ECE330: Power Circuits & Electromechanics

Lecture 3. Power Factor Correction

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Schedule

- ~~Mon 8/26: Phasors~~
- ~~Wed 8/28: Complex power~~
- Fri 8/30: Power factor correction
- Mon 9/2: Labor day (no class)
- Wed 9/4: Three-phase power
- **Fri 9/6: Review + Quiz 1**

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From the last lecture

Phasors		Waves
$\overline{V}_{rms} = V\angle\theta_v$ [V]	Equivalence	$v(t) = \sqrt{2}V\cos(\omega t + \theta_v)$ [V]
$\overline{I}_{rms} = I\angle\theta_i$ [A]		$i(t) = \sqrt{2}I\cos(\omega t + \theta_i)$ [A]
↓ product		↓ product
Complex Power [VA]		Instantaneous Power [W]
$\overline{S} = \overline{V}_{rms}\overline{I}^*_{rms}$		$p(t) = VI\left[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)\right]$
↓ real part		↓ average
Real Power [W]	Equal	Average Power [W]
$P = \mathrm{Re}\{\overline{V}_{rms}\overline{I}^*_{rms}\}$		$\langle p(t)\rangle = VI\cos(\theta_v - \theta_i)$

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Power Triangle

$$\overline{S} = P + jQ = S\angle\phi$$

Complex Power [VA]

$$\overline{S} = \overline{V}_{rms}\overline{I}^*_{rms}$$

Reactive Power [VAR]

$$Q = \mathrm{Im}\{\overline{V}_{rms}\overline{I}^*_{rms}\}$$

Apparent Power [VA]

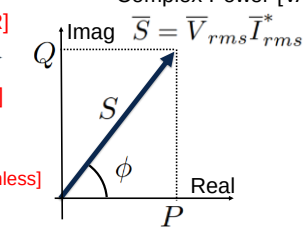
$$S = |\overline{S}|$$

Power Factor [Dimensionless]

$$\cos\phi = P/S$$

$\phi > 0$ Lagging

$\phi < 0$ Leading



Real Power [W]

$$P = \mathrm{Re}\{\overline{V}_{rms}\overline{I}^*_{rms}\}$$

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Today

- Conservation of complex power
- Reactive loads and reactive power
- Power factor correction

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Sum of sinusoids of same frequency

Add two cosine waves of the same frequency

$$A\cos(\omega t + \alpha) + B\cos(\omega t + \beta)$$

Which trig identity? Product-sum formula?

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Source: SOS math

Only valid if A = B

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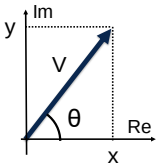
Review: Complex numbers



$$V\angle\theta$$ Polar (Engineer)

$$= x + yj$$ Rectangular

$$= Ve^{j\theta}$$ Polar (Mathematician)

$$x = V\cos\theta \qquad x + jy = V(\cos\theta + j\sin\theta)$$
$$y = V\sin\theta \qquad = Ve^{j\theta}$$

Theorem (Euler). $e^{j\theta} = \cos\theta + j\sin\theta$

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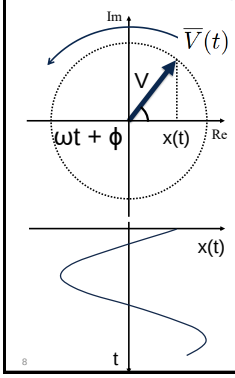
Review: Rotating arrow revisited



$$\overline{V}_{rms} = \frac{V}{\sqrt{2}}\angle\phi$$

$$\overline{V}(t) = V\angle(\omega t + \phi)$$
$$= Ve^{j(\omega t+\phi)}$$
$$= (Ve^{j\phi})e^{j\omega t}$$
$$= \overline{V}_{rms}\cdot\sqrt{2}e^{j\omega t}$$

$$x(t) = \mathrm{Re}\{\overline{V}_{rms}\cdot\sqrt{2}e^{j\omega t}\}$$
$$= V\cos(\omega t + \phi)$$

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Review: Analytic representation

Theorem.

$$\overline{V}_{rms} = \frac{V}{\sqrt{2}}\angle\phi$$

$$V\cos(\omega t + \phi) = \mathrm{Re}\{\overline{V}_{rms}\cdot\sqrt{2}e^{j\omega t}\}$$

Real signal — Analytic representation

We will use this result to add / subtract / differentiate / integrate sinusoids via their phasors

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Sum of sinusoids of same frequency

Add two cosine waves of the same frequency

$$A\cos(\omega t + \alpha) + B\cos(\omega t + \beta)$$

Define phasors

$$\overline{A} = \frac{A}{\sqrt{2}}\angle\alpha,\ \overline{B} = \frac{B}{\sqrt{2}}\angle\beta, \quad \overline{C} = \overline{A} + \overline{B} = \frac{C}{\sqrt{2}}\angle\gamma$$

Analytic representation

$$= \mathrm{Re}\{\overline{A}\cdot\sqrt{2}e^{j\omega t}\} + \mathrm{Re}\{\overline{B}\cdot\sqrt{2}e^{j\omega t}\}$$

$$= \mathrm{Re}\{(\overline{A} + \overline{B})\cdot\sqrt{2}e^{j\omega t}\}$$ Same frequency

$$= \mathrm{Re}\{\overline{C}\cdot\sqrt{2}e^{j\omega t}\} \quad = C\cos(\omega t + \gamma)$$

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Sum of sinusoids of same frequency

Theorem.

$$A\cos(\omega t + \alpha) \xleftrightarrow{\text{Equivalent}} \overline{A} = \frac{A}{\sqrt{2}}\angle\alpha,$$

$$+ B\cos(\omega t + \beta) \xleftrightarrow{\text{Equivalent}} \overline{B} = \frac{B}{\sqrt{2}}\angle\beta,$$

$$= C\cos(\omega t + \gamma) \xleftrightarrow{\text{Equivalent}} \overline{C} = \overline{A} + \overline{B}$$

Same frequency $\qquad = \frac{C}{\sqrt{2}}\angle\gamma$

We just simply add the phasors

(What about subtraction?)

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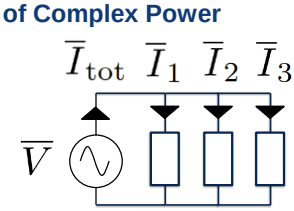
Conservation of Complex Power

Recall: We just simply add the phasors



$$\overline{I}_{tot} = \overline{I}_1 + \overline{I}_2 + \overline{I}_3$$

$$\overline{S}_{tot} = \overline{V}\overline{I}^*_{tot} = \overline{V}\overline{I}^*_1 + \overline{V}\overline{I}^*_2 + \overline{V}\overline{I}^*_3$$

$$\overline{S}_{tot} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3$$

Exercise: Repeat for components in series

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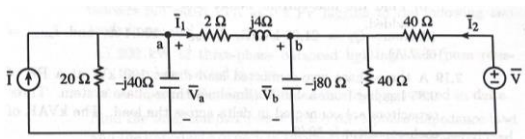
Complex KCL and KVL (Memorize this slide)

Complex voltages, currents, and impedances.



Same circuit analysis as resistor networks

View "impedance" as "complex resistance".

View "reactance" as "imaginary resistance".

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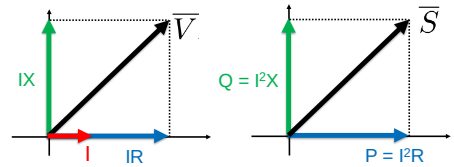
What does an impedance mean?

$\bar{I} = I\angle 0°$

$\bar{Z} = R + jX$

$$\bar{V} = \bar{Z}\bar{I} = (R + jX)\cdot I$$
$$= IR + jIX$$
$$\bar{S} = \bar{V}\bar{I}^* = (IR + jIX)\cdot I$$
$$= I^2R + jI^2X$$



Model for arbitrary linear P-Q load

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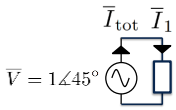
Exercise 1



Hint 1: Complex Ohm's law

$$\bar{V} = (1 + 1j)\bar{I}_1$$

Hint 2:

$$\cos 45° = \sin 45° = 1/\sqrt{2}$$

Load 1 is 1 + 1j Ohm.

What is $\bar{S}_{tot} = \bar{V}\bar{I}^*_{tot}$?

A) $\frac{1}{\sqrt{2}} + 1j$ [VA]

B) $1 + \frac{1}{\sqrt{2}}j$ [VA]

C) $\frac{1}{\sqrt{2}} + \frac{1}{2}j$ [VA]

D) $\frac{1}{2} + \frac{1}{2}j$ [VA]

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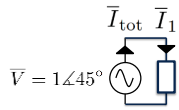
Exercise 1



Hint 1: Complex Ohm's law

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~~B) $1 + \frac{1}{\sqrt{2}}j$ [VA]~~

~~C) $\frac{1}{\sqrt{2}} + \frac{1}{2}j$ [VA]~~

D) $\frac{1}{2} + \frac{1}{2}j$ [VA]

$$\bar{V} = 1\angle 45° = 1e^{j(45°)}$$
$$\bar{Z} = \sqrt{2}\angle 45° = \sqrt{2}e^{j(45°)}$$
$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{1e^{j(45°)}}{\sqrt{2}e^{j(45°)}} = \frac{1}{\sqrt{2}}$$
$$\bar{S} = \bar{V}\bar{I}^* = (1/\sqrt{2})\angle 45°$$
$$= (1/\sqrt{2})(\cos 45° + j\sin 45°)$$
$$= 1/2 + j/2$$

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Today

- ~~Conservation of complex power~~
- Reactive loads and reactive power
- Power factor correction

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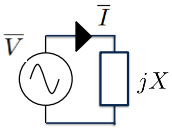
Reactance as an imaginary resistor



$\bar{I} = I\angle\theta,$ Complex Ohm's law

$$\bar{V} = jX \cdot Ie^{j\theta} = IXe^{j(\theta+90°)}$$
$$V = IX$$

$$\bar{S} = \bar{V}\bar{I}^* = (IX\angle(\theta + 90°))(I\angle\theta)^*$$
$$= I^2X\angle(\theta + 90° - \theta) = I^2X\angle 90°$$
$$= 0 + jI^2X = 0 + j\frac{V^2}{X}$$

Reactive power law $\boxed{\bar{S} = jI^2X = j\frac{V^2}{X}}$

Used to model capacitors and inductors

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Differentiation & Indefinite Integral

$$V\cos(\omega t+\phi) = \mathrm{Re}\{\underline{\overline{V}_{rms}} \cdot \sqrt{2}e^{j\omega t}\}$$

Trigonometric identities

$$\frac{\mathrm{d}}{\mathrm{d}t}\cos\omega t = -\omega\sin\omega t \qquad \frac{\mathrm{d}}{\mathrm{d}t}\sin\omega t = +\omega\cos\omega t$$

Trivial differentiation

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Re}\{\overline{V}_{rms}\cdot\sqrt{2}e^{j\omega t}\} = \mathrm{Re}\{\underline{(j\omega\overline{V}_{rms})}\cdot\sqrt{2}e^{j\omega t}\}$$

Differentiation = Multiply phasor by jω

Indefinite Integration = Divide phasor by jω

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Back to lecture 1

$$x(t) = V\cos(\omega t+\phi), \qquad \frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\omega V\sin(\omega t+\phi)$$

What is the phasor for dx/dt?

$$\underline{j}\omega\cdot\frac{V}{\sqrt{2}}e^{j\phi}$$

Write down the phasor for x(t)

Multiply phasor by jω

$$= \cancel{\frac{j\omega V}{\sqrt{2}}\angle\phi}$$

Magnitude Must be positive

$$\underbrace{\cos 90^\circ}_{=0} + j\underbrace{\sin 90^\circ}_{=1} = e^{j(90^\circ)}$$

$$= e^{j(90^\circ)}\omega\cdot\frac{V}{\sqrt{2}}e^{j\phi} = \left(\omega\cdot\frac{V}{\sqrt{2}}\right)\left(e^{j(90^\circ)}e^{j\phi}\right)$$

$$= \frac{\omega V}{\sqrt{2}}e^{j(\phi+90^\circ)} = \frac{\omega V}{\sqrt{2}}\angle(\phi+\underline{90^\circ})$$

Multiply by j adds 90° to angle

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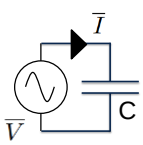
Reactive power of a capacitor



$$\overline{V} = V\boxed{\angle\theta} \qquad i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

$$\overline{I} = C\cdot j\omega\cdot Ve^{j\theta}$$

$$= C\cdot e^{j(90^\circ)}\omega\cdot Ve^{j\theta}$$

Current leads voltage by 90°

$$= \omega CV\boxed{e^{j(\theta+90^\circ)}}$$

$$\overline{S} = \overline{V}\overline{I}^* = (V\angle\theta)(\omega CV\angle(\theta+90^\circ))^*$$

$$= \omega CV^2\angle(\theta-\theta-90^\circ)$$

No real power

$$= \boxed{0 - j\omega CV^2}$$

Capacitors supply reactive power

$$\boxed{X_C = \frac{-1}{\omega C}}$$

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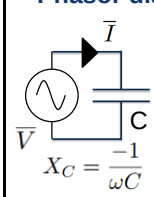
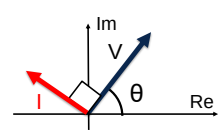
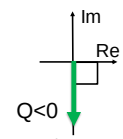
Phasor diagram





Current leads voltage (Leading PF)



Reactive power supplied

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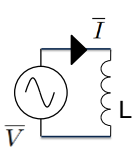
Reactive power of an inductor



$$\overline{I} = I\boxed{\angle\theta} \qquad v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$\overline{V} = L\cdot j\omega\cdot Ie^{j\theta}$$

$$= L\cdot e^{j(90^\circ)}\omega\cdot Ie^{j\theta}$$

Current lags voltage by 90°

$$= \omega LI\boxed{e^{j(\theta+90^\circ)}}$$

$$\overline{S} = \overline{V}\overline{I}^* = (\omega LI\angle(\theta+90^\circ))(I\angle\theta)^*$$

$$= \omega LI^2\angle(\theta+90^\circ-\theta)$$

No real power

$$= \boxed{0 + j\omega LI^2}$$

Inductors consume reactive power

$$\boxed{X_L = +\omega L}$$

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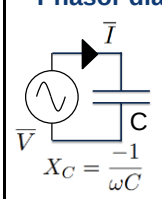
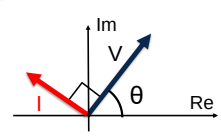
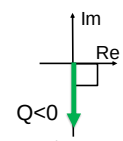
Phasor diagram

(Memorize this slide)

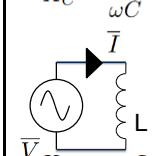


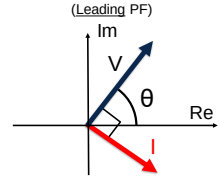


Current leads voltage (Leading PF)

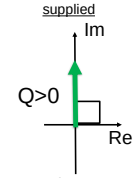


Reactive power supplied





Current lags voltage (Lagging PF)



Reactive power consumed

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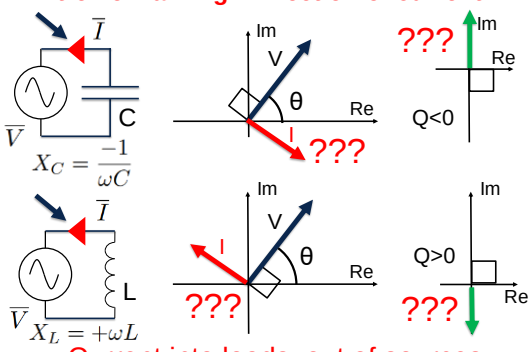
Extreme warning!! Direction of current!!



Current into loads, out of sources

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Today

- ~~Conservation of complex power~~
- ~~Reactive loads and reactive power~~
- Power factor correction

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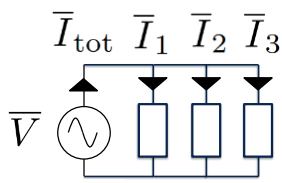
Conservation of Complex Power

Recall: We just simply add the phasors



$$\overline{I}_{\text{tot}} = \overline{I}_1 + \overline{I}_2 + \overline{I}_3$$

$$\overline{S}_{\text{tot}} = \overline{V}\overline{I}^*_{\text{tot}} = \overline{V}\overline{I}^*_1 + \overline{V}\overline{I}^*_2 + \overline{V}\overline{I}^*_3$$

$$\overline{S}_{\text{tot}} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3$$

Exercise: Repeat for components in series

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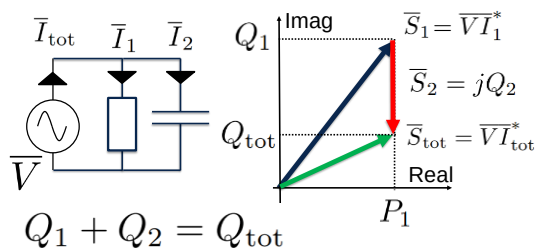
Power factor correction

(Memorize this slide)



$$Q_1 + Q_2 = Q_{\text{tot}}$$

$$Q_2 = Q_{\text{tot}} - Q_1 < 0$$

Report as: Add $|Q_2|$ VARs of capacitance

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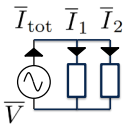
Exercise 2



$\overline{V} = \sqrt{2}\angle 45^\circ$

$\overline{I}_1 = \frac{1}{\sqrt{2}}\angle 45^\circ$

$\overline{I}_2 = \sqrt{2}\angle -45^\circ$

What is $\overline{S}_{\text{tot}} = \overline{V}\overline{I}^*_{\text{tot}}$?

A) $2 + 1j$ [VA]

B) $1 + 2j$ [VA]

C) $\frac{1}{\sqrt{2}} + \sqrt{2}j$ [VA]

D) $\sqrt{2} + \frac{1}{\sqrt{2}}j$ [VA]

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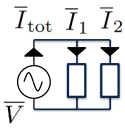
Exercise 2



$\overline{V} = \sqrt{2}\angle 45^\circ$

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$\overline{I}_2 = \sqrt{2}\angle -45^\circ$

What is $\overline{S}_{\text{tot}} = \overline{V}\overline{I}^*_{\text{tot}}$?

~~A) $2 + 1j$ [VA]~~

B) $1 + 2j$ [VA]

~~C) $\frac{1}{\sqrt{2}} + \sqrt{2}j$ [VA]~~

~~D) $\sqrt{2} + \frac{1}{\sqrt{2}}j$ [VA]~~

$$\overline{S}_1 = \overline{V}\overline{I}^*_1 = (\sqrt{2}\angle 45^\circ)\left(\frac{1}{\sqrt{2}}\angle 45^\circ\right)^* = 1\angle 0^\circ = 1 + 0j$$

$$\overline{S}_2 = \overline{V}\overline{I}^*_2 = (\sqrt{2}\angle 45^\circ)\left(\sqrt{2}\angle -45^\circ\right)^* = 2\angle 90^\circ = 0 + 2j$$

$$\overline{S}_{\text{tot}} = \overline{S}_1 + \overline{S}_2 = 1 + 2j$$

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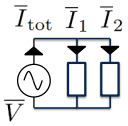
Exercise 3



$\overline{S}_{\text{tot}} = \overline{S}_1 + \overline{S}_2 = 1 + 2j$ [VA]

What is the power factor?

A) $1/\sqrt{3}$ leading

B) $2/\sqrt{5}$ leading

C) $1/\sqrt{5}$ lagging

D) $2/\sqrt{5}$ lagging

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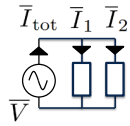
Exercise 3



$\overline{S}_{\text{tot}} = \overline{S}_1 + \overline{S}_2 = 1 + 2j$ [VA]

What is the power factor?

~~A) $1/\sqrt{3}$ leading~~

~~B) $2/\sqrt{5}$ leading~~

C) $1/\sqrt{5}$ lagging

~~D) $2/\sqrt{5}$ lagging~~

Q > 0 means ϕ > 0

cos ϕ = P/S

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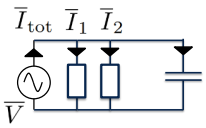
Exercise 4



$\overline{S}_1 + \overline{S}_2 = 1 + 2j$ [VA]

How much capacitance for

$\cos\phi = 1/\sqrt{2}$?

A) $1/\sqrt{2}$ [VAR]

B) 1 [VAR]

C) $\sqrt{2}$ [VAR]

D) 2 [VAR]

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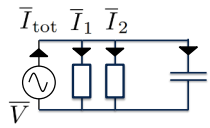
Exercise 4



$\overline{S}_1 + \overline{S}_2 = 1 + 2j$ [VA]

How much capacitance for

$\cos\phi = 1/\sqrt{2}$?

~~A) $1/\sqrt{2}$ [VAR]~~

B) 1 [VAR]

~~C) $\sqrt{2}$ [VAR]~~

~~D) 2 [VAR]~~

Compute new apparent power

$$\frac{P_{\text{tot}}}{S_{\text{tot}}} = \frac{1}{\sqrt{2}}, \qquad P_{\text{tot}} = 1 \text{ [W]}$$

$$S_{\text{tot}} = P/\cos\phi = \sqrt{2} \text{ [VA]}$$

Compute new reactive power

$$P_{\text{tot}}^2 + Q_{\text{tot}}^2 = S_{\text{tot}}^2$$

$$Q_{\text{tot}} = \sqrt{S_{\text{tot}}^2 - P_{\text{tot}}^2} = 1 \text{ [VAR]}$$

Compute compensation

$$Q_{\text{cap}} = Q_{\text{tot}} - Q_1 - Q_2$$
$$= 1 - 2 = -1 \text{ [VAR]}$$

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Schedule

- ~~Mon 8/26: Phasors~~
- ~~Wed 8/28: Complex power~~
- ~~Fri 8/30: Power factor correction~~
- Mon 9/2: Labor day (no class)
- Wed 9/4: Three-phase power
- **Fri 9/6: Review + Quiz 1**

You are ready to do Homework 1

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