ECE330: Power Circuits & Electromechanics
Lecture 3. Power Factor Correction

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Mon 8/26: Phasors
Wed 8/28: Complex power
Fri 8/30: Power factor correction
Mon 9/2: Labor day (no class)
Wed 9/4: Three-phase power
Fri 9/6: Review + Quiz 1

From the last lecture

Phasors
\[ V_{\text{rms}} = V \angle \theta [\text{V}] \]
\[ I_{\text{rms}} = I \angle \theta [\text{A}] \]

Equivalence

Waves
\[ v(t) = \sqrt{2} V \cos(\omega t + \theta_v) [\text{V}] \]
\[ i(t) = \sqrt{2} I \cos(\omega t + \theta_i) [\text{A}] \]

Complex Power [VA]
\[ S = V_{\text{rms}} I_{\text{rms}} \]

Instantaneous Power [W]
\[ p(t) = V I \cos(\omega t + \theta_v + \theta_i) + \cos(\theta_i - \theta_v) \]

Real Power [W]
\[ P = \text{Re} \{ V_{\text{rms}} I_{\text{rms}} \} \]

Average Power [W]
\[ p(t) = V I \cos(\theta_v - \theta_i) \]

Flux

Power Triangle

\[ S = P + jQ = S \angle \phi \]

Reactive Power [VAR]
\[ Q = \text{Im} \{ V_{\text{rms}} I_{\text{rms}} \} \]

Apparent Power [VA]
\[ S = |S| \]

Power Factor [Dimensionless]
\[ \cos \phi = P/S \]

\[ \phi > 0 \quad \text{Lagging} \]
\[ \phi < 0 \quad \text{Leading} \]

Today

- Conservation of complex power
- Reactive loads and reactive power
- Power factor correction

Sum of sinusoids of same frequency
Add two cosine waves of the same frequency
\[ A \cos(\omega t + \alpha) + B \cos(\omega t + \beta) \]

Which trig identity? Product-sum formula?

\[
\begin{align*}
\cos(\alpha) + \cos(\beta) &= 2 \cos \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right) \\
\cos(\alpha) - \cos(\beta) &= -2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right) \\
\sin(\alpha) + \sin(\beta) &= 2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right) \\
\sin(\alpha) - \sin(\beta) &= 2 \cos \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)
\end{align*}
\]

Only valid if \( A = B \)

Source: SOS math
Review: Complex numbers

\[ V \angle \theta = x + jy \]
\[ = x + jy = V e^{j\theta} \]
\[ x = V \cos \theta \]
\[ y = V \sin \theta \]

**Theorem (Euler).** \[ e^{j\theta} = \cos \theta + j \sin \theta \]

Review: Rotating arrow revisited

\[ V(t) = V \angle (\omega t + \phi) \]
\[ = V e^{j(\omega t + \phi)} \]
\[ = V e^{j\phi} e^{j\omega t} \]
\[ x(t) = \text{Re}\{V_{rms} \cdot \sqrt{2} e^{j\omega t}\} \]
\[ = V \cos(\omega t + \phi) \]

Review: Analytic representation

**Theorem.**

\[ V_{rms} = \frac{V}{\sqrt{2}} \angle \phi \]
\[ V \cos(\omega t + \phi) = \text{Re}\{V_{rms} \cdot \sqrt{2} e^{j\omega t}\} \]

We will use this result to add / subtract / differentiate / integrate sinusoids via their phasors

Sum of sinusoids of same frequency

Add two cosine waves of the same frequency

\[ A \cos(\omega t + \alpha) + B \cos(\omega t + \beta) \]

Define phasors

\[ \overline{A} = \frac{A}{\sqrt{2}} \angle \alpha, \quad \overline{B} = \frac{B}{\sqrt{2}} \angle \beta, \quad \overline{C} = \overline{A} + \overline{B} = \frac{C}{\sqrt{2}} \angle \gamma \]

Analytic representation

\[ = \text{Re}\{\overline{A} \cdot \sqrt{2} e^{j\omega t}\} + \text{Re}\{\overline{B} \cdot \sqrt{2} e^{j\omega t}\} \]
\[ = \text{Re}\{\overline{A} + \overline{B}\} \cdot \sqrt{2} e^{j\omega t}\} \]
\[ = \text{Re}\{\overline{C} \cdot \sqrt{2} e^{j\omega t}\} = C \cos(\omega t + \gamma) \]

Conservation of Complex Power

Recall: We just simply add the phasors

\[ \overline{I}_{tot} = \overline{I}_1 + \overline{I}_2 + \overline{I}_3 \]
\[ \overline{S}_{tot} = \overline{V}_{tot}^{*} = \overline{V_{I_1}^{*}} + \overline{V_{I_2}^{*}} + \overline{V_{I_3}^{*}} \]

Exercise: Repeat for components in series
Complex KCL and KVL

Complex voltages, currents, and impedances.

Same circuit analysis as resistor networks

View “impedance” as “complex resistance”.

View “reactance” as “imaginary resistance”.

What does an impedance mean?

\[ I = I_{\angle 0^\circ} \]
\[ V = \overline{Z}I = (R + jX) \cdot I \]
\[ \overline{Z} = R + jX \]
\[ \overline{S} = \overline{V}I^* = (1R + j1X) \cdot I \]

Model for arbitrary linear P-Q load

Exercise 1

\[ \overline{V} = (1 + 1j)\overline{I} \]

Exercise 1

\[ \overline{V} = (1 + 1j)\overline{I} \]

Conservation of complex power

• Reactive loads and reactive power
• Power factor correction

Reactance as an imaginary resistor

Used to model capacitors and inductors
Differentiation & Indefinite Integral

\[ V \cos(\omega t + \phi) = \text{Re}\{V_{\text{rms}} \cdot \sqrt{2}e^{j\omega t}\} \]

Trigonometric identities

\[
\frac{d}{dt} \cos \omega t = -\omega \sin \omega t \quad \frac{d}{dt} \sin \omega t = +\omega \cos \omega t
\]

Trivial differentiation

\[
\frac{d}{dt} \text{Re}\{V_{\text{rms}} \cdot \sqrt{2}e^{j\omega t}\} = \text{Re}\{(j\omega V_{\text{rms}}) \cdot \sqrt{2}e^{j\omega t}\}
\]

Differentiation = Multiply phasor by \(j\omega\)
Indefinite Integration = Divide phasor by \(j\omega\)

Back to lecture 1

\[ x(t) = V \cos(\omega t + \phi), \quad \frac{d}{dt} x(t) = -\omega V \sin(\omega t + \phi) \]

What is the phasor for \(dx/dt\)?

Writing down the phasor for \(x(t)\)
Multiply phasor by \(j\omega\)

Magnitude = \(0\)
Must be positive

\[
\text{Re} = 0, \quad \text{Im} = 1
\]

\[
\cos 90^\circ + j \sin 90^\circ = e^{j(90^\circ)}
\]

\[
\sqrt{2} \frac{e^{j\phi}}{\sqrt{2}} = (\omega \cdot \frac{V}{\sqrt{2}})(e^{j(90^\circ)}e^{j\phi})
\]

\[
\frac{\omega V}{\sqrt{2}} e^{j(\phi + 90^\circ)} = \frac{\omega V}{\sqrt{2}} (e^{j(90^\circ)}e^{j\phi})
\]

Multiply by \(j\)
Adds \(90^\circ\) to angle

Reactive power of a capacitor

\[
\vec{I} = \vec{V} \angle 0 \quad i(t) = C \frac{dv(t)}{dt}
\]

\[
\hat{I} = C \cdot j \omega \cdot V e^{j\theta}
\]

\[
\hat{S} = \sqrt{\hat{I}^*} \cdot (V \angle \theta)(\omega CV \angle (\theta + 90^\circ))^*
\]

\[
= \omega CV^2 \angle (\theta - \theta - 90^\circ)
\]

\[
= -j\omega CV^2
\]

No real power

Capacitors supply reactive power

\(X_C = -\frac{1}{\omega C}\)

Phasor diagram

Current leads voltage by \(90^\circ\)

Reactive power supplied

\(Q<0\)

Reactive power of an inductor

\[
\vec{I} = \vec{V} \angle \theta \quad v(t) = L \frac{di(t)}{dt}
\]

\[
\hat{I} = L \cdot j \omega \cdot I e^{j\theta}
\]

\[
\hat{S} = \sqrt{\hat{I}^*} \cdot (\omega LI \angle (\theta + 90^\circ))(I \angle \theta)^*
\]

\[
= \omega LI^2 \angle (\theta + 90^\circ - \theta)
\]

No real power

Inductors consume reactive power

\(X_L = +\omega L\)

Phasor diagram

(Memorize this slide)

Current leads voltage (Leading PF)

Reactive power supplied

\(Q<0\)

Current lags voltage (Lagging PF)

Reactive power consumed

\(Q>0\)
Exercise 2

\[ I_{tot} = I_1 + I_2 + I_3 \]
\[ V = \sqrt{2} \angle 45^\circ \]
\[ I_1 = \frac{1}{\sqrt{2}} \angle 45^\circ \]
\[ I_2 = \sqrt{2} \angle -45^\circ \]

What is \( S_{tot} = V I_{tot}^* \)?

A) \( 2 + j \) [VA]  
B) \( 1 + 2j \) [VA]  
C) \( \frac{1}{\sqrt{2}} + \sqrt{2}j \) [VA]  
D) \( \sqrt{2} + \frac{1}{\sqrt{2}} j \) [VA]
Exercise 3
\[
\bar{S}_{\text{tot}} = \bar{S}_1 + \bar{S}_2 = 1 + 2j \quad \text{[VA]}
\]
What is the power factor?
A) \(1/\sqrt{3}\) leading
B) \(2/\sqrt{5}\) leading
C) \(1/\sqrt{5}\) lagging
D) \(2/\sqrt{5}\) lagging

Exercise 4
\[
\bar{S}_{\text{tot}} = \bar{S}_1 + \bar{S}_2 = 1 + 2j \quad \text{[VA]}
\]
How much capacitance for
\(\cos \phi = 1/\sqrt{2}\)?
A) \(1/\sqrt{2}\) [VAR]
B) 1 [VAR]
C) \(\sqrt{2}\) [VAR]
D) 2 [VAR]

You are ready to do Homework 1