{\lambda} \Big|_{x = \text{CONST}} = \frac{\lambda^3}{3x}$ $\quad W_m \Big|_{\substack{x = .02 \\ \lambda = .2}} = \frac{.2^3}{3 \times .02} = \frac{.008}{.06}$ $0.133 = \frac{8}{60}$ J

b) $f^e = -\left(-\frac{\lambda^3}{3x^2}\right) = \frac{\lambda^3}{3x^2}$ $\quad f^e \Big|_{\substack{x = .02 \\ \lambda = .2}} = \frac{.2^3}{3 \times .02^2} = \frac{.008}{3 \times .0004}$ 6.66 $= \frac{80}{3 \times 4} = \frac{20}{3}$ N

c) $E_{FM\, a-b} = \int_{.02}^{.01} -f^e \Big|_{\lambda = 0.2} dx = -\left(-\frac{\lambda^3}{3x}\right)\Bigg|_{.02,\ \lambda = .2}^{.01} = \frac{.2^3}{3 \times .01} - \frac{.2^3}{3 \times .02}$

$= \frac{.008}{.03} - \frac{.008}{.06} = \frac{8}{30} - \frac{8}{60}$ J

0.133