Last Time

- Magnetic equivalent circuits
- Reluctance
- Inductance
Today

• Air gap reluctance
• Fringing effects
• Examples
Iron Core with Air Gap

\[ \oint H \cdot dl = \int_{\partial \Omega} J_f \cdot \hat{n} dA \]

\[ H_{iron} l + H_g g = Ni \]

\( l \) is the mean path length inside the iron. Calculated along center path of iron core.

Conservation of flux

\[ \oint_{\partial \Omega} B \cdot \hat{n} dA = 0 \]

Neglecting fringing:

\[ B_g A_g - B_{iron} A_{iron} = 0 \]

\[ B_g = B_{iron} \]
Iron Core with Air Gap

\[ \phi = B_{iron} A_{iron} = B_g A_g \]
\[ \phi \left( \frac{l}{\mu A_{iron}} + \frac{g}{\mu_0 A_g} \right) = Ni \]
\[ \phi (R_{iron} + R_g) = Ni \]
\[ \phi = \frac{N}{R_{iron} + R_g} i \]
Faraday’s Law

\[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \left( \int_{\partial \Omega} \mathbf{B} \cdot \hat{n} dA \right) \]

\[ v = \frac{d\lambda}{dt} \]

\[ \lambda = \frac{N^2}{\mathcal{R}_{\text{iron}} + \mathcal{R}_g} i = L_i \]

As \( \mu \to \infty \), \( \mathcal{R}_{\text{iron}} \to 0 \)

What does \( \mu \to \infty \) mean?

\[ B = \mu H \to H = \frac{B}{\mu} \]

\( B \) is infinite from conservation of flux

\( \mu \to \infty \), \( H \to 0 \) and reluctance \( \to 0 \)
Magnetic field spreads out.

\[ A_g = (a + g)(b + g) \]

As \( g \) decreases, fringing effects are reduced

Example: \( a=2 \) cm, \( b=3 \) cm, \( g=3 \) mm. What is the reluctance of the air gap?

a) \( 3.97 \times 10^6 \) At/Wb
b) \( 3.45 \times 10^6 \) At/Wb
c) \( 3.62 \times 10^6 \) At/Wb
d) \( 3.14 \times 10^6 \) At/Wb
What is the reluctance of the air gaps?

a) $1.99 \times 10^8$ At/Wb
b) $1.99 \times 10^6$ At/Wb
c) $1.99 \times 10^5$ At/Wb
d) $1.99 \times 10^7$ At/Wb
Example

What is the reluctance of the air gaps?

a) $1.99 \times 10^8$ At/Wb

b) $1.99 \times 10^6$ At/Wb

c) $1.99 \times 10^5$ At/Wb

d) $1.99 \times 10^7$ At/Wb
Example

What is the reluctance of the air gaps?

a) $1.99 \times 10^8 \text{ At/Wb}$

b) $1.99 \times 10^6 \text{ At/Wb}$

c) $1.99 \times 10^5 \text{ At/Wb}$

d) $1.99 \times 10^7 \text{ At/Wb}$

Where should the positive terminals be placed on the MMF sources?

a) right, left, right

b) left, left, right

c) right, right, left

d) right, right, right
Example

What is the reluctance of the air gaps?

a) $1.99 \times 10^8$ At/Wb
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Where should the positive terminals be placed on the MMF sources?

a) right, left, right
b) left, left, right
c) right, right, left
d) right, right, right
Ex.

Given: \(N_1 = 100\) \(I_1 = 25\text{A}\) \(A_0 = 4\text{cm}^2\)

\(N_2 = 50\) \(I_2 = 10\text{A}\) \(\mu = \infty\)

\(N_3 = 100\) \(I_3 = 15\text{A}\) \(g = 0.1\text{cm}\)

Find: Flux through the coils

Solution: All air gaps are the same.

\[
R_1 = R_2 = R_3 = R = \frac{g}{\mu A} = \frac{(0.001\text{m})}{(4\pi\times10^{-7}\text{H/m})(0.004\text{m}^2)} \Rightarrow R = 1.99\times10^6 \text{ At/Wb}
\]

Magnetic Circuit:

At point A: \(\phi_1 + \phi_2 + \phi_3 = 0\)

Let the MMF at A be \(f\)

\[
\phi_1 = \frac{2500 - f}{R} \quad \phi_2 = \frac{500 - f}{R} \quad \phi_3 = \frac{-1500 - f}{R}
\]

\[
\frac{2500 - f}{R} + \frac{500 - f}{R} - \frac{(1500 + f)}{R} = 0 \Rightarrow 3000 - 2f - 1500 - f = 0 \\
1500 - 3f = 0
\]

\[
f = 500\text{A}\text{t}
\]

\[
\phi_1 = 1\times10^{-3}\text{Wb} \\
\phi_2 = 0 \text{ Wb} \\
\phi_3 = -1\times10^{-3}\text{Wb}
\]
Example

What is the net reluctance of the circuit?

a) $1.47 \times 10^5 \text{ At/Wb}$

b) $1.15 \times 10^5 \text{ At/Wb}$

c) $1.99 \times 10^5 \text{ At/Wb}$

d) $6.46 \times 10^5 \text{ At/Wb}$
Example

What is the net reluctance of the circuit?

a) $1.47 \times 10^5 \text{ At/Wb}$

b) $1.15 \times 10^5 \text{ At/Wb}$

c) $1.99 \times 10^5 \text{ At/Wb}$

d) $6.46 \times 10^5 \text{ At/Wb}$
Ex:  

Given:  
- \( M = M_1 M_2 \)  
- \( N = 25 \)  
- \( i = 0.5 \text{A} \)  
- \( M_1 = M_2 = 2 \times 10^5 \)  
- \( l_1 = l_2 = 30 \text{cm} \)  
- \( A_1 = A_2 = 2 \text{cm}^2 \)  
- \( l_3 = 15 \text{cm} \)  
- \( l_4 = 10 \text{cm} \)  
- \( A_3 = 4 \text{cm}^2 \)  

Find:  
- \( B_1, B_2, B_3 \)

Solution:  
Magnetic circuit

\[
R_1 = R_5 = 5.31 \times 10^5 \text{At/m} \\
R_2 = 1.47 \times 10^5 \text{At/m} \\
\]

\[
\phi_1 = \frac{N_i}{R_1 + R_{\text{net}}} \\
\phi_1 = B_1 A_1 = 19.95 \times 10^{-6} \text{Wb} \\
B_1 = \frac{0.09675 \text{Wb/m}^2}{0.001} \\
\]

\[
N_i - R_1 \phi_1 = 5 \\
\phi_1 = 2.23 \text{At} \\
\phi_2 = \frac{5}{R_2} = \frac{5}{1.47 \times 10^5} \text{Wb} \\
B_2 = 0.0579 \text{Wb/m}^2 \\
\]

\[
\phi_1 = \phi_2 + \phi_3 \\
\phi_3 = \phi_1 - \phi_2 \\
\phi_3 = 2.23 - \frac{5}{1.47 \times 10^5} \text{Wb} \\
B_3 = 0.0209 \text{Wb/m}^2 \\
\]