ECE 430 Exam #1, Fall 2009 Name: Solution
90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. 25 / 25 2. 25 / 25

3. 25 / 25 4. 25 / 25 Total 100 / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle \theta$

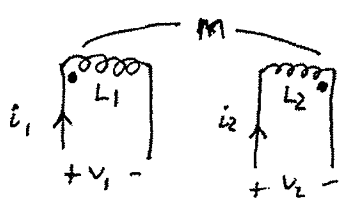
$0 < \theta < 180^\circ$ (lag)
$-180^\circ < \theta < 0$ (lead)

$I_L = \sqrt{3} I_\phi$ (delta)
$V_L = \sqrt{3} V_\phi$ (wye)

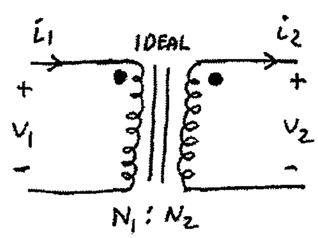
$\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi \Re$

$\phi = BA$ $\quad \lambda = Li = N\phi$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

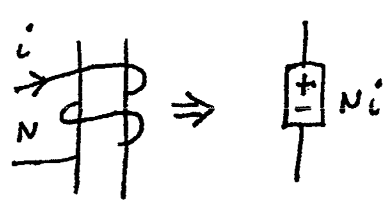


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$



$$\bar{V} = \frac{170}{\sqrt{2}} \angle -90^\circ$$

1. (25 points)

Three single-phase loads are connected in parallel. The load voltage is $v_{load} = 170\sin(120\pi t)$.
The load characteristics are:
Load #1: 80 Amps (RMS) and 7kW of real power (inductive load) $\quad S = 9618 \quad pf = .728$ lag
Load #2: 8 kVA at 0.7 power factor lag
Load #3: $(5 + j2)\Omega$

The impedance of each of the two wires that serves this combination of loads is $(0.1 + j0.1)\Omega$.

a) Find the total real and reactive powers provided by the source (10 points)
b) Find the source voltage and current time domain quantities (10 points)
c) Find the overall system power factor (5 points)

a) $\bar{S}_1 = \frac{170}{\sqrt{2}} \times 80 \angle 43.3^\circ = 9618 \angle 43.3^\circ = 7{,}000 + j6{,}596$

$\bar{S}_2 = 8000 \angle 45.6^\circ = 5{,}597 + j5{,}716$

$\bar{S}_3 = \frac{170}{\sqrt{2}} \angle -90^\circ \left(\frac{\frac{170}{\sqrt{2}} \angle -90}{5 + j2} \right)^* = \frac{14{,}450}{5.39 \angle -21.8^\circ} = 2{,}680 \angle +21.8^\circ$

$= 2488 + j995$

$\bar{S}_{load} = 15{,}085 + j13{,}307$

$= 20{,}115 \angle 41.4^\circ$

$\bar{I} = \frac{20{,}115 \angle -41.4^\circ}{120.2 \angle 90} = 167.35 \angle -131.4^\circ$

$\bar{V}_S = 167.35 \angle -131.4^\circ \times 2 \times (0.1 + j0.1) + 120.2 \angle -90^\circ$

$= 47.33 \angle -86.4^\circ + 120.2 \angle -90 = 3 - j44.3 - j120.2$

$= 3 - j167.4 = 167.46 \angle -89^\circ$

b) $v_S(t) = 237 \cos(120\pi t - 89^\circ)$ V

$i(t) = 237 \cos(120\pi t - 131^\circ)$ A

a) $\bar{S}_{TOT} = 167.46 \angle -89 \times 167.35 \angle 131.4 = 28{,}024 \angle 42.4^\circ$

$= 20{,}694 + j18{,}897$
W $\quad$ VARS

c) $PF = 0.738$ lag

2. (25 points)

The following three-phase, balanced loads are connected across a three-phase, wye-connected source of 480 Volts (RMS – line to line):

Load #1: Wye-connected load with line current of 58 Amps (RMS) at 0.85 PF lag;
Load #2: Delta-connected load with 80 KW (3-phase) at 0.8 PF lag;

Calculate the following:

a) The total complex power (3-phase) consumed by both loads (5 points)
b) Total source line current RMS magnitude (5 points)
c) The phase current RMS magnitude for each load. (10 points)
d) A delta-connected capacitor bank is added in parallel to make the overall power factor equal to unity. Determine the required VARS per phase. (5 points)

a) $S = \sqrt{3} \times 480 \times 58 \angle 31.8^\circ + \frac{80K}{0.8} \angle 36.9^\circ$

$= 40,981 + j\,25,409 + 80,000 + j\,60,000$

$= 120,981 + j\,85,409 = 148,091 \angle 35.2^\circ$

b) $|S| = \sqrt{3} \times 480 \times I = 148,091$ $\boxed{I = 178A}$

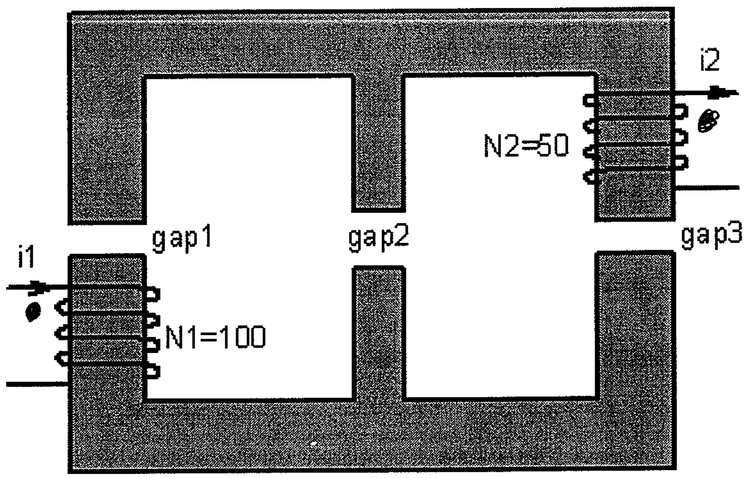
c) $I_{\phi_1} = 58A$ $I_{\phi_2} = \frac{80K/3}{0.8 \times 480} = 69.4A$

d) $Q_{cap} = -85,409$ VARS

OR 28,469 VARS/phase

3. (25 points)

A magnetic piece with two coils is shown below. The permeability of the magnetic core is infinitely large, hence the magnetic reluctance of the metal materials can be ignored. The dimensions of the air gaps are summarized in the figure. Ignore fringing effects.

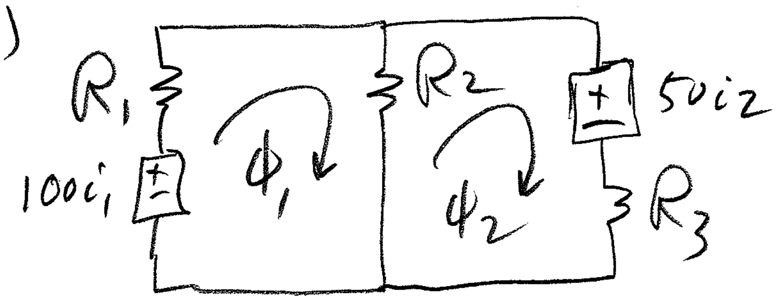


Gap 1: Distance: 5 mm; cross-sectional area: 1 cm^2
Gap 2: Distance: 8 mm; cross-sectional area: 0.5 cm^2
Gap 3: Distance: 5 mm; cross-sectional area: 1 cm^2

a) Label the dots on the two coils
b) Draw the magnetic equivalent circuit
c) Find the <u>magnitude</u> of the self-inductances and the mutual inductance

a) see figure

b)



$$R_1 = \frac{.005}{4\pi \times 10^{-7} \times 10^{-4}} = 3.98 \times 10^7 = R_3$$

$$R_2 = \frac{.008}{4\pi \times 10^{-7} \times .5 \times 10^{-4}} = 12.73 \times 10^7$$

$$-100 i_1 + 3.98\times10^7 \phi_1 + 12.73\times10^7(\phi_1 - \phi_2) = 0$$

$$50 i_2 + 12.73\times10^7(\phi_2 - \phi_1) + 3.98\times10^7 \phi_2 = 0$$

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$$\phi_2 = \frac{-100 i_1 + 3.98\times10^7 \phi_1}{12.73\times10^7} + \phi_1$$

$$50 i_2 + \left(-100 i_1 + 3.98\times10^7 \phi_1 + 3.98\times10^7 \left(\frac{-100 i_1 + 3.98\times10^7 \phi_1}{12.73\times10^7} + \phi_1\right) = 0$$

$$50 i_2 - 131.3 i_1 + 9.2\times10^7 \phi_1 = 0$$

$$\phi_1 = \frac{-50}{9.2\times10^7} i_2 + \frac{131.3}{9.2\times10^7} i_1$$

$$\lambda_1 = N_1 \phi_1 = \frac{-5000}{9.2\times10^7} i_2 + \frac{13130}{9.2\times10^7} i_1$$

$$\underline{M = 5.4\times10^{-5}\ H} \qquad \underline{L_1 = 1.43\times10^{-4}\ H}$$

$$\phi_2 = \frac{-100}{12.73\times10^7} i_1 + 1.31 \phi_1 = \frac{-100}{12.7\times10^7} i_1 + 1.87\times10^{-6} i_1 - 7.13\times10^{-7} i_2$$

$$\lambda_2 = N_2 \phi_2 = 5.4\times10^{-5} i_1 - 3.5\times10^{-5} i_2$$

$$\underline{L_2 = 3.5\times10^{-5}\ H}$$

4. (25 points)

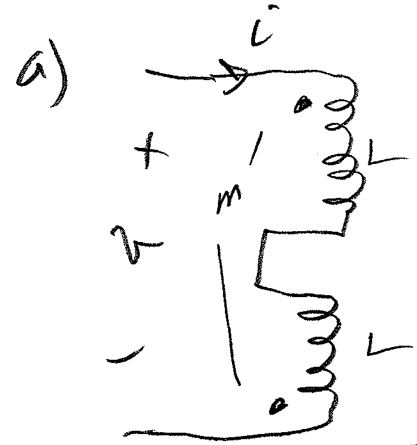
Two identical but mutually coupled lossless (R=0) coils are connected in two ways, and the inductance as seen by the combination is measured in each case.

a) *In series* with the undotted terminal of one coil connected to the undotted terminal of the other coil. L_{eq}=0.08 H.

b) *In parallel* with the dotted terminals connected together and the undotted terminals connected together. L_{eq} = 0.05 H.

Find the self inductance of each coil (L), the mutual inductance between the two coils (M), and the coefficient of coupling k.

a)

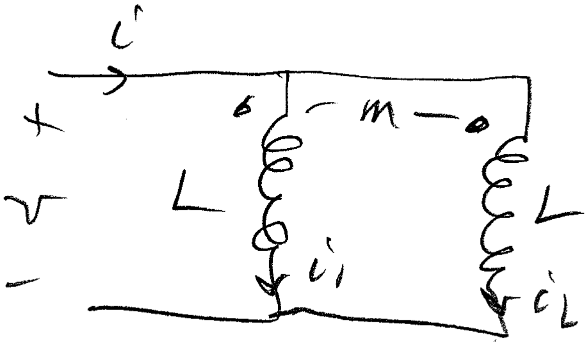


$$v = .08\frac{di}{dt} = L\frac{di}{dt} - m\frac{di}{dt} + L\frac{di}{dt} - m\frac{di}{dt}$$

$$2(L-m) = .08$$

$$\boxed{L - m = .04}$$

b)



$$v = .05\frac{di}{dt} = L\frac{di_1}{dt} + m\frac{di_2}{dt} = L\frac{di_2}{dt} + m\frac{di_1}{dt}$$

$$(L-m)\frac{di_1}{dt} = (L-m)\frac{di_2}{dt}$$

$$i_1 + i_2 = i \implies \frac{di_1}{dt} = \frac{di_2}{dt} \qquad \frac{di}{dt} = 2\frac{di_1}{dt}$$

$$v = .05\frac{di}{dt} = 0.1\frac{di_1}{dt} = (L+m)\frac{di_1}{dt}$$

$$\boxed{L+m = 0.1}$$

$2L = .14$

$$\boxed{\begin{array}{l} L = .07 \\ m = .03 \end{array}}$$

$$\boxed{k = \frac{3}{7}}$$