ECE 330 Exam #2, Fall 2013 Name: SOLUTION

90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. ________ / 25 2. ________ / 25

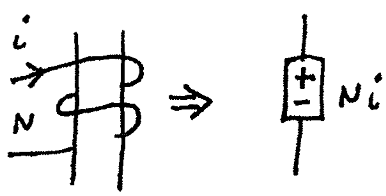
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



$W_m = \int_0^\lambda i \, d\hat{\lambda}$ $\quad W_m' = \int_0^i \lambda \, d\hat{i}$ $\quad W_m + W_m' = \lambda i$ $\quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\quad x \to \theta$

$f^e \to T^e$

$\underset{a \to b}{EFE} = \int_a^b i \, d\lambda$ $\quad \underset{a \to b}{EFM} = -\int_a^b f^e dx$ $\quad \underset{a \to b}{EFE} + \underset{a \to b}{EFM} = W_{mb} - W_{ma}$ $\quad \lambda = \frac{\partial W_m'}{\partial i}$ $\quad i = \frac{\partial W_m}{\partial \lambda}$

Problem 1. (25 points)

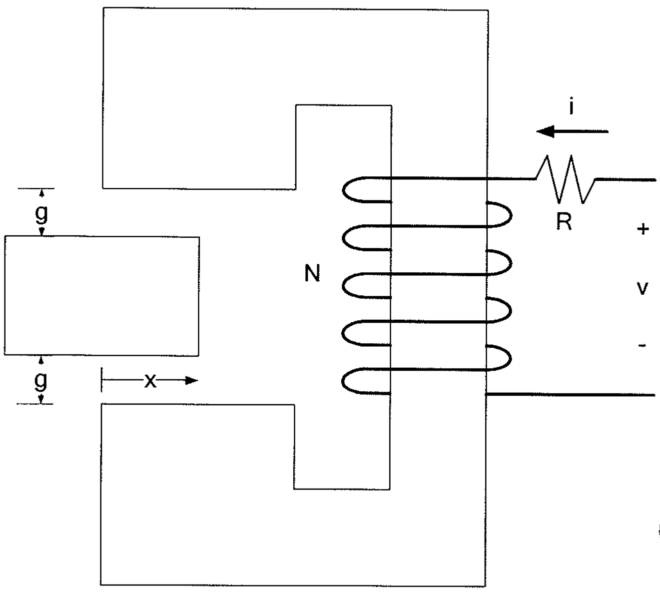
For the structure drawn below, the movable member is constrained to move left and right only as indicated in the figure where "x" is the distance to the right edge of the movable member. The large member with the coil is fixed, and the depth into the page for both members is 2cm. The gap g is 1mm, and the number of turns N =100. Find:

a) Total reluctance of the main flux path (through the two gaps).
b) Flux linkage, λ. (defined for the voltage polarity shown)
c) Co-energy, W_m'.
d) Force of electrical origin, f^e.
e) An expression for the voltage, v.

$g = 1\times 10^{-3}$

$A = 2\times 10^{-2} X$

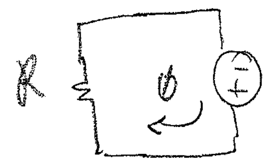
Express all of these as functions of current and/or position and/or velocity and/or time as appropriate. You may neglect fringing in the gap, and you may assume the iron is infinitely permeable.



ⓐ $R = \frac{l}{\mu A} = \frac{2g}{\mu X d} = \frac{250000}{\pi X}$

$\lambda = N\phi$

ⓑ



$\phi = \frac{Ni}{R}$

$\lambda = \frac{N^2 i}{R}$

$\lambda = \frac{N^2 i \mu x d}{2g} = 0.125 x i \text{ Wb}$

ⓒ $W_m' = \int \frac{N^2 i}{R} di = \frac{N^2 i^2}{2R} = 0.0628 x i^2$

ⓓ $f^e = \frac{\partial W_m'}{\partial x} = \frac{\partial}{\partial x}\left(\frac{N^2 i^2}{4g} \mu x d\right)$

$\frac{N^2 i^2 \mu d}{4g} = 0.0628 i^2 N$

ⓔ $v = Ri + \frac{d\lambda}{dt}$

$V = Ri + \frac{\partial \lambda}{\partial i}\frac{di}{dt} + \frac{\partial \lambda}{\partial x}\frac{dx}{dt}$ where $\frac{d\lambda}{di} = \frac{N^2}{R}$ $\frac{\partial \lambda}{\partial x} = \frac{N^2 i \mu d}{2g}$

$V = iR + 0.1257 x \frac{di}{dt} + 0.1257 i v$

Problem 2. (25 points.)

In class, generally we treated mutual inductances to be sinusoidal in rotational devices. In the system below, we have added a third harmonic (the 3θ terms). This gives the mutual inductance a more trapezoidal shape, which is common in some actual machines.

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & M(\cos\theta - 0.1\cos(3\theta)) \\ 0 & L_s & M(\sin\theta + 0.1\sin(3\theta)) \\ M(\cos\theta - 0.1\cos(3\theta)) & M(\sin\theta + 0.1\sin(3\theta)) & L_r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_r \end{bmatrix}$$

a) Find the co-energy in terms of the currents. $W_m' = \underbrace{\int_{\hat{i}_a=0}^{i_a} \lambda_a \, d\hat{i}_a}_{①} + \underbrace{\int_{\hat{i}_b=0}^{i_b} \lambda_b \, d\hat{i}_b}_{②} + \underbrace{\int_{\hat{i}_r=0}^{i_r} \lambda_r \, d\hat{i}_r}_{③}$

① $i_b = 0$, $i_r = 0$ ② i_a = const, $i_r = 0$ ③ i_a, i_b = const

① $= \frac{L_s i_a^2}{2}$

② $= \frac{L_s i_b^2}{2}$

③ $= M(\cos\theta - 0.1\cos(3\theta)) i_a i_r + M(\sin\theta + 0.1\sin(3\theta)) i_b i_r + \frac{L_r i_r^2}{2}$

$W_m' = \frac{L_s i_a^2}{2} + \frac{L_s i_b^2}{2} + \frac{L_r i_r^2}{2} + M i_r ([\cos\theta - 0.1\cos(3\theta)] i_a + [\sin\theta + 0.1\sin(3\theta)] i_b)$

b) Find an expression for the energy stored in the coupling field in terms of currents.

$W_m = \lambda_a i_a + \lambda_b i_b + \lambda_r i_r - W_m' = W_m'$ b/c linear

c) Find the torque of electric origin T^e in terms of the currents.

$T^e = \frac{\partial W_m'}{\partial \theta} = M i_r ([-\sin\theta + 0.3\sin(3\theta)] i_a + [\cos\theta + 0.3\cos(3\theta)] i_b)$

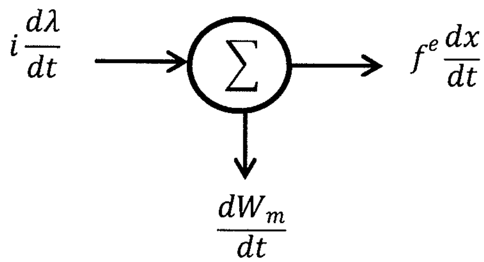
d) Suppose $i_a = i_b = i_r = 1$ A, $L_s = L_r = 1$ H, $M = 0.9$ H, and $\theta = 60°$. What is the contribution to the torque due to including the third harmonic (3θ) mutual inductance terms?

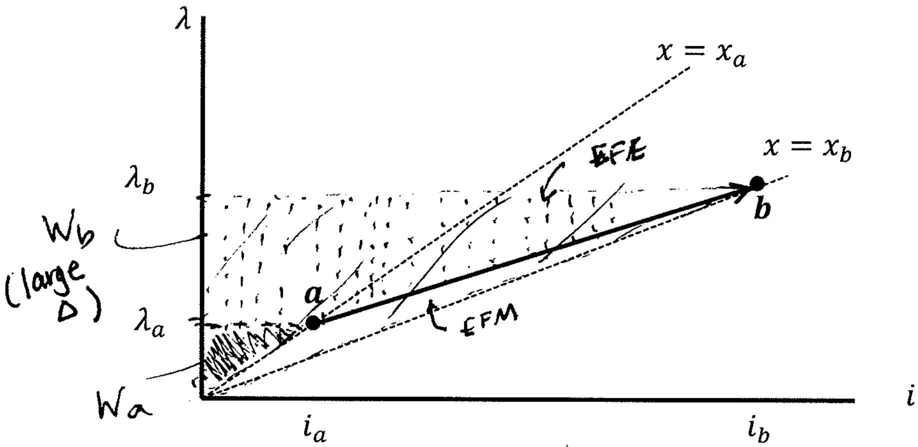
$T^e = \underbrace{M}_{0.9} \underbrace{i_r}_{1} (0.3\sin(\underbrace{3\theta}_{120})) \underbrace{i_a}_{1} + \underbrace{M}_{0.9} \underbrace{i_r}_{1} (0.3\cos(\underbrace{3\theta}_{120})) \underbrace{i_b}_{1}$

$= -0.27$ N·m

Problem 3. (25 points.)

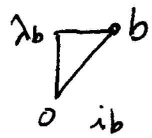
Use the block diagram and the graph to answer the questions below.





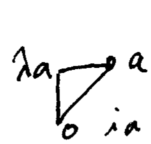
a) Find an expression for $W_{mb.}$ Draw and label it on the graph.

$$W_{mb} = \frac{\lambda_B i_B}{2}$$



b) Find an expression for W_{ma}. Draw and label it on the graph.

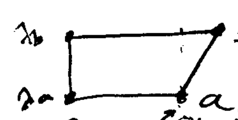
$$W_{ma} = \frac{\lambda_A i_A}{2}$$



c) Find an expression for the energy from electrical (EFE) on the path from a to b. Draw and label it on the graph.

$$\underset{a\to b}{EFE} = (\lambda_B - \lambda_A) i_a + \tfrac{1}{2}(\lambda_B - \lambda_A)(i_b - i_a)$$

$$= (\lambda_B - \lambda_A)\left(\frac{i_a + i_b}{2}\right)$$



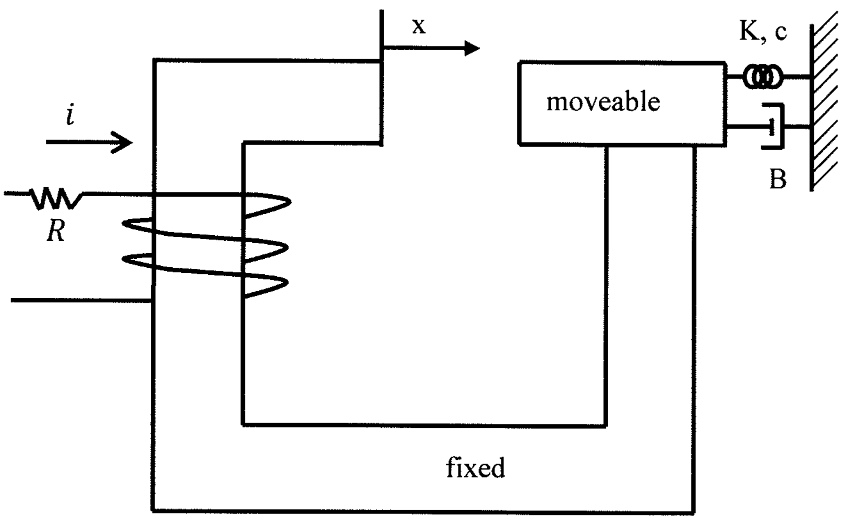
d) Find an expression for the energy from mechanical (EFM) on the path from a to b. Draw and label it on the graph.

$$\underset{a\to b}{EFM} = W_B - W_A - EFE = \frac{\lambda_B i_B}{2} - \frac{\lambda_A i_A}{2} - \left((\lambda_B - \lambda_a) i_a + \tfrac{1}{2}(\lambda_B - \lambda_a)(i_b - i_a)\right)$$

$$= \frac{i_b \lambda_a}{2} - \frac{i_a \lambda_b}{2}$$

Problem 4. (25 points.)

Consider the geometry given below. Both pieces are made of infinitely permeable material and have a constant width W and depth W into the page. The large air gap has permeability μ_0. Assume no fringing, and assume zero gap where the small piece touches the large piece.



At x= c, the spring exerts no force

$W = 2\,cm$ $\quad B = 2$

$c = 1\,cm$

$K = 0.2\,N/m$

$N = 100$

$m = 0.01\,kg$

$\frac{1}{W^2} = 2500$

$\mathcal{R} = \frac{x}{\mu_0 W^2}$

a) Compute λ (top conductor is positive) and f^e in terms of the current i.

Ni $\quad \phi = \frac{Ni}{\mathcal{R}}$ $\quad \lambda = \frac{N^2 i}{\mathcal{R}} = \frac{N^2 i \mu_0 W^2}{x} = \left(\frac{5i}{x}\right) \times 10^{-6}$

$i = x\lambda / (N^2 \mu_0 W^2)$

$W_m' = \int_{\hat{i}=0}^{i} \lambda\, d\hat{i} = \frac{N^2 i^2 \mu_0 W^2}{2x}$ $\quad f^e = \frac{\partial W_m'}{\partial x} = \frac{-N^2 i^2 \mu_0 W^2}{2x^2} = \left(\frac{-2.5 i^2}{x^2}\right) \times 10^{-6}$

$f^e(\lambda) = \frac{-N^2 \mu_0 W^2}{2\cancel{x^2}} \cdot \frac{\lambda^2 \cancel{x^2}}{N^4 \mu_0^2 W^4} = \frac{-\lambda^2}{2N^2 \mu_0 W^2}$

b) Write the equations of motion on both the electrical and mechanical side using flux linkage as a dynamic state variable.

Elect: $v = iR + \frac{d\lambda}{dt} = \frac{x\lambda}{N^2\mu_0 W^2} R + \frac{d\lambda}{dt}$

$M \frac{d^2x}{dt^2} = f^e - K(x - c) - B\frac{dx}{dt}$

Mech: $M \frac{d^2x}{dt^2} = \frac{-\lambda^2}{2N^2 \mu_0 W^2} - K(x - c) - B\frac{dx}{dt}$

$\frac{dx}{dt} = v$

$v = \frac{x_2 x_1 R}{N^2 \mu_0 W^2} + \dot{x}_1$

$M\dot{x}_3 = \frac{-x_1^2}{2N^2\mu_0 W^2} - K(x_2 - c) - Bx_3$

$\dot{x}_2 = x_3$

c) The system starts from a condition with λ~~(0) = 1~~ Wb, x(0) =0.01 m, and dx/dt$|_0$ = 0. The electrical input operates to maintain λ constant. Provide an estimate of the velocity dx/dt at time t = 0.02 s. (Hint: Use Euler's method with a step size of 0.01 s)

$x_1 \quad 0.005 \quad x_2 \quad x_3$

$\dot{x}_1 = v - \frac{x_2 x_1 R}{N^2 \mu_0 W^2}$

$\dot{x}_2 = x_3$

$\dot{x}_3 = \frac{1}{0.01}\left(\frac{-x_1^2}{1 \times 10^{-5}} - K(x_2 - c) - Bx_3\right)$

$X(0) = \begin{bmatrix} 0.005 \\ 0.01 \\ 0 \end{bmatrix}$

$X(0.01) = \begin{bmatrix} 0.005 \\ 0.01 \\ 0 \end{bmatrix} + (0.01)\begin{bmatrix} 0 \\ 0 \\ -250 \end{bmatrix}$

$X(0.01) = \begin{bmatrix} 0.005 \\ 0.01 \\ -2.5 \end{bmatrix}$

$X(0.02) = \begin{bmatrix} 0.005 \\ 0.01 \\ -2.5 \end{bmatrix} + (0.01)\begin{bmatrix} 0 \\ -2.5 \\ 250 \end{bmatrix} = \begin{bmatrix} 0.005 \\ -0.015 \\ 50 \end{bmatrix}$