ECE430
Exam #2
Fall 2005

Name Solution
(Print Name)

Section: (Circle One) 10 MWF 2 MWF

Problem 1 _______ **Problem 2** _______ **Problem 3** _______ **Problem 4** _______

TOTAL: ________

USEFUL INFORMATION

$$\oint_c \underline{H}\cdot d\underline{l} = \int_s \underline{J}\cdot\underline{n}\,da \qquad \oint_c \underline{E}\cdot d\underline{l} = -\frac{d}{dt}\int_s \underline{B}\cdot\underline{n}\,da \qquad \oint_s \underline{B}\cdot\underline{n}\,da = 0$$

$$mmf = Ni = \phi R \qquad R = \frac{l}{\mu A} \qquad \phi = BA \qquad B = \mu H \qquad \lambda = N\phi$$

$$W_m = \int i \Big|_{x=const} d\lambda \qquad W_m' = \int \lambda \Big|_{x=const} di \qquad W_m + W_m' = \lambda i$$

$$f^e = -\frac{\partial W_m}{\partial x} \qquad f^e = \frac{\partial W_m'}{\partial x}$$

for rotation, $x \rightarrow \theta$, $f \rightarrow T$

$$\underset{a-b}{EFE} = \int_{\lambda_a}^{\lambda_b} i \Big|_{path} d\lambda \qquad \underset{a-b}{EFM} = -\int_{x_a}^{x_b} f^e \Big|_{path} dx$$

$$W_{m_b} - W_{m_a} = \underset{a-b}{EFE} + \underset{a-b}{EFM} \qquad x(t_0+\Delta t) \approx x(t_0) + \frac{dx}{dt}\Big|_{t_0} \Delta t$$

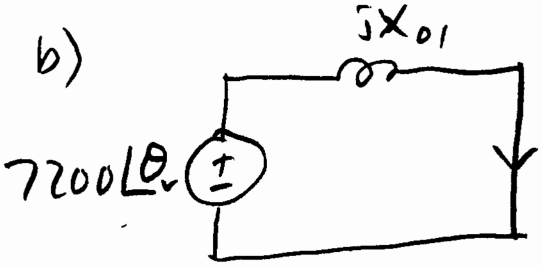
Problem 1 (25 pts.)

A single-phase transformer is rated for 7,200 Volts (RMS) on the primary (source) side and 240 Volts (RMS) on the secondary (load) side. It has a power rating of 50 KVA. Neglect all resistance and the shunt magnetizing reactance in the transformer.

a) What are the rated currents on the primary and secondary sides?
b) What should the series equivalent reactance as seen from the primary side be in order to limit the short circuit current (under rated voltage) to 8 times rated?
c) What should the series equivalent reactance as seen from the primary side be in order to limit the voltage drop across the transformer to 5% of rated when the transformer is loaded at rated current with unity power factor?

a) $I_p = \frac{50k}{7200} \approx 7A$ $\quad I_s = \frac{50k}{240} = 208A$

b)

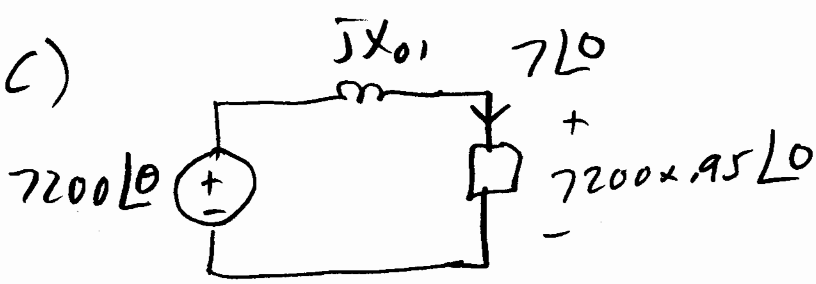


$I_{sc} = 7 \times 8 = 56A\angle\theta_i$

$7200\angle\theta_v = 56\angle\theta_i \times jX_{01}$

$56\, X_{01} = 7200$ $\quad \boxed{X_{01} = 129\,\Omega}$

c)



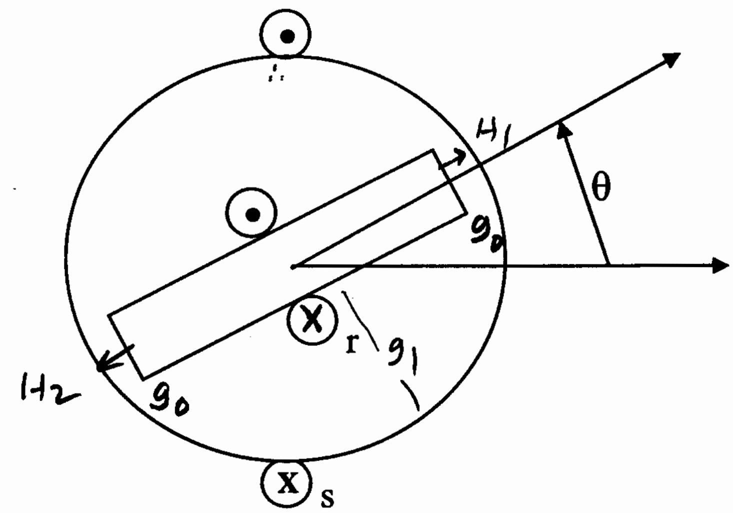
$7200\angle\theta = j7X_{01} + 6840\angle 0$

$7200^2 = 6840^2 + 49X_{01}^2$

$\boxed{X_{01} = 321\,\Omega}$

Problem 2 (25 pts)

A single-phase, salient-pole rotating machine is shown below:



a) Write a general form for the approximate self inductance of the rotor coil.
b) Write a general form for the approximate self inductance of the stator coil.
c) Write a general form for the approximate mutual inductance between the rotor and stator coils.
d) Derive an approximate expression for the self inductance of the rotor coil in terms of typical parameters.
e) Derive an approximate expression for the self inductance of the stator coil in terms of typical parameters.
f) Give an approximate expression for the mutual inductance between the rotor and stator coils in terms of typical parameters.

a) L_r b) $L_0 + L_1 \cos 2\theta$ c) $m \cos\theta$

d) $H_1 g_0 - H_2 g_0 = N_s i_s + N_r i_r$ (assumes only H through small g)

$\mu_0 H_1 A_0 + \mu_0 H_2 A_0 = 0$ $H_1 = -H_2 = \frac{N_s i_s + N_r i_r}{2 g_0}$

$$\lambda_r = N_r \mu_0 H_1 A_0 = \frac{\mu_0 A_0 N_r N_s}{2 g_0} i_s + \frac{\mu_0 A_0 N_r^2}{2 g_0} i_r$$

$$\boxed{L_r = \frac{\mu_0 A_0 N_r^2}{2 g_0}}$$

e) For the stator,

$$L_{ss} = L_0 + L_1 \cos 2\theta$$

$$L_{max} = \frac{\mu_0 A_0 N_s^2}{2 g_0} \quad (\theta = 0)$$

$$L_{min} = \frac{\mu_0 A_1 N_s^2}{2 g_1} \quad \left(\theta = \frac{\pi}{2}\right)$$

$L_0 + L_1 = L_{max}$

$L_0 - L_1 = L_{min}$

$$L_0 = \frac{L_{max} + L_{min}}{2} \qquad L_1 = \frac{L_{max} - L_{min}}{2}$$

f) $m = \frac{\mu_0 A_0 N_s N_r}{2 g_0}$

Problem 3 (25 pts.)

A mathematical model of an electromechanical system is:

$\lambda_1 = (a/x)\, i_1 + (b/x)\, i_3$

$\lambda_2 = (c/x)\, i_2 + (d/x)\, i_3$

$\lambda_3 = (e/x)\, i_1 + (f/x)\, i_2 + (g/x)\, i_3$

a) If this relationship came from a conservative coupling field, what can you say about the constants a, b, c, d, e, f, g?
b) Find an expression for the force of electrical origin in the positive x direction
c) Find an expression for the energy stored in the coupling field in terms of the currents plus x and the given parameters

a) Symmetric L-matrix: $L_{13} = L_{31}$ so $\boxed{b = e}$
$L_{23} = L_{32}$ so $\boxed{d = f}$

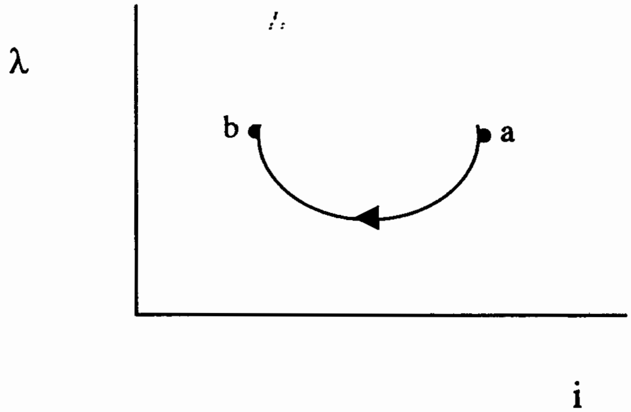
b) $W_m' = \frac{a}{2x} i_1^2 + \frac{c}{2x} i_2^2 + \frac{e}{x} i_1 i_3 + \frac{f}{x} i_2 i_3 + \frac{g}{2x} i_3^2$

$$\boxed{f^e = -\frac{a}{2x^2} i_1^2 - \frac{c}{2x^2} i_2^2 - \frac{e}{x^2} i_1 i_3 - \frac{f}{x^2} i_2 i_3 - \frac{g}{2x^2} i_3^2}$$

c) Linear system so $W_m = W_m'$ (above)

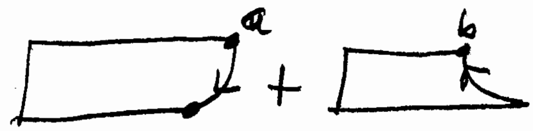
Problem 4 (25 pts.)

An electromechanical system with $\lambda = L(x)i$ is operated through the transition from a to b as shown below:



a) Find the energy transferred from the electrical system into the coupling field as the system moves from a to b as shown. (give a graphical answer)

b) Find the energy transferred from the mechanical system into the coupling field as the system moves from a to b as shown (give a graphical answer)

a) $EFE_{a-b} = \int_{path} i\,d\lambda = -$ 

$= -$ 

b) $EFM_{a-b} = W_{mb} - W_{ma} - EFE_{a-b} =$