1. Problem 3.10

a. Assuming an ideal (lossless) transformer, the primary side current $I_1$ may be estimated to be

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{N_2 P_2}{N_1 V_2} = \frac{120 \text{ kVA}}{240 \text{ V}} = 20 \text{ A}$$

(1)

b. Keeping the same ideal transformer assumptions, the load impedance referred to the primary side is:

$$R_1 = \frac{V_1}{I_1} = \frac{240}{20} = 12 \Omega$$

(2)
2. Problem 3.12

a. Comparing the circuits in Figure 2 with Figure 1:

\[ R_1 = 0 \]  \hspace{1cm} (3)
\[ L_1 - aM = 45 \ H \]  \hspace{1cm} (4)
\[ aM = 15 \ H \]  \hspace{1cm} (5)
\[ a^2R_2 = 0 \]  \hspace{1cm} (6)
\[ a^2L_2 - aM = 0 \]  \hspace{1cm} (7)

\[ a = \frac{N_1}{N_2} = 5 \]  \hspace{1cm} (8)

From the equations, we can obtain:

\[ R_1 = 0 \]  \hspace{1cm} (9)
\[ M = 3 \ H \]  \hspace{1cm} (10)
\[ L_1 = 60 \ H \]  \hspace{1cm} (11)
\[ L_2 = 0.6 \ H \]  \hspace{1cm} (12)
\[ R_2 = 0 \]  \hspace{1cm} (13)
\[ k = \frac{M}{\sqrt{L_1L_2}} = 0.5 \rightarrow \text{slightly coupled} \]  \hspace{1cm} (14)

Figure 1: Complete equivalent circuit of the physical transformer from Figure 3.32

Figure 2: Problem 3.12
b. Given the circuit, we have

\[ v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad (15) \]
\[ v_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad (16) \]
\[ v_2 = 5i_2 \quad (17) \]

Since \( v_1(t) = \sqrt{2}100 \sin(10t) \), the voltage source has a form of \( \bar{V}_1 = 100 \angle -90 \) V and \( \omega = 10 \):

\[ \bar{V}_1 = j \omega L_1 \bar{I}_1 - j \omega M \bar{I}_2 = (j 600) \bar{I}_1 - (j 30) \bar{I}_2 \rightarrow \text{loop 1} \quad (18) \]
\[ 5\bar{I}_2 = -j \omega L_2 \bar{I}_2 + j \omega M \bar{I}_1 = -(j 6) \bar{I}_2 + (j 30) \bar{I}_1 \rightarrow \text{loop 2} \quad (19) \]
3. Problem 3.14

a. Using the material in Section 3.3.3: Writing equations with mutually coupled coils, the two loop equations can be derived while paying close attention to current direction and location of dot markings:

\[ v_s = R_1i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = i_1 + \frac{di_1}{dt} + 0.5 \frac{di_2}{dt} \]  
\[ 0 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + R_2i_2 = \frac{di_2}{dt} + 0.5 \frac{di_1}{dt} + 2i_2 \]

Figure 3: Problem 3.14

b. Since \( v_s(t) = 100 \cos(2t) \), the voltage source has a form of \( V_s = 50\sqrt{2} \angle 0 \) V and \( \omega = 2 \):

\[ 50\sqrt{2} \angle 0 = \bar{I}_1 + (j2)\bar{I}_1 + (j)\bar{I}_2 \]
\[ 0 = j2\bar{I}_2 + j\bar{I}_1 + 2\bar{I}_2 \]

Solving the above equations, we have \( \bar{V} = 23.2495\angle 170.538 \) V and \( v(t) = 23.2495\sqrt{2} \cos(2t + 170.538) \) V
4. Problem 3.17
According to Figs. 4, the appropriate loop equations can be derived for each case. Note that the 1st and 2nd terms on the right hand of each loop equation are the interactions from the top and bottom coils respectively.

\[
V = \left[ L \frac{di}{dt} + M \frac{di}{dt} \right] + \left[ L \frac{di}{dt} + M \frac{di}{dt} \right] = 2L \frac{di}{dt} + 2M \frac{di}{dt} \to \text{part a} \quad (24)
\]

\[
V = \left[ L \frac{di}{dt} - M \frac{di}{dt} \right] + \left[ L \frac{di}{dt} - M \frac{di}{dt} \right] = 2L \frac{di}{dt} - 2M \frac{di}{dt} \to \text{part b} \quad (25)
\]

\[L = 0.02875 \, H\] \quad (26)

\[M = 0.01125 \, H\] \quad (27)

\[k = \frac{M}{\sqrt{LL}} = 0.39 \to \text{loosely coupled}\] \quad (28)
5. Special Problem

a. The complex power at the load is

$$\bar{S}_2 = 4.8[0.8 + j\sin(\arccos(0.8))] \text{ kVA} = 4.8\angle36.87 \text{ kVA} \quad (29)$$

Assuming an ideal (lossless) transformer, where the input power is equal to the output power ($\bar{S}_1 = \bar{S}_2$), the primary side current is

$$\bar{I}_1 = (\frac{\bar{S}_1}{V_1})^* = \frac{4.8\angle - 36.87 \text{ kVA}}{480\angle 0 \text{ V}} = 10\angle - 36.87 \text{ A} \quad (30)$$

b. With the primary side voltage angle as the reference angle:

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}_2} = \frac{\bar{V}_2\bar{V}_2^*}{\bar{S}_2^*} = \frac{240^2}{4800\angle - 36.87} = 12\angle36.87 \Omega \quad (31)$$

c. With the primary side voltage angle as the reference angle:

$$\bar{Z}_1 = \frac{\bar{V}_1}{\bar{I}_1} = \frac{480\angle 0}{10\angle - 36.87} = 48\angle36.87 \Omega \quad (32)$$