Final Exam SP2005

Problem 1 (25 pts.) (Sauer + Liu)

The following three-phase, balanced loads are connected across a three-phase, wye-connected source (60 Hz and 480 V – line to line). The nature of the three loads are described below:

Load #1: Wye-connected load with 100 kVA at 0.9 PF lag;
Load #2: Wye-connected load with 60 kW at 0.7 PF lead;
Load #3: Delta-connected load, with 75 A phase current and 0.9 PF lag.

Calculate the following:

a) The total complex power (three phase) consumed by the three loads;
b) The magnitude of the total source line current;
c) The reactive power per phase needed to be added to the delta connected load to make the overall PF 0.9 lead (total for all loads).

a) $\bar{S} = 100k \angle \cos^{-1}.9 + \frac{60k}{.7} \angle -\cos^{-1}.7 + 3 \times 75 \times 480 \angle \cos^{-1}.9$

$= 90k + j43.6k + 60k - j61k + 97.2k + j47k$

$= 247.2k + j29.6k = 249k \angle 7^\circ$

b) $249k = \sqrt{3} \times 480 \times I$

$\boxed{I = 300A}$

c) $\bar{S}_{new} = 247.2k + j(29.6k + Q_{add\,3\phi}) = \frac{247.2k}{.9} \angle -\cos^{-1}.9$

$= 275 \angle -26^\circ = 247K - j120K$

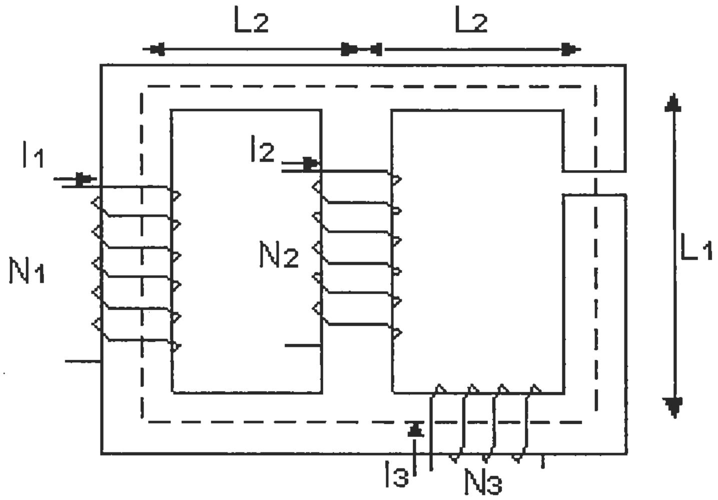
$29.6k + Q_{add\,3\phi} = -120k$

$Q_{add\,3\phi} = 150k$

$\boxed{Q_{add} = 50 kVAR \text{ per phase}}$

Problem 2 (25 pts.)

A multi-coil magnetic circuit is illustrated in the diagram below. The depth of the magnetic core is d. The width of the magnetic core is w. The permeability of the magnetic core is μ_m. The permeability of the air gap is μ_o. The length of the air gap is g.



Find:

a) The magnetic field intensity in the air gap in terms of the parameters and currents.
b) The magnetic flux density in the core of each coil in terms of the parameters and currents.
c) The three equations describing the three flux linkages for coils 1, 2, and 3 in terms of the parameters and the three currents (+ terminal of the coil is where the current enters).

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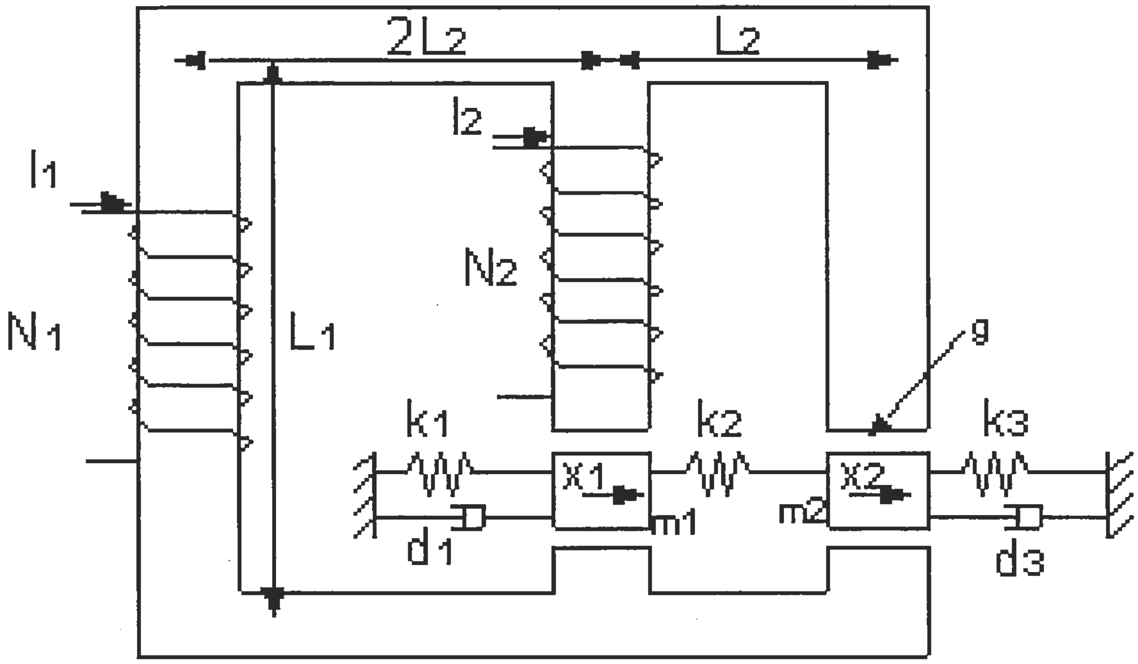
Problem 3 (25 pts):



An electromechanical system at rest is shown above. There are two moving pieces that are connected to walls and to each other through springs. The two pieces are labeled m1 and m2, and their respective displacements are designated x_1 and x_2. These displacements are relative to the equilibrium shown above. The cross-sectional area of the iron core in all locations is width w and depth d. All four gaps are length g and do not change (vertical motion is not allowed). The permeability of the magnetic core is μ_m. In the position shown (x1 = 0 and x2 = 0), the springs are exerting zero force. Neglect fringing.

a) Find the expressions for the two flux linkages in terms of the two currents, all parameters, and the two distances x1 and x2 (+ terminals of the coils are where the current enters).
b) Find the expressions for the two forces of electrical origin (one on m1 and one on m2).
c) Write the coupled electromechanical equations describing both the motions of the two pieces as well as the two currents. Include external mechanical forces acting on m1 and m2. You do not need to put these in state-space form.
d) Find the equations you would need to solve for the equilibrium points of the system for some applied voltages and/or external forces.

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Problem 4 (25 pts.)

An electromechanical system has the following flux linkage vs current relationship:

$$\lambda_1 = (0.1/x)i_1 + (0.05/x)\, i_2$$

$$\lambda_2 = (0.05/x)i_1 + 0.1\, i_2$$

Find the energy transferred from the electrical system into the coupling field (EFE) and the mechanical energy transferred from the mechanical system into the coupling field (EFM) for the following paths ($i_2 = 2$ Amps throughout):

a) The system moves from x = 0.1 m and $i_1 = 5$ Amps to x = 0.15 m with constant current.
b) The system moves from x = 0.15 m and $i_1 = 5$ Amps back to x = 0.1 m along a constant force of electrical origin path (using current that is always positive).

$$W_m = \frac{.05}{x} i_1^2 + \frac{.05}{x} i_1 i_2 + .05 i_2^2$$

$$f^e = -\frac{.05}{x^2} i_1^2 - \frac{.05}{x^2} i_1 i_2$$

a) $$EFE_{a-b} = \int_6^4 5\, d\lambda_1 + \int_{2.7}^{1.866} 2\, d\lambda_2 = 5(4-6) + 2(1.866-2.7) = -11.67$$

$$EFM_{a-b} = \int_{.1}^{.15} \left(\frac{.05 \times 25}{x^2} + \frac{.05 \times 10}{x^2} \right) dx = -\frac{1.25}{x} - \frac{.5}{x} \Big|_{.1}^{.15} = 5.833\ J$$

$$W_{mb} = W'_{mb} = \frac{.05 \times 25}{.15} + \frac{.05 \times 10}{.15} + .05 \times 4 = 11.867$$

$$W_{ma} = W'_{ma} = \frac{.05 \times 25}{.1} + \frac{.05 \times 10}{.1} + .05 \times 4 = 17.7$$

$$11.8667 - 17.7 \stackrel{?}{=} -11.67 + 5.833 \qquad \text{yes}$$

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b) $EFm_{b-c} = -\int_{.15}^{.1} \left(-\frac{.05 \times 25}{.15^2} - \frac{.05 \times 5 \times 2}{.15^2}\right)dx = 77.78\,(0.1 - 0.15)$

$= -3.89\ J$

for constant f^e_1 $\quad -\frac{.05}{x^2} i_1^2 - \frac{.05}{x^2} \times 2 i_1 = \text{constant}$

at $x_1 = .15$ $\quad i_1 = 5A, \quad f' = -55.5 - 22.2 = -77.7\ N$

at $x_1 = .1$ $\quad f' = -77.7\ N$ $\quad$ Solve for i_1

$-77.7 = -\frac{.05}{.1^2} i_1^2 - \frac{.05}{.1^2} 2 i_1$

$0.777 = .05 i_1^2 + .1 i_1$

$i_1^2 + 2 i_1 - 15.54 = 0$

$i_1 = -1 \pm 4$ $\quad$ choose $\boxed{i_1 = +3}$

$u_{m_c} = w'_{m_c} = \frac{.05}{.1} 3^2 + \frac{.05}{.1} 3 \times 2 + .2 = 7.7\ J$

$u_{m_c} - u_{m_b} = EFE_{b-c} + EFm_{b-c}$

$7.7 - 11.867 = EFE_{b-c} - 3.89$

$EFE_{b-c} = -0.3\ J$

Problem 5 (25 pts.)

A 3-phase, 4-pole, 60Hz, round rotor, Y-connected synchronous machine is delivering 5,700 Watts (3-phase) to a network at 480 Volts (line to line). When the rotor (field) current is 5 Amps, the machine is overexcited (delivering VARS). In this condition, the generator line current is 8 Amps, and the torque angle is 30 degrees (electrical).

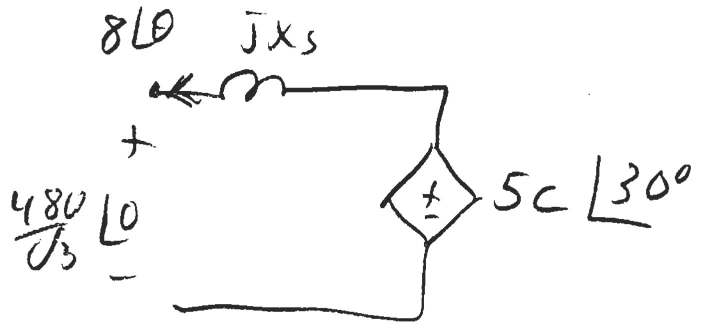
a) What is the maximum amount of real power this machine can deliver if the network voltage remains at 480 Volts and the rotor (field) current remains at 5 Amps?
b) What is the synchronous reactance of this machine?

a) $5700 = \dfrac{3 \times \frac{480}{\sqrt{3}} \times 5C}{X_s} \sin 30°$

$\frac{C}{X_s} = 2.74$ $\quad P_{max} = 3 \times \frac{480}{\sqrt{3}} \times 5 \times 2.74$

$= 11,390$

b)



$3 \times \frac{480}{\sqrt{3}} \times 8 \cos\theta = 5700$ $\quad \theta = -31°$

lagging load

$5C \angle 30° = 8\angle -31° + jX_s + 277\angle 0$

$4.33C = 277 + 4.12 X_s$

$2.5C = 0 + 6.86 X_s$

$X_s = 35.7\Omega$

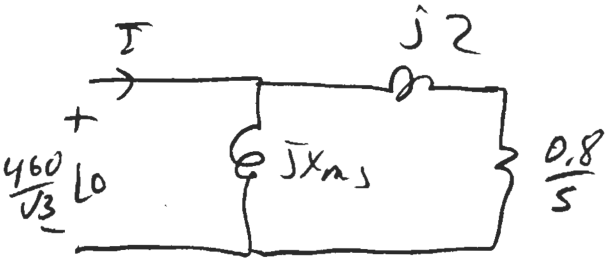
Problem 6 (25 pts.)

A 460 Volt (line to line), 60 Hz, 3-phase, Y-connected, 4 pole Induction motor is running at rated voltage, current and speed. The speed is 1720 RPM. The stator resistance and leakage reactance may be neglected. When the motor runs at no load, the speed is approximately 1800 RPM and the stator line current (magnitude) is 6 Amps. The rotor resistance referred to the stator (R_r') is 0.8 Ohms. The rotor leakage reactance referred to the stator (X_{lr}') is 2.0 Ohms.

a) What is the rated line current?
b) What is the starting line current if started at full rated voltage?
c) What is the rated torque?
d) What is the maximum possible torque this motor can deliver?
e) What is the speed at which this maximum torque occurs?

$S_{FL} = 0.0444$

a)



No load – $S=0$

$$\frac{460/\sqrt{3}}{X_{ms}} = 6 \qquad X_{ms} = 44.3$$

$$\bar{I}_{rated} = \left(\frac{460/\sqrt{3}}{j44.3} + \frac{460/\sqrt{3}}{(18+j2)}\right)$$

$$= -j6 + 14.7 = 15.9\angle\theta \qquad I_{rated} = 15.9A$$

b) $$I_{start} = -j6 + \frac{460/\sqrt{3}}{(j2+.8)} = -j6 + 123\angle{-68^\circ} \qquad I_{start} = 125A$$

c) $$T_{rated} = \frac{P_{AG}}{\omega_s \frac{2}{P}} = \frac{3 \times 14.7^2 \left(\frac{.8}{.0444}\right)}{377 \times \frac{2}{4}} = 62\ Nm$$

e) $$S_{max\,T} = \frac{.8}{2} = .4$$

$N = 1080$ RPM

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d) $$T^e_{max} = \frac{4}{2 \times 377} \times \frac{3}{2}\frac{(265.6)^2}{2} = 281\ Nm$$