ECE 430 Exam #2, Spring 2008 Name: Solution
90 Minutes

Section (Check One) MWF 10 ______ TTH 1 ______ MWF 3 ______

1. ________ / 30 2. ________ / 20

3. ________ / 30 4. ________ / 20 Total ________ / 100

Useful information

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m $\quad \int_C \mathbf{H}\cdot d\mathbf{l} = \int_S \mathbf{J}\cdot \mathbf{n}da \quad \int_C \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B}\cdot \mathbf{n}da$

$\Re = \frac{l}{\mu A} \quad MMF = Ni = \phi\Re \quad \phi = BA \quad \lambda = N\phi$ (always) $\quad \lambda = Li$ (if linear)

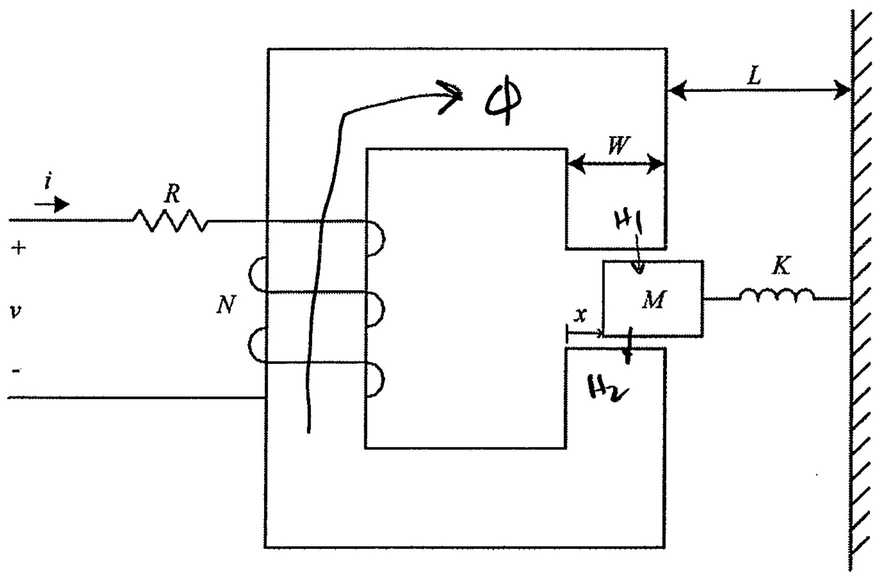
$W_m = \int_0^{\lambda} i\,d\hat{\lambda} \quad W_m' = \int_0^{i} \lambda\,d\hat{i} \quad W_m + W_m' = \lambda i \quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x} \quad x \to \theta$

$f^e \to T^e$

$\underset{a\to b}{EFE} = \int_a^b i\,d\lambda \quad \underset{a\to b}{EFM} = -\int_a^b f^e dx \quad \underset{a\to b}{EFE} + \underset{a\to b}{EFM} = W_{mb} - W_{ma} \quad \lambda = \frac{\partial W_m'}{\partial i} \quad i = \frac{\partial W_m}{\partial \lambda}$

1. **(30 points)**

The figure below shows a magnetic strucuture composed of an iron core and a moveable iron plunger. The plunger is connected to a solid structure through a spring with stiffness coefficient K. The spring exerts no force when the plunger is located at x = C. The plunger is seperated from the core by a constant gap of length *g* above and below the plunger. The mean path length of the core (excluding the plunger) is l_c. The vertical length of the plunger is *P*, and the horisontal length of the plunger is *W*. The plunger and the core both have a depth of *D*. The horizontal length of the plunger is *W*.



Depth: D

$\mu \to \infty$

For v = dλ/dt,

a) Find the flux linkage λ in terms of current i and position x plus μ_0, N, W, g, and D (state all of your assumptions clearly).

b) Find the force of electric origin either in terms of flux linkage λ and position x, or in terms of current i and position x (whichever you prefer).

c) Select either flux linkage λ or current i as an independent state and write the three differential equations that describe all of the dynamics of this electromechanical system with voltage v as the only input.

$H_1 g + H_2 g = Ni$ $\qquad \mu_0 H_1 (w-x) = \mu_0 H_2 (w-x)$

$H_1 = H_2 = \frac{Ni}{2g}$ $\qquad \phi = \frac{\mu_0 (w-x) D N}{2g} i$

$\boxed{\lambda = \frac{\mu_0 (w-x) D N^2}{2g} i}$

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b) $w_m' = \dfrac{\mu_0 (w-x) D N^2 i^2}{4g}$

$$\boxed{f^e = -\frac{\mu_0 D N^2 i^2}{4g}}$$

c) Select i

$$v = iR + \frac{\mu_0 (w-x) D N^2}{2g}\frac{di}{dt} - \frac{\mu_0 D N^2 i v}{2g}$$

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = -\frac{\mu_0 D N^2 i^2}{4g} - K(x-c)$$

OR

Select λ

$$v = \left(\frac{2g\lambda}{\mu_0 (w-x) D N^2}\right) R + \frac{d\lambda}{dt}$$

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = -\frac{\mu_0 D N^2}{4g}\left(\frac{2g\lambda}{\mu_0 (w-x) D N^2}\right)^2 - K(x-c)$$

$$= -\frac{g\lambda^2}{\mu_0 (w-x)^2 D N^2}$$

2. (20 points)

The co-energy of a device is given by

$$Wm'(i,x) = \frac{i^5}{24x} + \frac{i^3}{6x} + \frac{i}{x}$$

Find:

a) The force of electrical origin $f^e(i,x)$

b) The energy stored in the coupling field Wm as a function of i and x.

a) $f^e = -\dfrac{i^5}{24x^2} - \dfrac{i^3}{6x^2} - \dfrac{i}{x^2}$ N

b) $W_m = \lambda i - W_m' = \left(\dfrac{5i^4}{24x} + \dfrac{3i^2}{6x} + \dfrac{1}{x}\right)i - W_m'$

$$= \frac{5i^5}{24x} + \frac{3i^3}{6x} + \frac{i}{x} - \frac{i^5}{24x} - \frac{i^3}{6x} - \frac{i}{x}$$

$$= \frac{4i^5}{24x} + \frac{2i^3}{6x} = \boxed{\frac{i^5}{6x} + \frac{i^3}{3x} \text{ J}}$$

3. **(30 points)**

The flux linkage vs current expression for a rotational electromechanical system is:

$$\lambda = (L_0 + L_1 \cos 2\theta)i$$

If the system is operated with constant current i = 1 Amp as the angle θ is changed from zero degrees to 90 degrees, find:

a) The energy stored in the coupling field when the angle is at zero degrees
b) The energy stored in the coupling field when the angle is at 90 degrees
c) The energy transferred from the electrical system to the coupling field during this change
d) The energy transferred from the mechanical system to the coupling field during this change
e) The torque of electric origin when the angle is 45 degrees

$$W_m = W_m' = \frac{1}{2}(L_0 + L_1 \cos 2\theta)i^2$$

a) $W_{m_a} = \frac{1}{2}(L_0 + L_1)$ J

b) $W_{m_b} = \frac{1}{2}(L_0 - L_1)$ J

c) $$E_{FE_{a-b}} = \int_{L_0+L_1}^{L_0-L_1} 1 \, d\lambda = -2L_1 \text{ J}$$

d) $$E_{FM_{a-b}} = W_{m_b} - W_{m_a} - E_{FE_{a-b}} = \frac{1}{2}(L_0 - L_1) - \frac{1}{2}(L_0 + L_1) - (-2L_1)$$
$$= L_1 \text{ J}$$

e) $$\boxed{T^e = -L_1 \sin 2\theta \, i^2 = -L_1 \quad \text{Nm}}$$

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4. **(20 points)**

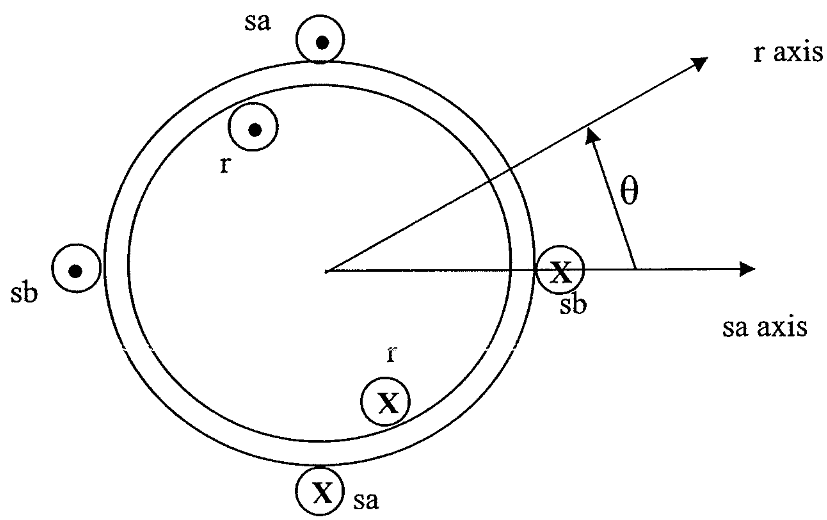
The rotating device shown below has two coils on the stator (sa and sb) plus one coil on the rotor (r). The magnetic axes are shown for sa and r. The three flux linkage vs current equations are:

$$\lambda_{sa} = L_s i_{sa} + (M \cos\theta)\, i_r$$

$$\lambda_{sb} = L_s i_{sb} + (M \sin\theta)\, i_r$$

$$\lambda_r = (M \cos\theta)\, i_{sa} + (M \sin\theta)\, i_{sb} + L_r i_r$$

Compute the torque of electric origin in the positive θ direction in terms of the three currents i_{sa}, i_{sb}, i_r, and the angle θ.



$$W_m' = \tfrac{1}{2} L_s i_{sa}^2 + \tfrac{1}{2} L_s i_{sb}^2 + M\cos\theta\, i_{sa} i_r + M\sin\theta\, i_{sb} i_r + \tfrac{1}{2} L_r i_r^2$$

$$\boxed{T^e = -M\sin\theta\, i_{sa} i_r + M\cos\theta\, i_{sb} i_r}$$