

Section (Check One) MWF 10am \_\_\_\_\_ MF 2pm \_\_\_\_\_

1. \_\_\_\_\_ / 25    2. \_\_\_\_\_ / 25

3. \_\_\_\_\_ / 25    4. \_\_\_\_\_ / 25    Total \_\_\_\_\_ / 100

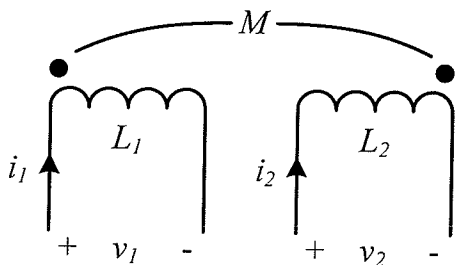
Useful Information

$$\sin(x) = \cos(x - 90^\circ) \quad \vec{V} = \vec{Z} \cdot \vec{I} \quad \vec{S} = \vec{V} \cdot \vec{I}^* \quad \vec{S}_{3\phi} = \sqrt{3}V_L I_L \angle \theta$$

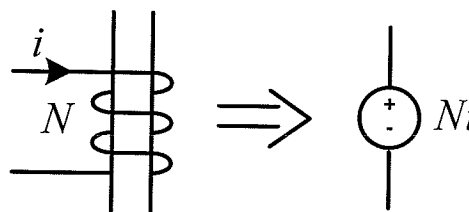
$$\begin{aligned} 0^\circ < \theta < 90^\circ \text{ (lag)} & \quad \vec{I}_L = \sqrt{3}I_\phi \angle (\theta_i - 30^\circ) & \quad \vec{Z}_Y = \vec{Z}_\Delta / 3 & \quad \mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am} \\ -90^\circ < \theta < 0^\circ \text{ (lead)} & \quad \vec{V}_L = \sqrt{3}V_\phi \angle (\theta_v + 30^\circ) \end{aligned}$$

$$\oint_c \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot \vec{n} ds \quad \oint_c \vec{E} \cdot d\vec{l} = - \int_s \frac{d}{dt} \vec{B} \cdot \vec{n} ds \quad \mathcal{R} = \frac{l}{\mu A} \quad mmf = Ni = \mathcal{R}\phi$$

$$\phi = BA \quad \lambda = N\phi \quad L = \frac{\partial \lambda}{\partial i} \quad M_{12} = \frac{\partial \lambda_1}{\partial i_2} \quad k = \frac{M}{\sqrt{L_1 L_2}} \quad 1 \text{ hp} = 746 \text{ Watts}$$



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$



## Problem 1 (25 points)

Complete the following sub-problems below.

- a) Write the following time-domain voltage and current in phasor notation:

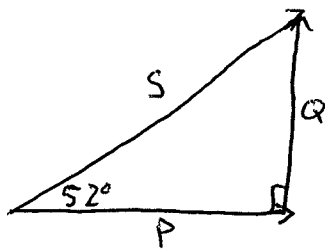
$$v(t) = 181 \cos(377t - 33^\circ) \text{ V} \quad \underline{\frac{181}{\sqrt{2}} \angle -33^\circ \text{ V}}$$

$$i(t) = 12.7 \sin(377t + 5^\circ) \text{ A} \quad \underline{\frac{12.7}{\sqrt{2}} \angle -85^\circ \text{ A}}$$

- b) Assuming that the quantities in a) are measured in a load, what is the complex power delivered to that load?

$$\begin{aligned} \bar{S} &= \bar{V} \bar{I}^* = \frac{181 \cdot 12.7}{2} \angle -33 + 85^\circ \text{ VA} \\ &= 1149.35 \angle 52^\circ \text{ VA} \end{aligned}$$

- c) Draw the corresponding power triangle, indicate the various quantities, and calculate the power factor angle?



$$\begin{aligned} S &= 1149.35 \text{ VA} \\ P &= 1149.35 \cos(52) = 707.61 \text{ W} \\ Q &= 1149.35 \sin(52) = 905.7 \text{ VAR} \end{aligned}$$

- d) Calculate the power factor of the load

$$\text{pf} = \cos(52^\circ) = \frac{P}{S} = 0.616 \text{ lag}$$

- e) How much capacitance or inductance do you have to add in parallel to the load to achieve 0.98 lagging power factor?

$$\begin{aligned} Q_{\text{new}} &= \left( \frac{707.61}{0.98} \right) \left( \sin(\cos^{-1}(0.98)) \right) \\ &= 143.69 \text{ VAR} \end{aligned}$$

$$\begin{aligned} \Delta Q &= 905.7 \text{ VAR} - 143.69 \text{ VAR} \\ &= 762.01 \text{ VAR} \end{aligned}$$

$$\Delta Q = \frac{V^2}{X} = \frac{V^2}{\frac{1}{\omega C}} = V^2 \omega C$$

$$C = \frac{762.01 \text{ VAR}}{\left( \frac{181}{\sqrt{2}} \right)^2 (377)}$$

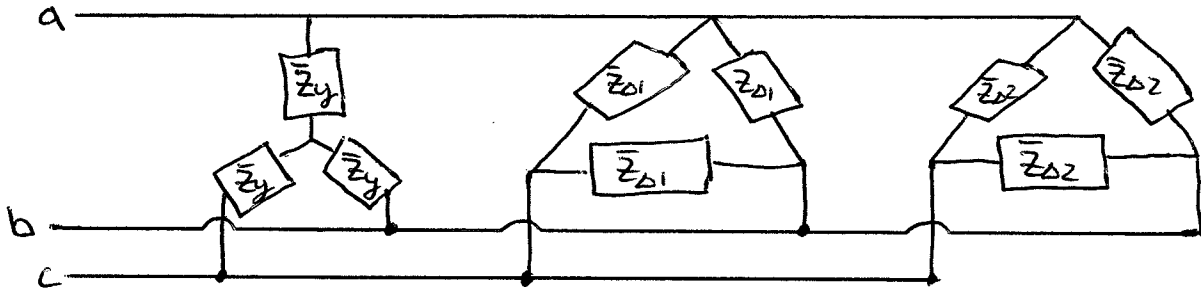
$$\boxed{C = 123.4 \mu\text{F}}$$

## Problem 2 (25 points)

Three three-phase loads are connected across a three-phase source whose line-to-line voltage is 208 V. One of these loads is connected as Y, while each of the other two is connected as  $\Delta$ . The load impedances per phase are as follows:

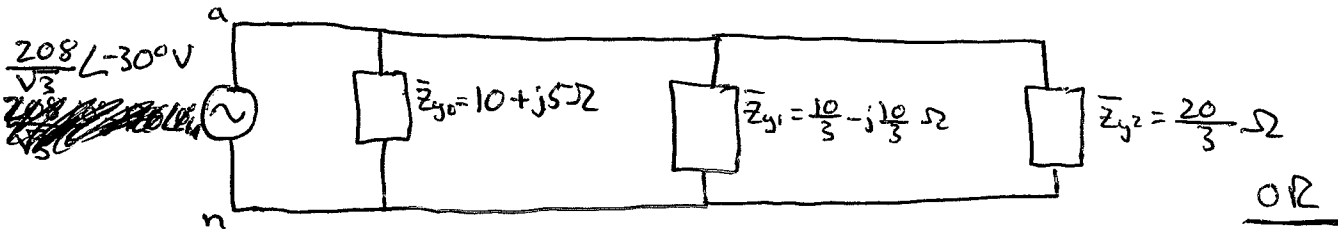
- $\bar{Z}_Y = 10 + j5 \Omega$
- $\bar{Z}_{\Delta 1} = 10 - j10 \Omega$
- $\bar{Z}_{\Delta 2} = 20 \Omega$

a) Draw the three-phase circuit.



b) Find the per-phase equivalent circuit with a source  $\bar{V}_{ab} = 208V \angle 0^\circ$ . Draw the equivalent circuit.

$$\bar{Z}_{\Delta 1} \rightarrow \bar{Z}_{y1} = \frac{1}{3} [10 - j10] \Omega, \quad \bar{Z}_{\Delta 2} \rightarrow \bar{Z}_{y2} = \frac{1}{3} [20] \Omega$$



c) Compute the three-phase complex, real, and reactive power.

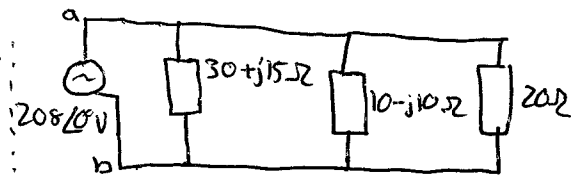
$$\begin{aligned} \bar{Z}_{eq} &= \bar{Z}_{y0} \parallel \bar{Z}_{y1} \parallel \bar{Z}_{y2} \\ &= 2.43 - j0.70 \Omega \end{aligned}$$

$$\bar{S}_{3\phi} = 3 \bar{V} \bar{I}^* = 3 \bar{V} \left( \frac{\bar{V}}{\bar{Z}_{eq}} \right)^* = 3 \frac{|\bar{V}|^2}{\bar{Z}_{eq}^*}$$

$$= \frac{3 \left( \frac{208}{\sqrt{3}} \right)^2}{2.43 + j0.70}$$

$$\bar{S}_{3\phi} = (16.44 - j4.74) \text{ KVA}_3$$

$$\begin{aligned} &\rightarrow P_{3\phi} = 16.44 \text{ kW} \\ &\rightarrow Q_{3\phi} = 4.74 \text{ KVAR} \\ &\rightarrow \text{pf} = 0.96 \text{ lead} \end{aligned}$$



### Problem 3 (25 points)

A common-mode choke inductor is shown in Fig. 1a below. The legs of the inductor core have all the same thickness (1 cm). The coupling coefficient between the coil 1 and coil 2 is  $k = 0.95$ .

- Determine the dot polarities (draw into Fig. 1a)
- Calculate the self inductance of each coil and the mutual inductance between the two coils. Assume that the flux through the core and gap make up the total amount of flux coupled between the two coils (i.e. the mutual inductance). The self inductances have equal amounts of leakage (hence  $k < 1$  as given above). *Neglect fringing in the air gap and remember that the value mutual inductance is defined as a positive number.*
- The choke inductor is wired to a resistive load as shown in Fig. 1b. Write the loop equation of the circuit.
- What is the total effective inductance of the circuit?
- (2 bonus points) What could this circuit arrangement be used for (imagine how the inductor behaves if a fault occurs at the source forces positive currents to enter coil terminals a and d and exits to earth ground at the load)?

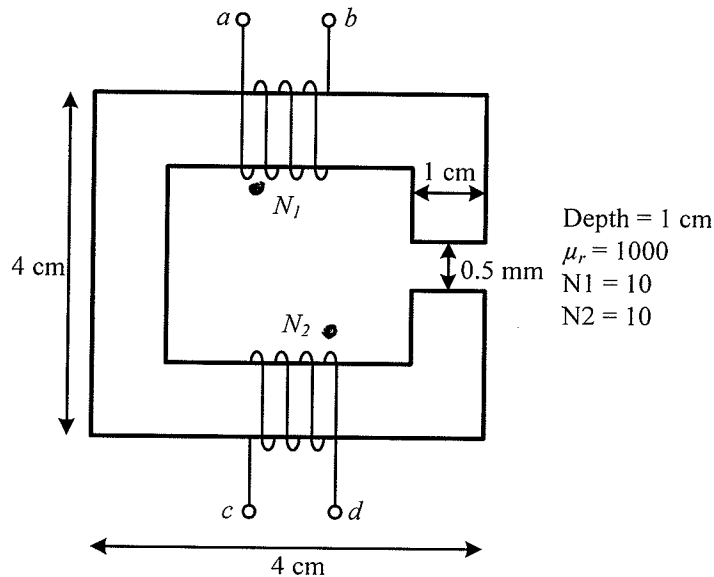


Fig. 1a

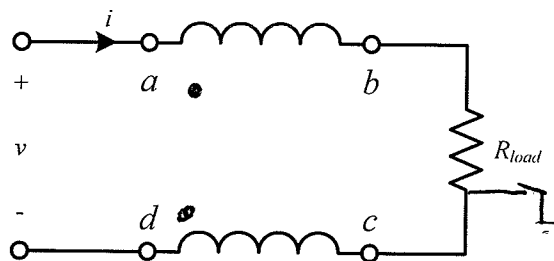


Fig. 1b

$$b) \ell_c = 3\text{cm} + 3\text{cm} + 3\text{cm} + 2.95\text{cm} = 11.5 \times 10^{-2}\text{m}$$

$$R_c = \frac{\ell_c}{\mu A} = \frac{11.95 \times 10^{-2}\text{m}}{(4\pi \times 10^{-7}\text{H/m})(1000)(1 \times 10^{-4}\text{m}^2)} = \frac{9.509 \times 10^5 \text{At/Wb}}{9.509 \times 10^5 \text{At/Wb}}$$

$$R_g = \frac{g}{\mu_0 \mu_r A_g} = \frac{0.5 \times 10^{-3}\text{m}}{(4\pi \times 10^{-7}\text{H/m})(1 \times 10^{-4}\text{m}^2)} = \frac{3.979 \times 10^5 \text{At/Wb}}{3.979 \times 10^5 \text{At/Wb}}$$

$$R_{TOT} = R_c + R_g = 4.93 \times 10^6 \frac{\text{At}}{\text{Wb}}$$

$$\frac{\Phi = \pm N_1 i_1 \pm N_2 i_2}{R_{TOT}} \rightarrow L_{11} = L_{22} = \frac{N^2}{R_{TOT}} = \frac{20.28 \text{ } \cancel{20.28 \mu\text{H}}}{20.28 \mu\text{H}}$$

$$0.95 = k = \frac{M}{\sqrt{L_{11}L_{22}}} \rightarrow M = 19.27 \mu\text{H}$$

$$c) v(t) = V_{ab}(t) + i(t)R_{load} + v_{cd}(t)$$

$$v(t) = \left( L_{11} \frac{di}{dt} - M \frac{di}{dt} \right) + i(t)R_{load} + \left( L_{22} \frac{di}{dt} - M \frac{di}{dt} \right)$$

$$v(t) = iR_{load} + (L_1 + L_2 - 2M) \frac{di}{dt}$$

$$d) L_{eq} = L_1 + L_2 - 2M = 2.02 \mu\text{H}$$

e) It may be used for noise suppression. For currents flowing from a → d, the inductor presents a small impedance (L<sub>1</sub> + L<sub>2</sub> - 2M). However, for currents flowing into a + d at the same time, the inductance of the circuit is much higher.

### Problem 4 (25 points)

Given the following coupled inductors shown in Fig. 2a. The inductors are placed in a circuit shown in Fig. 2b where the common secondary terminals are 2' and 3 (connected to  $R_3$ ).

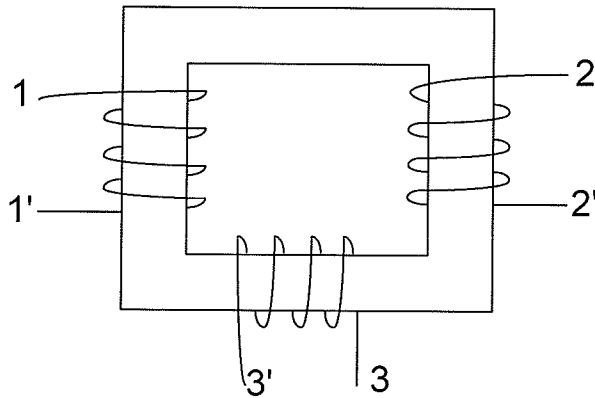


Fig. 2a

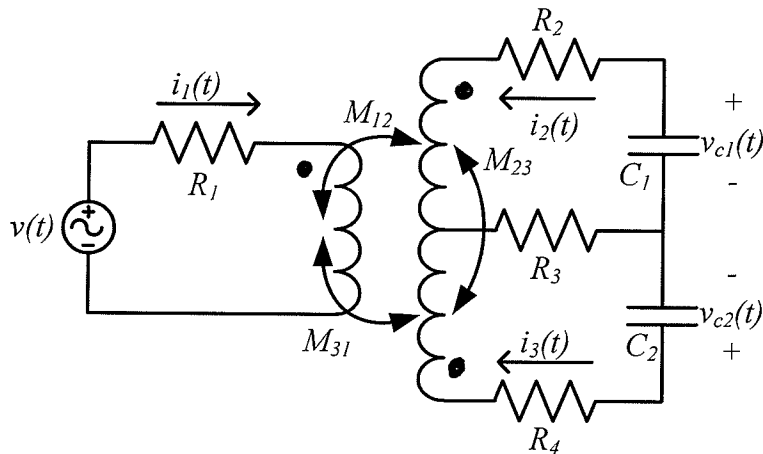


Fig. 2b

The following values are given:  $R_1=5 \Omega$ ,  $L_1=1 \text{ mH}$ ,  $M_{12}=2 \text{ mH}$ ,  $M_{23}=2 \text{ mH}$ , and  $M_{31}=2 \text{ mH}$ . Other values are unknown.

- Place the dots on Fig. 2a first then on Fig. 2b.
- Write the three KVL equations that govern the circuit in Fig. 2b assuming that the three coils have self-inductances  $L_1$ ,  $L_2$ , and  $L_3$ . Note that the coupling between the coils is on the sides with dots.
- Find  $v(t)$  given that  $i_1(t) = 2\cos(377t) \text{ A}$ ,  $i_2(t) = 0.5\cos(377t) \text{ A}$ , and  $i_3(t) = 0.5\cos(377t) \text{ A}$ .
- What is  $v(t)$  at  $t = 0$ ?

$$b) \quad v(t) = i_1 R_1 + \left( L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{31} \frac{di_3}{dt} \right)$$

$$v_{c1}(t) = i_2 R_2 + \left( L_2 \frac{di_2}{dt} + M_{23} \frac{di_3}{dt} + M_{12} \frac{di_1}{dt} \right) + R_3 (i_2 + i_3)$$

$$v_{c2}(t) = i_3 R_4 + \left( L_3 \frac{di_3}{dt} + M_{31} \frac{di_1}{dt} + M_{23} \frac{di_2}{dt} \right) + R_3 (i_2 + i_3)$$

$$c) \quad v(t) = 5(2 \cos(377t)) + 1 \text{mH} (-2.377 \sin(377t)) \\ + 2 \text{mH} (-0.5 \cdot 377 \sin(377t)) \\ + 2 \text{mH} (-0.5 \cdot 377 \sin(377t))$$

$$v(t) = 10 \cos(377t) - 1.508 \sin(377t)$$

$$d) \quad v(0) = 10$$