Today

- Recap of Week 1
- Advantages of three phase
- Standard three phase models
- Exercises

Schedule

- Mon 8/26: Phasors
- Wed 8/28: Complex power
- Fri 8/30: Power factor correction
- Mon 9/2: Labor day (no class)
- Wed 9/4: Three-phase power
- Fri 9/6: Review + Quiz 1

Derivation of the Real Power

\[ 
\overline{S} = P + jQ = S \angle \phi 
\]

- Reactive Power [VAR]
  \[ Q = \text{Im} \{ \overline{V}_{\text{rms}} \overline{I}_{\text{rms}}^* \} \]
- Apparent Power [VA]
  \[ S = |\overline{S}| \]
- Power Factor [Dimensionless]
  \[ \cos \phi = \frac{P}{S} \]

\[ P = \text{Re} \{ \overline{V}_{\text{rms}} \overline{I}_{\text{rms}}^* \} \]

Real Power [W]

\[ \overline{V}_{\text{rms}} = V \angle \theta_v \] [V]
\[ \overline{I}_{\text{rms}} = I \angle \theta_i \] [A]

Average Power [W]

\[ p(t) = VI \cos(\omega t + \phi_v - \phi_i) \]

Impedance is complex resistance

\[ \overline{Z} = R + jX \]

Model for arbitrary linear P-Q load
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Power ripple in 1φ
\[ v(t) = \cos(\omega t) \]
\[ i(t) = \frac{v(t)}{R} = \cos(\omega t) \]
\[ p(t) = v(t)i(t) = \frac{1}{2} \cos(2\omega t) + \frac{1}{2} \]

Power ripple cancelation in 2φ
\[ v_1(t) = \cos(\omega t) \quad v_2(t) = \cos(\omega t - 90°) \]
\[ v_2(t) = \cos(\omega t - 90°) \]
\[ i_2(t) = \frac{v_2(t)}{R} = \cos(\omega t - 90°) \]
\[ p_2(t) = \frac{1}{2} \cos(2\omega t - 180°) + \frac{1}{2} = -\frac{1}{2} \cos(2\omega t) + \frac{1}{2} \]
\[ p_1(t) = +\frac{1}{2} \cos(2\omega t) + \frac{1}{2} \]

Total instantaneous power is constant

Power ripple in 2φ
\[ v_1(t) = \cos(\omega t) \quad v_2(t) = \cos(\omega t - 90°) \]
\[ p_1(t) = \max \quad p_2(t) = 0 \]
\[ p_1(t) = p_2(t) \]

Power factor correction
\[ P_1 = \overline{S_1 = VI_1^*} \]
\[ Q_1 = \overline{S_2 = jQ_2} \]
\[ Q_1 + Q_2 = Q_{tot} \]
\[ Q_2 = Q_{tot} - Q_1 < 0 \]

Report as: Add $|Q_2|$ VARs of capacitance

Reactance is imaginary resistance

Current leads voltage (Leading PF)
\[ X_L = \omega L \]

Current lags voltage (Lagging PF)
\[ X_C = -\frac{1}{\omega C} \]

Reactance is imaginary resistance

Total instantaneous power is constant
Transmitting 2φ with 90° offset

\[ V_{an} = V \angle 0^\circ \]
\[ I_a = I \angle 0^\circ \]
\[ V_{bn} = V \angle (-90^\circ) \]
\[ I_b = I \angle (-90^\circ) \]

\[ S_{tot} = S_a + S_b = 2 \cdot S_a \]
\[ |I_n| = |I_a + I_b| = \sqrt{2} |I_a| \]

Big neutral cable

Transmitting 2φ with 180° offset

\[ V_{an} = V \angle 0^\circ \]
\[ I_a = I \angle 0^\circ \]
\[ V_{bn} = V \angle (-180^\circ) \]
\[ I_b = I \angle (-180^\circ) \]

\[ S_{tot} = S_a + S_b = 2 \cdot S_a \]
\[ |I_n| = |I_a + I_b| = 0 \]

No neutral cable

Power ripple in 2φ with 180° offset

\[ v_1(t) = \cos(\omega t) \]
\[ v_2(t) = \cos(\omega t - 180^\circ) \]

2φ with 180° offset is the same as 1φ with twice the voltage

Power ripple cancelation in 3φ

\[ v_1(t) = \cos(\omega t) \]
\[ v_2(t) = \cos(\omega t - 120^\circ) \]
\[ v_3(t) = \cos(\omega t - 240^\circ) \]

\[ p_1(t) = p_2(t) \]
\[ p_3(t) = 0 \]

Summary: Advantages of 3φ

<table>
<thead>
<tr>
<th></th>
<th>Ripple</th>
<th>Power</th>
<th>Cables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-phase</td>
<td>Y</td>
<td>1x</td>
<td>2x</td>
</tr>
<tr>
<td>2-phase (90° offset)</td>
<td>N</td>
<td>2x</td>
<td>3.414x</td>
</tr>
<tr>
<td>2-phase (180° offset)</td>
<td>Y</td>
<td>2x</td>
<td>2x</td>
</tr>
<tr>
<td>3-phase (120° offset)</td>
<td>N</td>
<td>3x</td>
<td>3x</td>
</tr>
</tbody>
</table>
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Standard 3φ model (Wye-Wye)

Assumption 1 (Balanced)
• Same phase-neutral voltage
• Same phase current

Assumption 2 (Positive sequence)
• Phase b lags phase a by 120°
• Phase c leads phase a by 120°

\[
\begin{align*}
V_{an} &= V \angle \theta_v & I_a &= I \angle \theta_i \\
V_{bn} &= V \angle (\theta_v - 120^\circ) & I_b &= I \angle (\theta_i - 120^\circ) \\
V_{cn} &= V \angle (\theta_v + 120^\circ) & I_c &= I \angle (\theta_i + 120^\circ)
\end{align*}
\]

Implications (Assumption 1)
• Zero neutral current
• Identical load per phase
• Total power = 3x power per phase

Standard 3φ model (Wye-Delta)

Assumption 1 (Balanced)
• Same phase-neutral voltage
• Same phase current

Assumption 2 (Positive sequence)
• Phase b lags phase a by 120°
• Phase c leads phase a by 120°

\[
\begin{align*}
V_{an} &= V \angle \theta_v \\
V_{ab} &= \sqrt{3}V \angle (\theta_v + 30^\circ) \\
V_{bc} &= \sqrt{3}V \angle (\theta_v - 90^\circ) \\
V_{ca} &= \sqrt{3}V \angle (\theta_v + 150^\circ)
\end{align*}
\]

Source-Load Connections

<table>
<thead>
<tr>
<th>Source</th>
<th>Wye</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>Wye – Wye</td>
<td>Delta – Wye</td>
</tr>
<tr>
<td>Wye</td>
<td>Wye – Delta</td>
<td>Delta – Delta</td>
</tr>
</tbody>
</table>

Wye-Wye and Wye-Delta the most common
Want to ground the generator neutral for safety

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