ECE 330: Power Circuits and Electromechanics

Lecture 8

2019-09-18

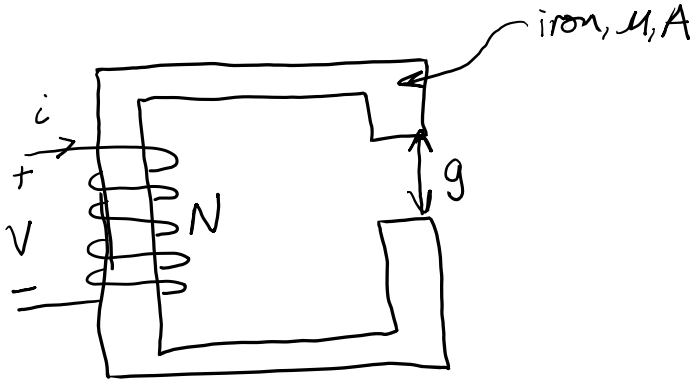
Last Time

- Magnetic equivalent circuits
- Reluctance
- Inductance

Today

- Air gap reluctance
- Fringing effects
- Examples

Iron Core with Air Gap



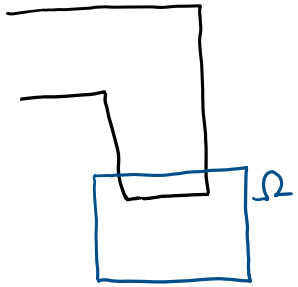
$$\oint_C \underline{H} \cdot \underline{dl} = \int_{\partial\Omega} \underline{J_f} \cdot \hat{n} dA$$

$$H_{iron} l + H_g g = Ni$$

l is the mean path length inside the iron.
Calculated along center path of iron core.

Conservation of flux

$$\oint_{\partial\Omega} \underline{B} \cdot \hat{n} dA = 0$$

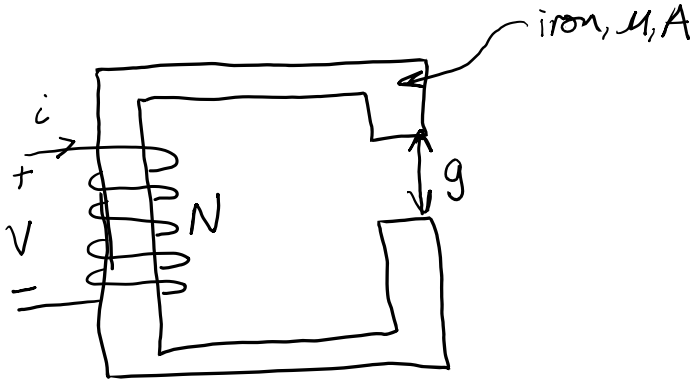


$B_g A_g$- B_{iron} A_{iron}=0

Neglecting fringing:

$B_g = B_{iron}$

Iron Core with Air Gap



$$\mu H_{iron} = B_{iron}, \mu_0 H_g = B_g$$

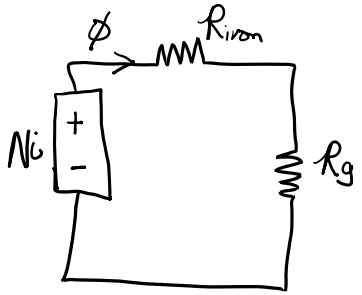
$$B_{iron}\left(\frac{l}{\mu}\right) + B_g\left(\frac{g}{\mu_0}\right) = Ni$$

$$\phi = B_{iron}\, A_{iron} = B_g\, A_g$$

$$\phi\left(\frac{l}{\mu A_{iron}} + \frac{g}{\mu_0 A_g}\right) = Ni$$

$$\phi\left(\mathcal{R}_{iron} + \mathcal{R}_g\right) = Ni$$

$$\phi = \frac{N}{\mathcal{R}_{iron} + \mathcal{R}_g}\, i$$



Faraday's Law

$$\oint_C \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int_{\partial\Omega} \underline{B} \cdot \hat{n} dA\right)$$

$$v = \frac{d\lambda}{dt}$$

$$\lambda = \frac{N^2}{\mathcal{R}_{iron} + \mathcal{R}_g} i = Li$$

As μ→∞, $\mathcal{R}_{iron}$ →0

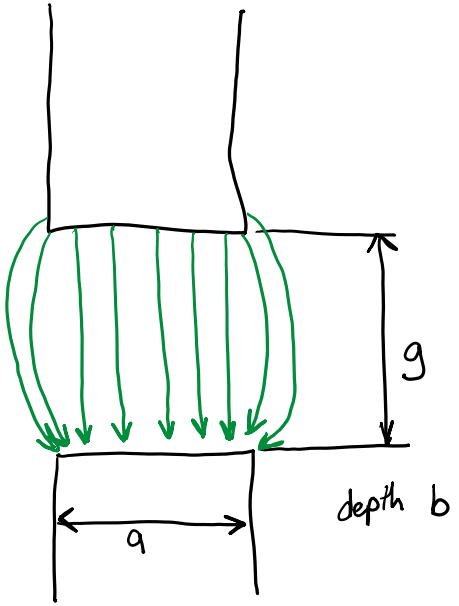
What does μ→∞ mean?

$$B = \mu H \rightarrow H = \frac{B}{\mu}$$

B is infinite from conservation of flux

μ→∞, H→0 and reluctance→0

Fringing Effects



Magnetic field spreads out.

$$A_g = (a + g)(b + g)$$

As g decreases, fringing effects are reduced

Example: a=2 cm, b=3 cm, g=3 mm. What is the reluctance of the air gap?

a) 3.97×10^6 At/Wb

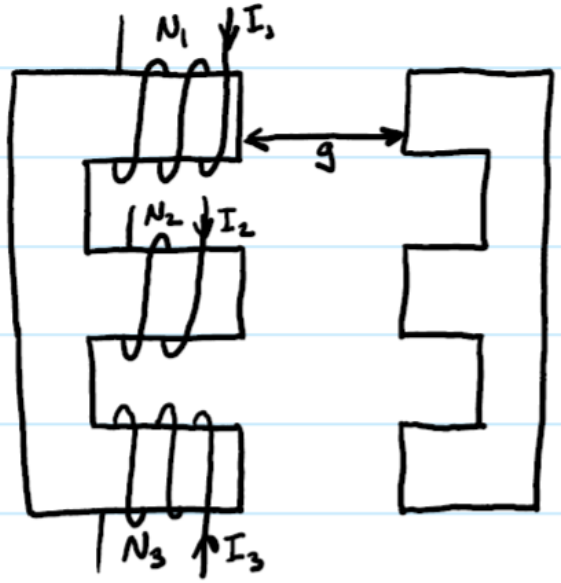
b) 3.45×10^6 At/Wb

c) 3.62×10^6 At/Wb

d) 3.14×10^6 At/Wb

Example

Ex



Given: $N_1 = 100$ $I_1 = 25A$ $A_g = 4cm^2$

$N_2 = 50$ $I_2 = 10A$ $\mu = \infty$

$N_3 = 100$ $I_3 = 15A$ $g = 0.1cm$

Find: Flux through the coils

What is the reluctance of the air gaps?

a) 1.99×10^8 At/Wb

b) 1.99×10^6 At/Wb

c) 1.99×10^5 At/Wb

d) 1.99×10^7 At/Wb

Example

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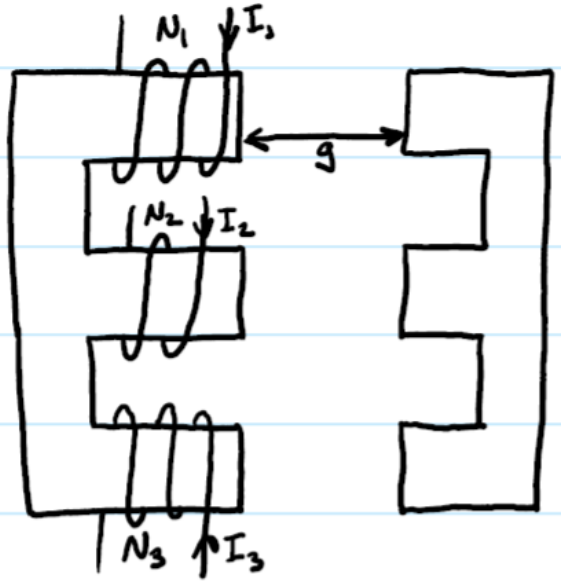

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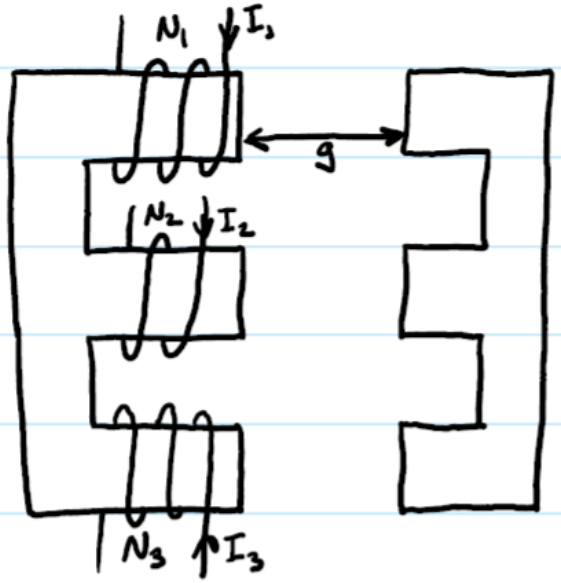

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d) 1.99×10^7 At/Wb

Where should the positive terminals be placed on the MMF sources?

a) right, left, right

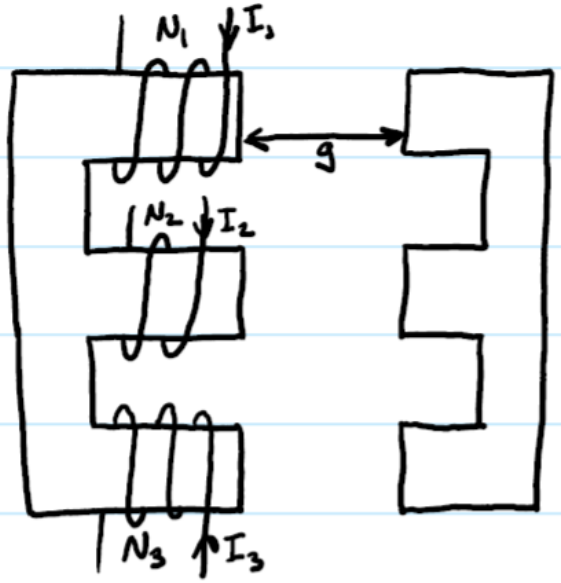
b) left, left, right

c) right, right, left

d) right, right, right

Example

Ex



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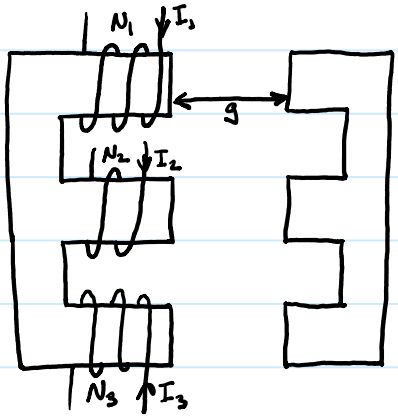

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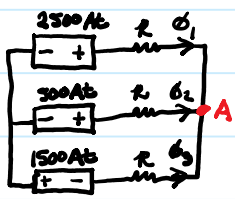
Find: Flux through the coils

Solution: All air gaps are the same

$$\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = \mathcal{R} = \frac{g}{\mu_0 A} = \frac{(0.001m)}{(4\pi\times10^{-7}\ H/m)(.0004m^2)} \Rightarrow \mathcal{R} = 1.99\times10^6\ At/Wb$$

Magnetic Circuit:



at point A: $\phi_1 + \phi_2 + \phi_3 = 0$

Let the MMF at A be f

$$\phi_1 = \frac{2500 - f}{\mathcal{R}} \qquad \phi_2 = \frac{500 - f}{\mathcal{R}} \qquad \phi_3 = \frac{-1500 - f}{\mathcal{R}}$$

$$\frac{2500 - f}{\mathcal{R}} + \frac{500 - f}{\mathcal{R}} - \frac{(1500 + f)}{\mathcal{R}} = 0 \Rightarrow 3000 - 2f - 1500 - f = 0$$

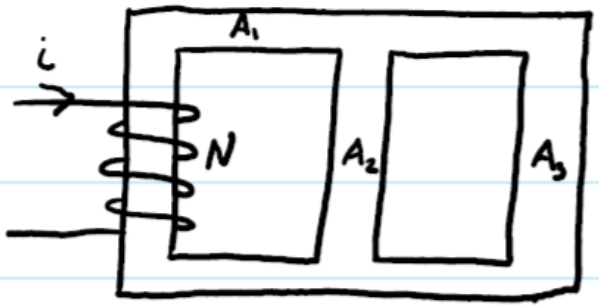
$$1500 - 3f = 0$$

$$\boxed{f = 500At}$$

$$\boxed{\begin{aligned} \phi_1 &= 1\times10^{-3}\ Wb \\ \phi_2 &= 0\ Wb \\ \phi_3 &= -1\times10^{-3}\ Wb \end{aligned}}$$

Example

Ex



Given: $\mu = \mu_o \mu_r$ $\quad N = 25 \quad i = 0.5A$

$\mu_1 = \mu_3 = 2250 \quad l_1 = l_3 = 30cm \quad A_1 = A_3 = 2cm^2$

$\mu_2 = 1350 \quad l_2 = 10cm \quad A_2 = 4cm^2$

Find: B_1, B_2, B_3

What is the net reluctance of the circuit?

a) 1.47×10^5 At/Wb

b) 1.15×10^5 At/Wb

c) 1.99×10^5 At/Wb

d) 6.46×10^5 At/Wb

Example

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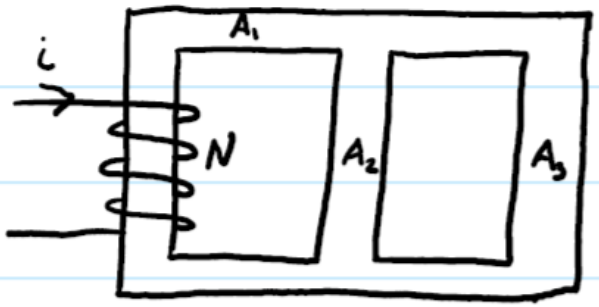

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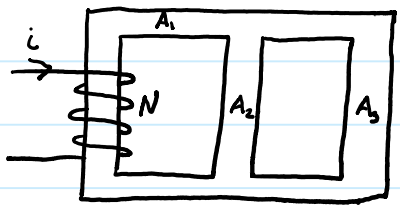

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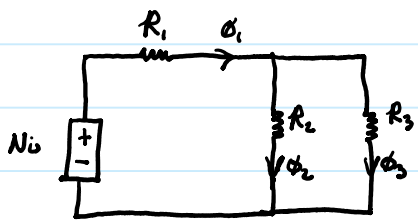
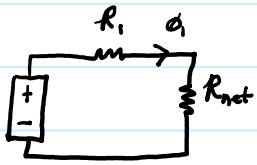
Solution: Magnetic circuit



$R_1 = R_3 = 5.31 \times 10^5$ At/Wb

$R_2 = 1.47 \times 10^5$ At/Wb



$$\frac{1}{R_{net}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$R_{net} = 1.15 \times 10^5$ At/Wb

$$\phi_1 = \frac{Ni}{R_1 + R_{net}} \Rightarrow \phi_1 = B_1 A_1 = 19.35 \times 10^{-6} \text{ Wb}$$

$$\boxed{B_1 = 0.09675 \text{ Wb/m}^2}$$

$Ni - R_1 \phi_1 = f$

$f = 2.23$ At

$$\phi_2 = \frac{f}{R_2} \Rightarrow \phi_2 = B_2 A_2 = 15.17 \times 10^{-6} \text{ Wb}$$

$$\boxed{B_2 = 0.0379 \text{ Wb/m}^2}$$

$\phi_1 = \phi_2 + \phi_3$

$$\phi_3 = \phi_1 - \phi_2 \Rightarrow \phi_3 = B_3 A_3 = 4.18 \times 10^{-6} \text{ Wb}$$

$$\boxed{B_3 = 0.0209 \text{ Wb/m}^2}$$