

ECE 330 Exam #2, Fall 2011 Name: Solution PWS  
 90 Minutes

Section (Check One) MWF 10am \_\_\_\_\_ MWF 2pm \_\_\_\_\_

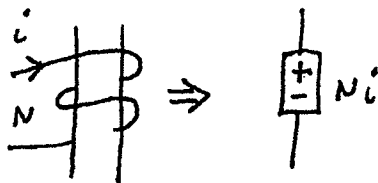
1. \_\_\_\_\_ / 25    2. \_\_\_\_\_ / 25  
 3. \_\_\_\_\_ / 25    4. \_\_\_\_\_ / 25    Total \_\_\_\_\_ / 100

Useful information

$$\sin(x) = \cos(x - 90^\circ) \quad \bar{V} = \overline{ZI} \quad \bar{S} = \overline{VI^*} \quad \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$$

$$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da \quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da \quad \mathfrak{R} = \frac{l}{\mu A} \quad MMF = Ni = \phi \mathfrak{R}$$

$$\mathfrak{R} = \frac{l}{\mu A} \quad B = \mu H \quad \phi = BA \quad \lambda = N\phi \quad \lambda = Li \text{ (if linear)}$$



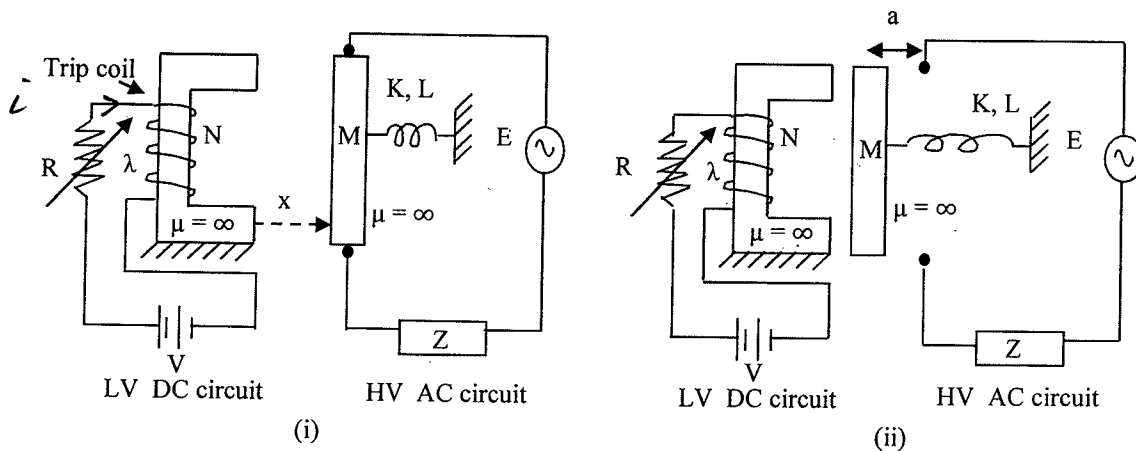
$$W_m = \int_0^\lambda id\hat{\lambda} \quad W_m' = \int_0^i \lambda d\hat{i} \quad W_m + W_m' = \lambda i \quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x} \quad x \rightarrow \theta$$

$$f^e \rightarrow T^e$$

$$EFE_{a \rightarrow b} = \int_a^b id\lambda \quad EFM_{a \rightarrow b} = -\int_a^b f^e dx \quad EFE_{a \rightarrow b} + EFM_{a \rightarrow b} = W_{mb} - W_{ma} \quad \lambda = \frac{\partial W_m'}{\partial i} \quad i = \frac{\partial W_m}{\partial \lambda}$$

**Problem 1. (25 points)**

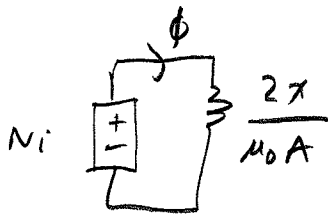
Figure (i) shows the operation of an electromechanical circuit breaker. The bar of mass  $M$  is conductive and completes a high voltage ac (HVAC) electric circuit. The trip coil is energized with a low voltage dc source  $V$  through a variable resistance  $R$ . When  $R$  is very large (tending to infinity), there is no current in the trip coil. In this condition, the mass is at  $x = L$ . Now  $R$  is decreased very slowly. As a result, current in the coil of the electromagnet increases. The electromagnet pulls the bar towards it and is opposed by the spring force. As the bar moves towards the electromagnet, an electric arc with zero resistance maintains the current in the HVAC circuit. However, when the bar is at a distance 'a' from the electric terminals, as shown in figure (ii), the arc extinguishes and the HVAC circuit trips. Assume this is also the new equilibrium position. Find an expression for the value of resistance  $R$  that will produce this result. Assume that the winding resistance of the trip coil itself is zero and area of cross section of the electromagnet and the bar is  $A$ . Also assume that both the electromagnet and the bar have infinite permeability.



$$V = iR + \frac{d\lambda}{dt}$$

$$\lambda = N\phi$$

$$\phi = \frac{Ni}{\frac{2x}{\mu_0 A}} = \frac{\mu_0 ANi}{2x}$$



$$\lambda = \frac{\mu_0 AN^2 i}{2x}$$

$$W_m = \frac{\mu_0 AN^2 i^2}{4x}$$

$$f^e = - \frac{\mu_0 AN^2 i^2}{4x^2}$$

$$m \frac{dx}{dt} = f^e - k(x-L)$$

at equilibrium,  $x = L - a$        $i = \frac{V}{R}$

$$0 = - \frac{\mu_0 AN^2 \left(\frac{V}{R}\right)^2}{4(L-a)^2} + ka$$

$$\mu_0 AN^2 \left(\frac{V}{R}\right)^2 = 4(L-a)^2 ka$$

$$\left(\frac{V}{R}\right)^2 = \frac{4(L-a)^2 ka}{\mu_0 AN^2}$$

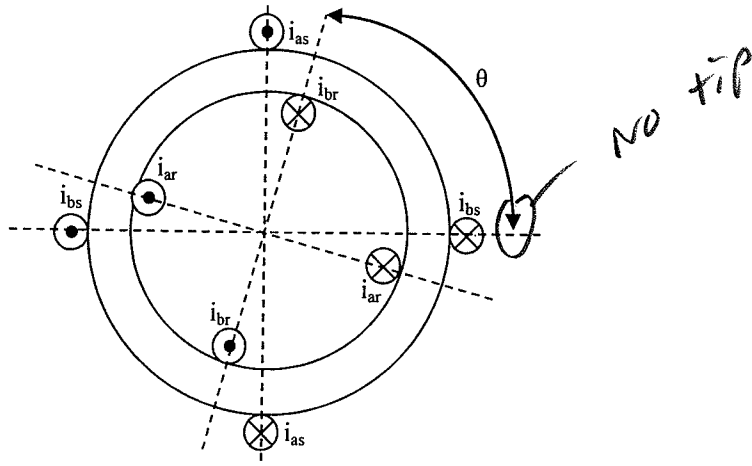
$$\frac{V}{R} = \frac{2(L-a)\sqrt{ka}}{N\sqrt{\mu_0 A}}$$

$$R = \frac{NV}{2(L-a)} \sqrt{\frac{\mu_0 A}{ka}}$$

**Problem 2. (25 points.)**

Consider a machine with two stator coils and two rotor coils as shown below. Coils  $a_s$  and  $b_s$  are mechanically displaced from each other by 90 degrees. Coils  $a_r$  and  $b_r$  are also mechanically displaced by 90 degrees from each other. The angle between the stator 'a' axis and rotor 'a' axis is  $\theta$ . Write down the flux linkage equations  $\lambda_{a_s}(i_{a_s}, i_{b_s}, i_{a_r}, i_{b_r}, \theta)$ ,  $\lambda_{b_s}(i_{a_s}, i_{b_s}, i_{a_r}, i_{b_r}, \theta)$ ,  $\lambda_{a_r}(i_{a_s}, i_{b_s}, i_{a_r}, i_{b_r}, \theta)$ ,  $\lambda_{b_r}(i_{a_s}, i_{b_s}, i_{a_r}, i_{b_r}, \theta)$  by inspection using inductance parameters  $L_s$ ,  $L_r$ , and  $M$ . You may use sinusoidal approximations for position effects as appropriate.

Find the co-energy and the torque of electric origin.



$$\begin{bmatrix} \lambda_{a_s} \\ \lambda_{b_s} \\ \lambda_{a_r} \\ \lambda_{b_r} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M \cos \theta & -M \sin \theta \\ 0 & L_s & M \sin \theta & M \cos \theta \\ M \cos \theta & M \sin \theta & L_r & 0 \\ -M \sin \theta & M \cos \theta & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{a_s} \\ i_{b_s} \\ i_{a_r} \\ i_{b_r} \end{bmatrix}$$

$$w'_m = \left( \frac{1}{2} L_s i_{a_s}^2 \right) + \left( \frac{1}{2} L_s i_{b_s}^2 \right) + \left( M \cos \theta i_{a_s} i_{a_r} + M \sin \theta i_{b_s} i_{a_r} + \frac{1}{2} L_r i_{a_r}^2 \right) + \left( -M \sin \theta i_{a_s} i_{b_r} + M \cos \theta i_{b_s} i_{b_r} + \frac{1}{2} L_r i_{b_r}^2 \right)$$

$$T^e = -M \sin \theta i_{a_s} i_{a_r} + M \cos \theta i_{b_s} i_{a_r} - M \cos \theta i_{a_s} i_{b_r} - M \sin \theta i_{b_s} i_{b_r}$$

**Problem 3. (25 points.)**

An electromechanical system is described by the following flux-linkage vs current characteristic:

$$\lambda = \frac{.01}{x - .01} i$$

- Find the energy transferred from the electrical system into the coupling field when the system moves from  $x = 0.03$  meters to  $x = 0.02$  meters while the current is held constant at 4 Amps.
- Find the energy transferred from the mechanical system into the coupling field when the system moves from  $x = 0.03$  meters to  $x = 0.02$  meters while the current is held constant at 4 Amps.
- Find the energy transferred from the electrical system into the coupling field when the system moves from  $i = 4$  Amps to  $i = 0$  Amps while the mass is held fixed at  $x = 0.03$  meters.
- Find the energy transferred from the mechanical system into the coupling field when the system moves from  $i = 4$  Amps to  $i = 0$  Amps while the mass is held fixed at  $x = 0.03$  meters.

$$a) \quad E_{FE} = \int_{.03}^{.02} 4 \, d\lambda = \boxed{8 \text{ J}}$$

$$b) \quad w'_m = \frac{.005 i^2}{x - .01} \quad f^e = -\frac{.005 i^2}{(x - .01)^2}$$

$$E_{FM} = - \int_{.03}^{.02} -\frac{.005 \times 16}{(x - .01)^2} \, dx$$

$$= - \left( \frac{.08}{x - .01} \right) \Big|_{.03}^{.02} = -8 + 4 = \boxed{-4 \text{ J}}$$

Check this!

$$w_{m1} = w'_{m1} = \frac{.005 \times 16}{.03 - .01} = 4 \text{ J}$$

$$w_{m2} = w'_{m2} = \frac{.005 \times 16}{.02 - .01} = 8 \text{ J}$$

$$w_{m2} - w_{m1} = 8 - 4 = 4 \text{ J} \stackrel{?}{=} E_{FE} + E_{FM} = 8 - 4 = 4 \text{ J} \checkmark$$

$$c) \quad E_{FE} = \int_{.03}^{.02} 2\lambda \, d\lambda = \lambda^2 \Big|_{.03}^{.02} = \boxed{-4 \text{ J}}$$

$$E_{FM} = \boxed{0} \quad (x \text{ is fixed})$$

$$w_{m2} = 0$$

$$w_{m1} = \frac{.08}{.03 - .01} = 4$$

$$w_{m2} - w_{m1} = 0 - 4 = -4 \checkmark$$

$$= E_{FE} + E_{FM}$$

**Problem 4. (25 points.)**

A dynamic system has the following equations:

$$\begin{aligned} \frac{dx}{dt} &= v & x(0) &= 0.5 \quad \text{assume meters} \\ \frac{dv}{dt} &= 1 - xv + x^2 & v(0) &= 0 \quad \text{assume meters/sec} \end{aligned}$$

a) Find all of the equilibrium points for this system.

$$\begin{aligned} 0 &= v_e \\ 0 &= 1 - x_e v_e + x_e^2 \end{aligned}$$

$$\begin{aligned} v_e &= 0 \text{ m/s} \\ x_e &= \pm j \text{ (No equilibrium for real } x \text{)} \end{aligned}$$

b) Use Euler's method with time step  $\Delta t = 0.1$ s to compute  $x(t)$  and  $v(t)$  at  $t = 0.1$  and  $0.2$ s

$$\begin{aligned} x(0.1) &= 0.5 + v(0) \times 0.1 = 0.5 + 0 \times 0.1 = 0.5 \text{ m} \\ v(0.1) &= 0 + (1 - x(0)v(0) + x(0)^2) \times 0.1 = 0 + (1 - 0 + 0.5^2) \times 0.1 = 0.125 \text{ m/s} \\ x(0.2) &= 0.5 + 0.125 \times 0.1 = 0.5125 \text{ m} \\ v(0.2) &= 0.125 + (1 - 0.5 \times 0.125 + 0.5^2) \times 0.1 = 0.24375 \text{ m/s} \end{aligned}$$