Equations for HW 7

- Energy change between two points
  - $W_{mb} - W_{ma} = EFE|_{a\to b} + EFM|_{a\to b}$
  - $W_{ma} = \int_0^{\lambda_a} i(\lambda, x) d\lambda$
  - $W_{mb} = \int_0^{\lambda_b} i(\lambda, x) d\lambda$
  - $EFE|_{a\to b} = \int_C i(\lambda, x) d\lambda$
  - $EFM|_{a\to b} = \int_C -f^e(\lambda, x) dx$

- Energy change over a cycle
  - $0 = EFE|_{cycle} + EFM|_{cycle}$
  - $EFE|_{cycle} = \oint i(\lambda, x) d\lambda$
  - $EFM|_{cycle} = \oint -f^e(\lambda, x) dx$
  - EFE>0 (EFM<0): Motor
  - EFE<0 (EFM >0): Generator

Example

Problem 3 (25 Points)

A certain system has flux linkage given as

$$\lambda = \frac{0.3}{x - 0.01} i^2 \tag{2}$$

with the constraint that $x>0.01$. For this system, find the energy from the mechanical field EFM and the energy from the electrical field EFE as the system moves from x=0.02 m to x=0.015 m while i is held constant at i=2 A.

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Example

Problem 3. (25 points.)

An electromechanical system has the following flux linkage-current relationship:

$$\lambda = \frac{0.09}{(0.02+x)}\, i$$

In the following questions, EFE stands for "Energy From the Electrical system into the coupling field", and EFM stands for "Energy From the Mechanical system into the coupling field".

Consider the following to be point a: x = .01 meters, i = 2 Amps, λ = 6 Wb-Tns.

a) Find the EFE and EFM as the system is moved along constant x from point a to point b which has i = 3 Amps.
b) Find the EFE and EFM as the system is moved along constant current from point b to point c which has x = 0.025 meters.
c) Find W_m at point c
d) Find the EFE and EFM as the system is moved along constant λ from point c to point a which has x = .01 meters.
e) For this cycle, is this a motor or a generator? Explain why.

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Equations for HW 8

- State Space
  - $\frac{d\bar{x}}{dt} = \bar{f}(\bar{x}, \bar{u}, t)$
  - Number of state variables correspond to order of ODE
- Equilibrium
  - $\frac{d\bar{x}}{dt} = \bar{f}(\bar{x}, \bar{u}, t) = 0$
  - List as a point in space
    $(x_1, x_2, x_3, \dots, x_n)$
- Euler's method
  - $\bar{x}^n = \bar{x}^{n-1} + \Delta t \bar{f}(\bar{x}^{n-1}, \bar{u}^{n-1}, t^{n-1})$
  - Note: this is NOT raising to a power
  - *n*: current time step
  - *n*-1: previous time step
  - Making a table helps solve

Example

Problem 4 (25 Points)

A spring pendulum has equations of motion given as

$$m\ddot{r} = -k\left(r - R_0\right) + mg\cos\left(\theta\right) + mr\dot{\theta}^2$$

$$mr\ddot{\theta} = -mg\sin\left(\theta\right) - 2m\dot{r}\dot{\theta}$$

where m is the mass of the attached object, R_0 is the unstretched length of the spring, and the dot notation signifies a time derivative, i.e. $\frac{dr}{dt} = \dot{r}$.

For this given system,

a) Find the equilibrium positions.
b) Rewrite the equations of motion in state space form
c) If the spring pendulum initially starts at $r\left(0\right) = R_0$, $\dot{r}\left(0\right) = 0$, $\theta\left(0\right) = \frac{\pi}{12}$, $\dot{\theta}\left(0\right) = 0$, use $\Delta t = 0.001$ s to determine the state variables at $t = 0.002$ s using R_0=1 m, k=200 N/m, m=2 kg, and g=9.81 m/s^2.

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Example

Problem 4 (25 points)

The equation of motion governing the angle of a pendulum connected to a torsional spring under gravity is given as

$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{ml^2}\theta + \frac{g}{l}\sin(\theta)$$

For this given system:

a) Write the equation of motion in state space form. (7 points)

b) Write the equation(s) you would have to solve (BUT DO NOT SOLVE) for determining the equilibrium positions for the pendulum. (7 points)

c) The function $\frac{\sin(\theta)}{\theta} \approx 1 - \frac{\theta^2}{6}$. Using this approximation, what are the equilibrium positions for the pendulum? (7 points)

d) Assuming that the pendulum starts at an initial position of $\theta = 0.01$ rad, $\dot{\theta} = 0$ rad/s, m=1 kg, l=0.1 m, $\kappa = 10$ Nm/rad, and g=9.8 m/s^2, find the state variables at t=0.002 s using a step size $dt = 0.001$ s. (4 points)