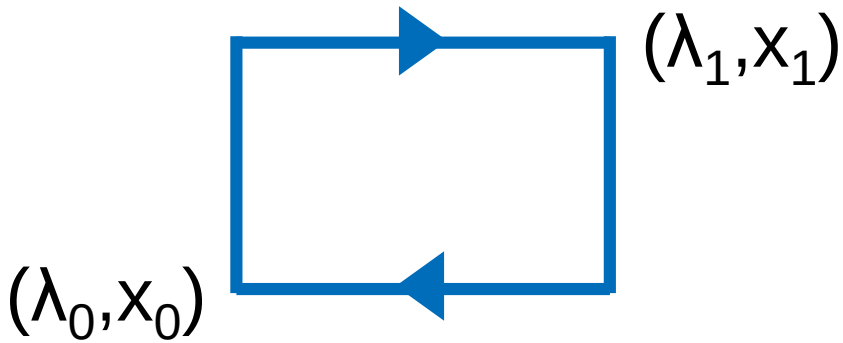
Today

- (Review) Energy, EFE, EFM
- Additional problems

Energy line integral over a closed loop

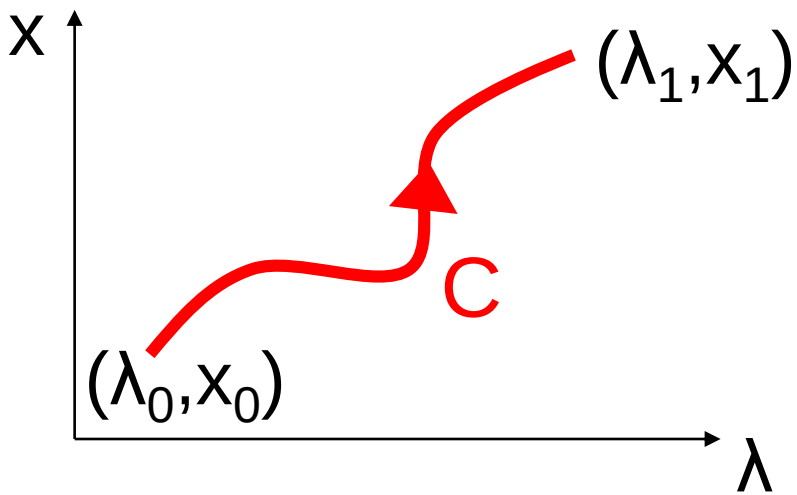


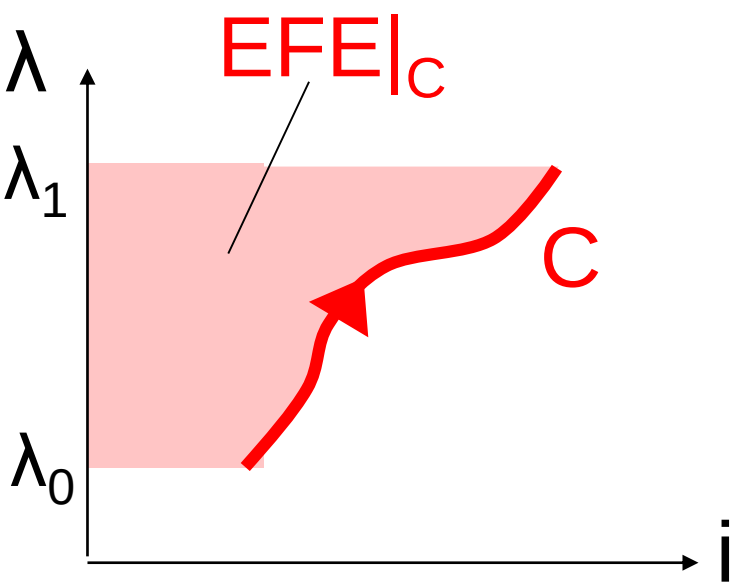
$$\text{Energy}(\lambda_1, x_1) - \text{Energy}(\lambda_1, x_1)$$

$$= \underbrace{\oint_C \underline{F}(\underline{r}) \cdot \mathrm{d}\underline{r}}_{=\,0} = \underbrace{\oint_C i(\lambda, x)\,\mathrm{d}\lambda}_{\mathrm{EFE}|_{\mathrm{cycle}}} + \underbrace{\oint_C -f(\lambda, x)\,\mathrm{d}x}_{\mathrm{EFM}|_{\mathrm{cycle}}}$$

Generator mech → elec	$\mathrm{EFE}\|_{\mathrm{cycle}} < 0,$	$\mathrm{EFM}\|_{\mathrm{cycle}} > 0$
Motor elec → mech	$\mathrm{EFE}\|_{\mathrm{cycle}} > 0,$	$\mathrm{EFM}\|_{\mathrm{cycle}} < 0$

EFE / EFM: Monotone electrical path





1. Integrate trajectory for EFE

$$\mathrm{EFE}|_C = \int_C i(\lambda', x')\,\mathrm{d}\lambda'$$
$$= \int_{\lambda_0}^{\lambda_1} i(\lambda')\,\mathrm{d}\lambda'$$

⚠ Assumes $d\lambda/dt > 0$ or $d\lambda/dt < 0$

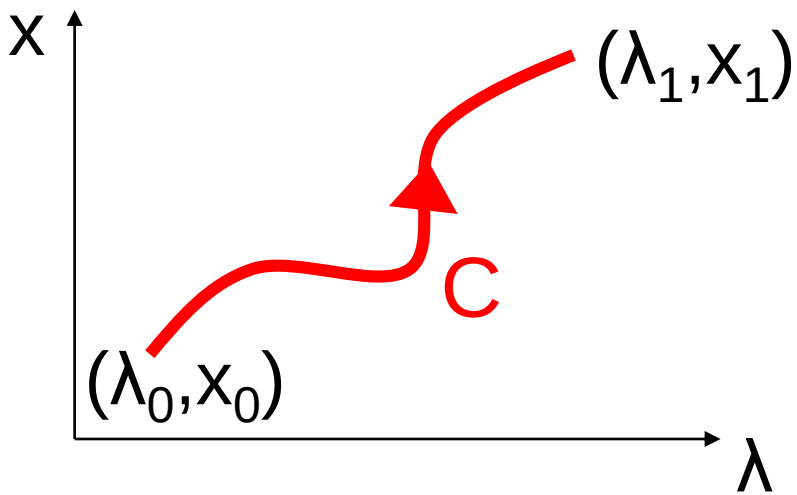
2. Compute expression for energy

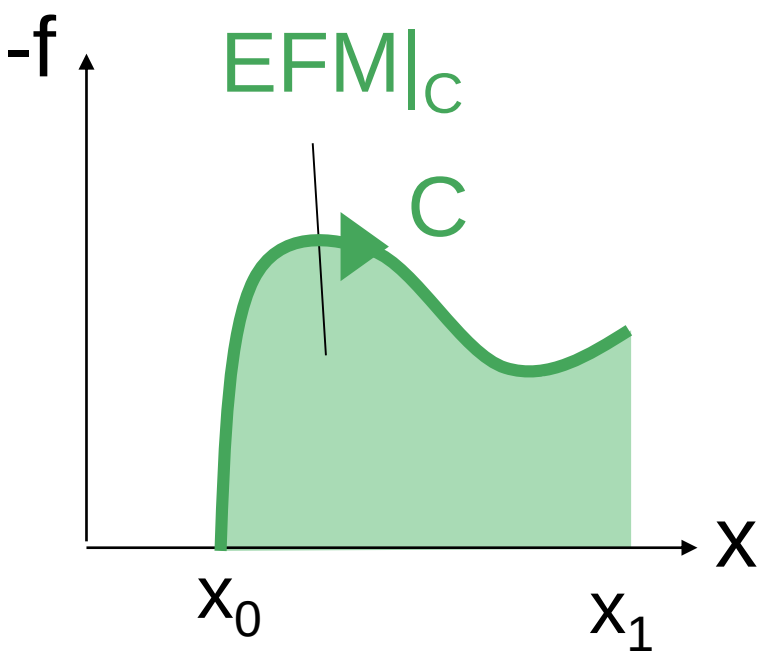
$$\mathrm{Energy} = \int_0^{\lambda} i(\lambda', x)\,\mathrm{d}\lambda'$$

3. Infer EFM from energy

$$\mathrm{EFM}|_C = \mathrm{Energy}(\lambda_1, x_1) - \mathrm{Energy}(\lambda_0, x_0) - \mathrm{EFE}|_C$$

EFE / EFM: Monotone mechanical path





1. Integrate trajectory for EFM

$$\mathrm{EFM}|_C = \int_C -f(\lambda', x')\,\mathrm{d}x' = \int_{x_0}^{x_1} -f(x')\,\mathrm{d}x'$$

⚠ Assumes dx/dt > 0 or dx/dt < 0

2. Compute expression for energy

$$\mathrm{Energy} = \int_0^{\lambda} i(\lambda', x)\,\mathrm{d}\lambda'$$

3. Infer EFE from energy

$$\mathrm{EFE}|_C = \mathrm{Energy}(\lambda_1, x_1) - \mathrm{Energy}(\lambda_0, x_0) - \mathrm{EFM}|_C$$

Conservative electromagnetic systems

Flux & current relations

$$\lambda = \lambda(i, x), \;\; i = i(\lambda, x)$$

Energy & Co-energy

$$\text{Co-energy} = \int_0^i \lambda(i', x)\, \mathrm{d}i'$$

function of current

$$\text{Energy} = \int_0^\lambda i(\lambda', x)\, \mathrm{d}\lambda'$$

function of flux

$$\text{Co-energy}(i, x) = \lambda i - \text{Energy}(\lambda, x)$$

Force of electric origin

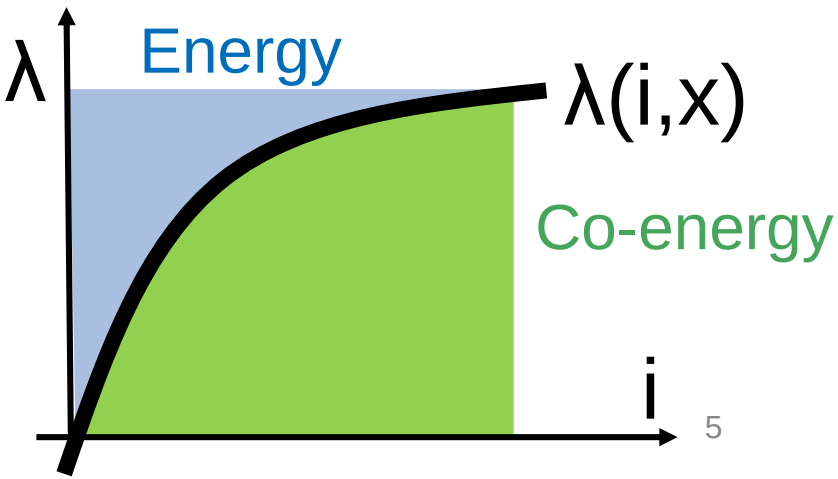
(direction of increasing x)

$$f^e = +\frac{\partial}{\partial x}\,[\text{Co-energy}]$$

$$= -\frac{\partial}{\partial x}\,[\text{Energy}]$$

$$\lambda = +\frac{\partial}{\partial i}\,[\text{Co-energy}]$$

$$i = \frac{\partial}{\partial \lambda}[\text{Energy}]$$



Three types of trajectories

A) x fixed, $\lambda_0 \rightarrow \lambda_1$ and $i_0 \rightarrow i_1$

$$\mathrm{EFE} = \int_{\lambda_0}^{\lambda_1} i(\lambda', x)\,\mathrm{d}\lambda',$$

$$\mathrm{EFM} = 0.$$

B) λ fixed, $x_0 \rightarrow x_1$ and $i_0 \rightarrow i_1$

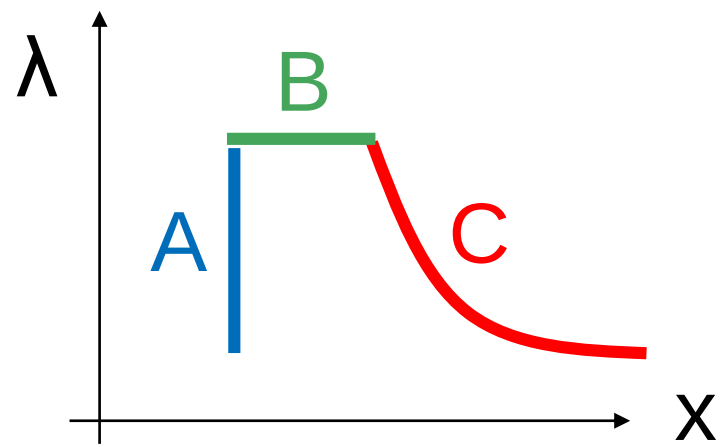
$$\mathrm{EFE} = 0,$$

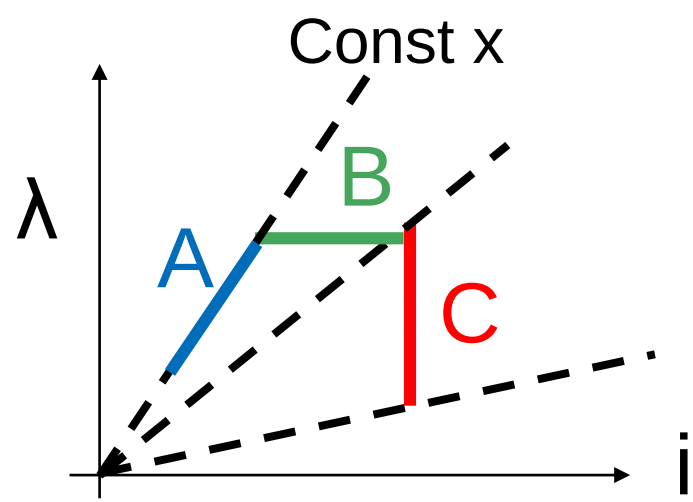
$$\mathrm{EFM} = W_m(\lambda, x_1) - W_m(\lambda, x_0).$$

C) i fixed, $x_0 \rightarrow x_1$ and $\lambda_0 \rightarrow \lambda_1$

$$\mathrm{EFE} = \int_{\lambda_0}^{\lambda_1} i\,\mathrm{d}\lambda',$$

$$\mathrm{EFM} = W_m(\lambda_1, x_1) - W_m(\lambda_0, x_0) - \mathrm{EFE}.$$





Problem 3. (25 points.)

An electromechanical system has the following flux linkage-current relationship:

$$\lambda = \frac{0.09}{(0.02+x)}\, i$$

$$W_m' = \frac{0.045}{(.02+x)}\, i^2 \qquad f^e = -\frac{0.045\, i^2}{(.02+x)^2}$$

In the following questions, EFE stands for "Energy From the Electrical system into the coupling field", and EFM stands for "Energy From the Mechanical system into the coupling field".

Consider the following to be point a: x = .01 meters, i = 2 Amps, λ = 6 Wb-Tns.

a) Find the EFE and EFM as the system is moved along constant x from point a to point b which has i = 3 Amps.
b) Find the EFE and EFM as the system is moved along constant current from point b to point c which has x = 0.025 meters.
c) Find W_m at point c
d) Find the EFE and EFM as the system is moved along constant λ from point c to point a which has x = .01 meters.
e) For this cycle, is this a motor or a generator? Explain why.

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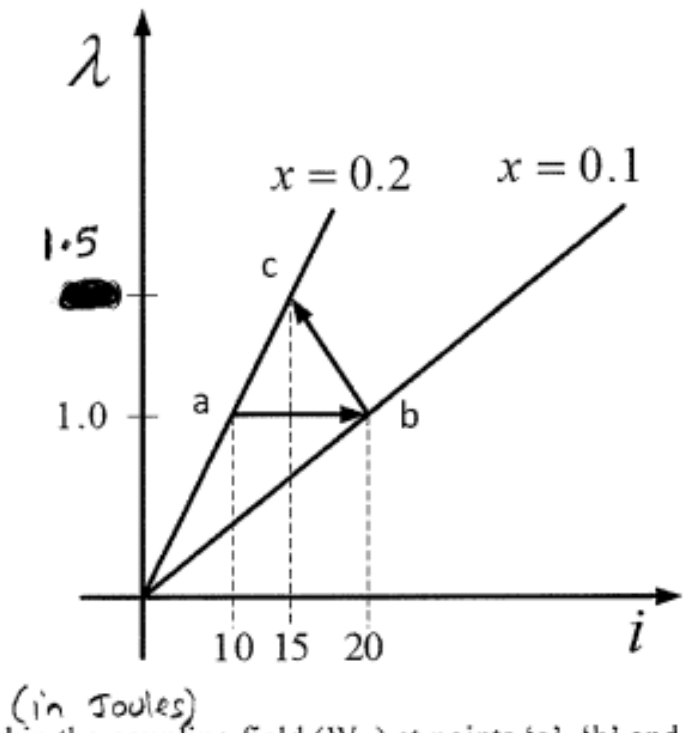
Problem 3. (25 points.)

An electromechanical device is operated over the path $a \to b \to c$ shown in the figure below. The system is known to be electrically linear, i.e., $\lambda = L(x)i$.



a) Calculate the energy stored in the coupling field (W_m) at points 'a', 'b' and 'c'. (10 Points)

b) Calculate $EFE\big|_{a\to b\to c}$ in Joules. (5 points)

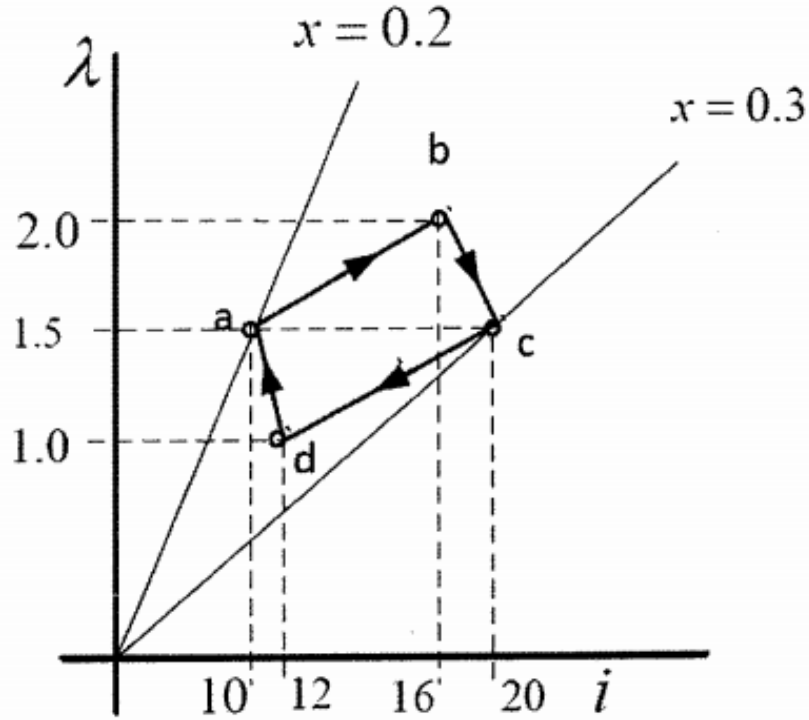
c) Calculate $EFM\big|_{a\to b\to c}$ in Joules. (5 Points)

d) Suppose the system takes the path $a \to c$ directly. Is $EFM\big|_{a\to c} = EFM\big|_{a\to b\to c}$? Explain your answer. If $EFM\big|_{a\to c} \neq EFM\big|_{a\to b\to c}$ then state the value of $EFM\big|_{a\to c}$ (5 Points).

Exam 2, Spring 2013

Problem 3. (25 points.)

An electromechanical device is operated over the cycle abcda shown in the figure below. The system is known to be electrically linear, i.e., $\lambda = L(x)i$.

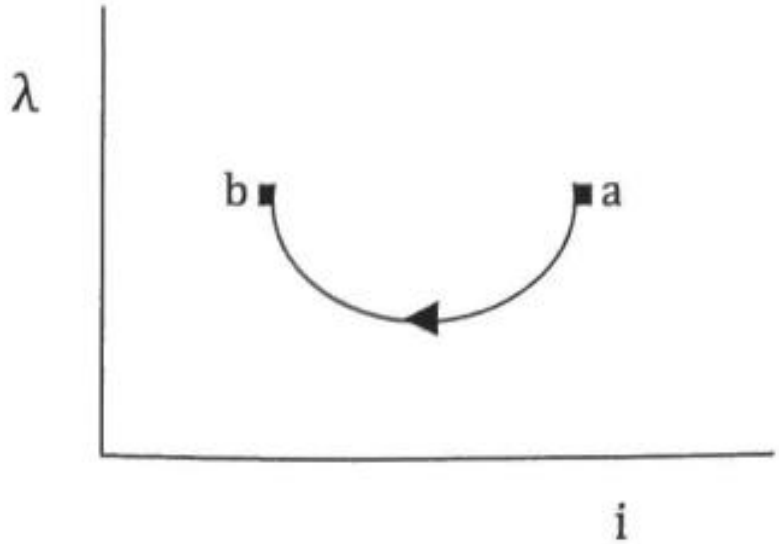


a) Calculate the energy stored in the coupling field (W_m) at points 'a' and 'd'.
b) Calculate EFE|cycle in Joules.
c) Calculate EFM|cycle in Joules.
d) Is the machine operating as a motor or a generator?

(Note : You must clearly show the steps for parts a) - c) and state the reason for your answer in part d))

Exam 2, Fall 2012

An electromechanical system with $\lambda = L(x)i$ is operated through the transition from a to b as shown below - note that the flux linkage at a is equal to the flux linkage at b:



a) Find the energy transferred from the electrical system into the coupling field as the system moves from a to b as shown. (give a graphical answer)
b) Find the energy transferred from the mechanical system into the coupling field as the system moves from a to b as shown (give a graphical answer)
c) If the device moves back from b to a along a constant flux linkage path, find the energy transferred from the electrical system into the coupling field for this motion and the energy transferred from the mechanical system into the coupling field during this motion.
d) If you consider the combined motion of a) to c) above to be one cycle, is this a motor or a generator? Explain why.

Exam 2, Spring 2019