Schedule
- Mon 9/23: Coupled coils
  - Wed 9/25: Ideal transformers
  - Friday 9/27: Review + Quiz 4
- Mon 9/30: Review
- Wed 10/2: Review
- Thursday 10/3: Mid-term Exam 1
  - Friday 10/4: No class

War of Currents (circa 1880-1900)
- Edison
- Westinghouse
- Tesla

1880 Edison receives patent for lightbulb, founds the Edison Illuminating Company. (Today: ConEd)
1888 Tesla joins Westinghouse to commercialize AC.
1888 Edison: “It will never be free from danger.”

IN MEMO CONCERNING THE WESTINGHOUSE AC SYSTEM.
1893 Westinghouse wins contract for Chicago World Fair.
1908 Edison: “Tell your father I was wrong.”

TO GEORGE STANLEY, THE SON OF WILLIAM STANLEY WHO HAD INVENTED AN AC TRANSFORMER FOR WESTINGHOUSE.

The Transformer: AC’s secret weapon

\[ v_1(t) = \frac{N_2}{N_1} v_2(t) \]

Winding 1 (Primary) \( N_1 \) : \( N_2 \)

Winding 2 (Secondary)

\[ v_1(t) \]
\[ v_2(t) \]

Turns ratio

\( N_2 > N_1 \) Step-up
\( N_2 < N_1 \) Step-down
\( N_2 = N_1 \) Isolation

Transformers in the AC power system

\[ P_{loss} = I^2 R \]

220 kV

11 kV

33 kV

110 V

Genius is one percent inspiration and ninety-nine percent perspiration.

--- Thomas Edison (c. 1903)

Just a little theory and calculation would have saved him ninety percent of his labor.

--- Nikola Tesla (1931)
Today

- Ideal transformer model
- Apparent impedance
- Practical transformer model

Transformer (ideal)

\[ i_1(t) \quad i_2(t) \]
\[ v_1(t) \quad v_2(t) \]

\[ N_1 : N_2 \]

Turns ratio

\[ a = \frac{N_1}{N_2} \]

Definition (ideal transformer)

\[ \frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = a \]

(Transformation ratio)

\[ P_{in} = P_{out} = 0 \]

(Power conservation)

Implies current transformation

\[ 0 = v_1(t)i_1(t) + v_2(t)i_2(t) \]

\[ = v_1(t)\left(i_1(t) + \frac{N_2}{N_1}i_2(t)\right) \]

\[ = i_1(t) + \frac{N_2}{N_1}i_2(t) \]

\[ \frac{i_1(t)}{i_2(t)} = -\frac{N_2}{N_1} = -\frac{1}{a} \]

Minus sign!!

Example 1. High voltage transmission

\[ + \quad \text{source} \quad 110 \text{ V} \quad 100 \text{ A} \quad + \]
\[ \text{wire} \quad 10 \Omega \quad + \quad \text{load} \quad 1 \Omega \quad - \]

\[ + \quad \text{source} \quad 100.1 \text{ V} \quad 100 \text{ A} \quad + \]
\[ \text{wire} \quad 0.001 \Omega \quad + \quad \text{load} \quad 100 \text{ V} \quad 100 \text{ A} \quad - \]

10x step-up yields 100x distance
(for the same efficiency)

Why Edison lost the war (1890s)

Why Edison lost the war (1960s)
Today

- Ideal transformer model
  - Apparent impedance
  - Practical transformer model

Ideal transformer with resistance

\[ \begin{align*}
  i_1(t) &= \frac{N_1}{N_2} i_2(t) \\
  v_1(t) &= a v_2(t) \\
  a &= \frac{N_1}{N_2}
\end{align*} \]

Example 2. Warm-up

\[ \begin{align*}
  \text{N}_1 &= 100, \text{ N}_2 = 10 \\
  \text{A)} &= 1 \Omega \\
  \text{B)} &= 10 \Omega \\
  \text{C)} &= 100 \Omega \\
  \text{D)} &= 1,000 \Omega \\
  \text{E)} &= 10,000 \Omega
\end{align*} \]

Ideal transformer with impedance

\[ \begin{align*}
  \frac{v_1(t)}{v_2(t)} &= a \\
  \frac{i_1(t)}{i_2(t)} &= - \frac{1}{a} \\
  a &= \frac{N_1}{N_2}
\end{align*} \]

Example 3. High voltage transmission

\[ \begin{align*}
  \frac{v_1}{v_2} &= -a \\
  \frac{i_1}{i_2} &= \frac{N_2}{N_1} \\
  a &= \frac{N_1}{N_2}
\end{align*} \]

Example 4. Power transfer

\[ \begin{align*}
  P &= |I_2|^2 R = \left( \frac{N_1}{N_2} \right)^2 |I_1|^2 R = |I_1|^2 R' \\
  R' &= \left( \frac{N_1}{N_2} \right)^2 R
\end{align*} \]
Maximum power transfer

\[ V_1 = 1 \text{ [V_{rms}]} \]

\[ 1 \text{ [Ω]} \]

\[ \text{Power} = ? \text{ [W]} \]

\[ 4 \text{ [Ω]} \]

Apparent resistor matches source resistor

High current

Low voltage

\[ P_{\text{load}} \]

\[ 1/2 \]

\[ N_1 / N_2 \]

Impedance matching

\[ V \]

\[ \text{Ideal} \]

\[ Z_{\text{source}} \]

\[ Z_{\text{load}} \]

Theorem. Maximum power transfer if ratio \( N_1 : N_2 \) satisfies

\[ |Z_{\text{source}}| = \left( \frac{N_1}{N_2} \right)^2 |Z_{\text{load}}| \]

Apparent load impedance

Today

- Ideal transformer model
- Apparent impedance
- Practical transformer model

So far: Ideal transformers

\[ v_1(t) + \]

\[ v_2(t) = \frac{N_2}{N_1} v_1(t) \]

Winding 1 (Primary)

Winding 2 (Secondary)

\[ N_1 : N_2 \]

Real transformers far from ideal

Must use practical model

Mutual inductance

\[ \phi = \frac{N_1 I_1 + N_2 I_2}{R_{\text{core}}} \]

\[ \lambda_1 = N_1 \phi = \frac{N_1^2 I_1}{R_{\text{core}}} + \frac{N_1 N_2 I_2}{R_{\text{core}}} \]

\[ -\text{emf}_1 = \frac{N_1^2}{R_{\text{core}}} \frac{di_1}{dt} + \frac{N_1 N_2}{R_{\text{core}}} \frac{di_2}{dt} \]

Self Inductance

Mutual Inductance

Circuit models of mutual inductance

Polarity (Important!)

\( v \) pos. terminal at dot

\( i \) flows into dot

\[ \lambda_1 = L_1 i_1 + M i_2 \]

\[ \lambda_2 = M i_1 + L_2 i_2 \]

\[ v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \]

\[ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]

\[ v_1(t) + \]

\[ v_2(t) + \]

\[ i_1(t) + \]

\[ i_2(t) + \]

\[ i_1(t) - \]

\[ i_2(t) - \]
Imperfect coupling ($k < 1$)

Coefficient of coupling

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

Winding and core loss

Lossless linear transformer model

Leakage inductance from leakage flux

Magnetizing inductance from coupled flux

Unique solution for each fixed $N_1:N_2$

Lossy linear transformer model

Design considerations

• Cost
• Size
• Weight
• Temperature
• Insulation rating
• Load current
• Short circuit current
• Load factor
• Target efficiency
• Target service life

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Winding and core loss

Rmagn

Rleak

N1

N2

Rcu

Rcore

mmf

Saturation

Hysteresis

Eddy loss