USEFUL INFORMATION

\[ \sin(x) = \cos(x - 90^\circ) \]
\[ \bar{V} = \bar{Z} \bar{I} \quad \bar{S} = \bar{V} \bar{I}^* = P + jQ \quad \bar{S}_{3\varphi} = \sqrt{3} V_L I_L \angle \theta \]

\( 0 < \theta < 180^\circ \) (lag)
\( I_L = \sqrt{3} I_\varphi \) (delta)
\( \bar{Z}_Y = \bar{Z}_\Delta / 3 \)

\( -180^\circ < \theta < 0 \) (lead)
\( V_L = \sqrt{3} V_\varphi \) (wye)
\( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)

**ABC phase sequence** has A at 0, B at -120°, and C at +120°

\[ \int H \cdot dl = \int J_f \cdot \hat{n} dA \quad \int E \cdot dl = -\frac{d}{dt} \left( \int B \cdot \hat{n} dA \right) \quad R = \frac{l}{\mu A} \quad Ni = N\varphi \]

\[ \varphi = BA \]
\[ \lambda = N\varphi = Li \) (if linear)
\[ v = \frac{d\lambda}{dt} \quad k = \frac{M}{\sqrt{L_1 L_2}} \]

1 hp = 746 W
Problem 1 (25 Points)

A single phase source is serving three loads connected in parallel, through a feeder with impedance $\bar{Z}_{\text{feeder}} = 1 + j2 \, \Omega$.

- Load 1 is an impedance of $30 + j40 \, \Omega$.
- Load 2 draws 200 W at a lagging PF of 0.8.
- Load 3 draws 1 A at a leading PF of 0.6.

The voltage across the loads is $\bar{V}_{\text{load}} = 100\angle0^\circ \, V$.

a) Compute the current phasors $\bar{I}_1$, $\bar{I}_2$, and $\bar{I}_3$ for each of the three loads (20 points total).

\[
\bar{V}_L = 100\angle0^\circ \, V
\]
\[
\bar{I}_1 = \frac{\bar{V}_L}{\bar{Z}_1}
\]
\[
\bar{Z}_1 = 30 + j40 \, \Omega = 50\angle53.13^\circ \, \Omega
\]
\[
\bar{I}_1 = \frac{100\angle0^\circ}{50\angle53.13^\circ} \Rightarrow \bar{I}_1 = 2\angle-53.13^\circ \, A
\]

\[
\bar{I}_2 = \frac{P_2}{\bar{V}(\cos\phi)}
\]
\[
\bar{I}_2 = \frac{200}{100(\cos0.8)} \Rightarrow \bar{I}_2 = 2.5\angle-36.87^\circ \, A
\]

\[
\bar{I}_3 = 1\angle\cos^{-1}(0.6)
\]
\[
\bar{I}_3 = 1\angle53.13^\circ \, A
\]
c) What is the source voltage $\overline{V}_{\text{source}}$? (3 points)

\begin{align*}
\overline{I}_{\text{tot}} &= \overline{I}_1 + \overline{I}_2 + \overline{I}_3 \\
&= (1.2 - j1.6) + (2 - j1.5) + (0.6 + j0.8) \\
&= 3.8 - j2.3 \text{ A} \\
&= 4.44 \angle -31.18^\circ \text{ A}
\end{align*}

\begin{align*}
\overline{V}_0 &= \overline{V}_{\text{load}} + \overline{Z}_{\text{line}} \overline{I}_{\text{tot}} \\
&= 100 \angle 0^\circ + (2.24 \angle 63.43^\circ)(4.44 \angle -31.18^\circ) \\
&= 100 \angle 0^\circ + 9.94 \angle 32.25^\circ \\
\overline{V}_0 &= 108.41 + j307 \text{ V} \\
&= 108.54 \angle 2.86^\circ \text{ V}
\end{align*}

d) What is the power factor at the source? State whether it is leading or lagging. (1 point)

\begin{align*}
\theta_s &= (280^\circ - [-31.18^\circ]) = 33.98^\circ \\
\text{PF}_s &= 0.829 \text{ lagging}
\end{align*}

e) What is the power factor at the load? State whether it is leading or lagging. (1 point)

\begin{align*}
\theta &= 31.18^\circ \\
\text{PF}_l &= 0.856 \text{ lagging}
\end{align*}
Problem 2 (25 Points)

A Wye-connected three-phase generator delivers 1200 kVA at 0.6 PF lagging and 4160 volt (line-to-line) to the following loads in parallel.
- Load 1 is connected in Wye, and draws a total of 300 kW at unity power factor.
- Load 2 is an impedance load connected in Delta, with unknown per-phase impedance $\bar{Z}$.

a) Draw the three-phase circuit diagram. Label all relevant instances of the following:
- Phase-to-neutral voltage phasor (e.g. $10\angle 50^\circ$ V).
- Per-phase impedance for load 2 as symbolic expressions (e.g. $\bar{Z}/\sqrt{3}$ Ω).
Include polarity markers (+/-) in all of your voltage labels. (11 points)

b) Draw the single-phase equivalent circuit for phase a. Label one instance of the following:
- Voltage phasor (e.g. $10\angle 50^\circ$ V).
- Power for the generator in terms of kVA and PF (e.g. 10 kVA at 0.1 PF leading).
- Power for load 1 in terms of kVA and PF (e.g. 10 kVA at 0.1 PF leading).
- Impedance for load 2 as a symbolic expression (e.g. $\bar{Z}/\sqrt{3}$ Ω).
All quantities should be stated in per-phase values. (6 points)
d) How much three-phase kVARs do we need to bring the combined power factor to unity? How would you connect the capacitors (in Wye or Delta) and why? (3 points)

\[
\bar{S}_0 = 1200 \angle 33.15^\circ \text{ kVA} = 720 + j 960 \text{ kVA}
\]

Add 960 kVAR of capacitance

- **Delta**: lower capacitance needed to get given power, lower phase current
- **Wye**: lower phase voltage
- Either complex power equation is the same for Wye and Delta

\[\bar{Z} = \frac{3V^2}{\bar{I}_0}\]

\[\bar{Z} = 49.5 + 4 \angle 60.57^\circ \Omega\]

e) Compute the numerical value for the unknown impedance \(\bar{Z}\). (5 points)
A coil is wrapped 300 times around an iron core with the given dimensions and a depth of 2 cm into the page. The iron has a permeability \( \mu = 2000\mu_0 \) and contains an air gap of \( g = 2 \) mm. Fringing effects can be neglected.

a) Draw the magnetic path through the iron core on the figure above and draw the corresponding magnetic equivalent circuit. (15 points).
b) What is the total flux \( \phi \) through the circuit? (7 points)

\[
N_i = \phi R_{tot} \\
R_{tot} = \left( \frac{1}{R_3} + \frac{1}{R_{tot} R_0} \right)^{-1} + R_i \\
= 4.538 \times 10^4 \text{ A}^2/\text{Wb} \\
300i = 4.538 \times 10^4 \phi \\
\phi = 6.601 \times 10^{-4} \text{ Wb}
\]

\[
N_i = \phi R_{tot} \\
R_{tot} = \left( \frac{1}{R_3} + \frac{1}{R_{tot} R_0} \right)^{-1} + R_i \\
= 4.538 \times 10^4 \text{ A}^2/\text{Wb} \\
300i = 4.538 \times 10^4 \phi \\
\phi = 6.601 \times 10^{-4} \text{ Wb}
\]

c) What is the inductance of the coil? (3 points)

\[
\lambda = N \phi = L i \\
\lambda = 300 (6.601 \times 10^{-4}) i \\
\lambda = 0.198 i \\
L = 0.198 \text{ H}
\]

\[
\lambda = N \phi = L i \\
\lambda = 300 (6.601 \times 10^{-4}) i \\
\lambda = 0.198 i \\
L = 0.198 \text{ H}
\]
Two coils are wrapped around an iron core with the given dimensions and depth of 3 cm into the page. Coil 1 contains 400 turns and Coil 2 contains 200 turns. The iron has a permeability $\mu = 1500\mu_0$ and contains an air gap of $g = 4$ mm. Fringing effects can be neglected.

a) Draw on the figure above where the dot marks should go for each coil. (2 points)

b) What is the self-inductance for each coil ($L_1$ and $L_2$) and the mutual inductance between the two coils ($M$)? (18 points)
c) What is the open circuit voltage measured across Coil 2 if a current of \( i_1 = \sqrt{2}(10)\cos(377t) \) is applied across Coil 1? Write your answer as a cosine function. (5 points)

\[
\begin{align*}
V_1 &= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\
V_2 &= -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \\
\text{Open circuit: } \quad i_2 &= 0 \\
V_2 &= -0.0667 \left(-\frac{1}{2}(10)(377)\sin(377t)\right) \\
&= \frac{1}{2}(251.6)\sin(377t) \\
V_2 &= \frac{1}{2}(251.6)\cos(377t - 90) \\
\end{align*}
\]
(Blank page for extra work)