ECE 330 Exam #2, Spring 2015 Name: Solutions

90 Minutes

Section (Check One) MWF 10am _______ TTh 9:30am _______

1. ________ / 25 2. ________ / 25

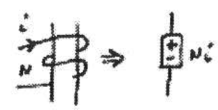
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



$$W_m = \int_0^{\lambda} i \, d\hat{\lambda} \qquad W_m' = \int_0^{i} \lambda \, d\hat{i} \qquad W_m + W_m' = \lambda i \qquad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x} \qquad x \to \theta$$

$$f^e \to T^e$$

$$\underset{a \to b}{EFE} = \int_a^b i \, d\lambda \qquad \underset{a \to b}{EFM} = -\int_a^b f^e dx \qquad \underset{a \to b}{EFE} + \underset{a \to b}{EFM} = W_{mb} - W_{ma} \qquad \lambda = \frac{\partial W_m'}{\partial i} \qquad i = \frac{\partial W_m}{\partial \lambda}$$

$M \frac{dv}{dt} = \sum forces\ in + x\ direction$ $\quad \underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u})$ $\quad$ Equilibrium: $\underline{f}(\underline{x}_{eq}, \underline{u}_{eq}) = 0$

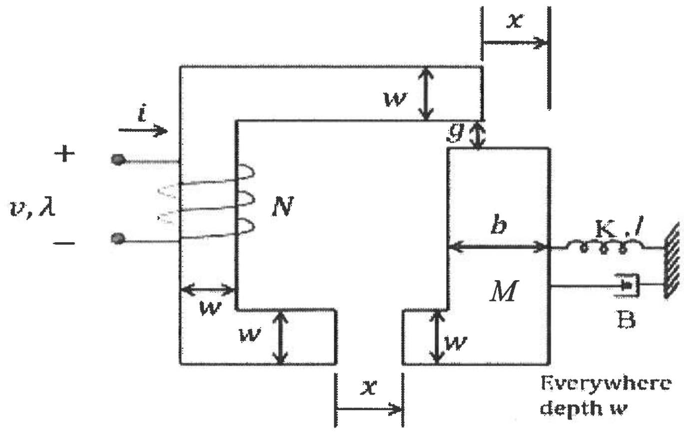
$\underline{x}(t_{n+1}) = \underline{x}(t_n) + \Delta t \cdot \underline{f}\left(\underline{x}(t_n), \underline{u}(t_n)\right)$

Linearization:
$$\Delta \dot{x}_1 = \frac{\partial f_1}{\partial x_1} \Delta x_1 + \frac{\partial f_1}{\partial x_2} \Delta x_2 + \cdots + \frac{\partial f_1}{\partial u} \Delta u$$
$$\Delta \dot{x}_2 = \frac{\partial f_2}{\partial x_1} \Delta x_1 + \frac{\partial f_2}{\partial x_2} \Delta x_2 + \cdots + \frac{\partial f_2}{\partial u} \Delta u$$

Stability: $A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$ $\quad |\lambda I - A| = 0$ $\quad$ stable if $\mathrm{Re}\{\lambda\} < 0$

Problem 1. **(25 points)**

Consider the magnetic circuit for the electromechanical system with $\mu = \infty$ in the iron path. There is no fringing of the flux. Mass M only moves in the x direction. The spring force is zero when $x = l$.



a) Find the flux $\lambda(i,x)$ linked by the electrical terminal pairs;

b) Compute energy $W_m(i,x)$ and co-energy $W_m'(i,x)$;

c) Compute the force of electric origin $f_x^e(i,x)$. What would happen when $x \to 0$?

a) $\lambda = N\phi = \frac{N^2 i}{\mathcal{R}}$

$$\mathcal{R} = \frac{x}{\mu_0 w^2} + \frac{g}{\mu_0 (b-x) w}$$

$$\lambda = \frac{N^2 i \mu_0}{\frac{x}{w^2} + \frac{g}{(b-x)w}}$$

b) $W_m'(i,x) = \int_0^i \lambda \, d\hat{i}$

$$= \frac{N^2 i^2 \mu_0}{2\left(\frac{x}{w^2} + \frac{g}{(b-x)w}\right)}$$

$W_m(i,x) = \lambda i - W_m'$

$$= \frac{N^2 i^2 \mu_0}{\frac{x}{w^2} + \frac{g}{(b-x)w}} - W_m'$$

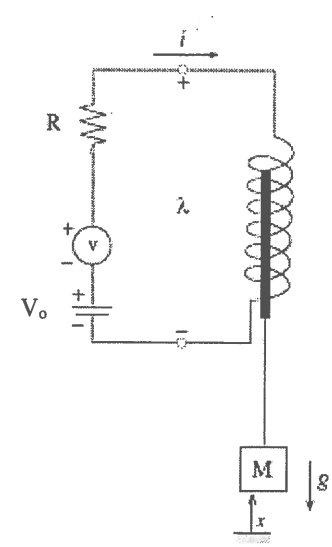
$$= \frac{N^2 i^2 \mu_0}{2\left(\frac{x}{w^2} + \frac{g}{(b-x)w}\right)}$$

c) $f^e = \frac{\partial W_m'}{\partial x}$

$$= -\frac{N^2 i^2 \mu_0}{2\left(\frac{x}{w^2} + \frac{g}{w(b-x)}\right)^2} \times \left(\frac{1}{w^2} + \frac{g}{w(b-x)^2}\right)$$

When $x \to 0$, the gap with length x disappears. We are only left with the gap of length g. The flux remains finite.

Problem 2. (25 points.)



An electromechanical system with one electrical and one mechanical terminal pair is shown here. The electrical terminal relation is given by

$$\lambda(i,x) = \frac{x}{(1-x)^2}\, i$$

The system is driven by the voltage $V_0 + v(t)$, where V_0 is a constant dc source. The mass of the plunger is included in M, and gravity acts on mass M as shown.

a) Find the co-energy $W_m'(i,x)$;

b) Compute the force of electric origin $f^e(i,x)$;

c) Write the complete mechanical and electrical system equations in terms of i and x;

d) Assuming $v(t)$=0, how many equilibrium points are there at most?

a) $W_M' = \int_0^{i} \lambda \, d\hat{i}$

$= \frac{x i^2}{2(1-x)^2}$

b) $f^e = \frac{\partial W_M'}{\partial x}$

$= \frac{i^2}{2}\left(\frac{1}{(1-x)^2} + \frac{2x}{(1-x)^3}\right)$

$= \frac{i^2(1+x)}{2(1-x)^3}$

c) $\uparrow f^e$ $\boxed{M}$ $\downarrow mg$

$m\frac{d^2x}{dt^2} = \frac{i^2(1+x)}{2(1-x)^3} - mg$

$V_0 + v = iR + \frac{d\lambda}{dt}$

$= iR + \frac{x}{(1-x)^2}\frac{di}{dt} + i\left(\frac{1}{(1-x)^2} + \frac{2x}{(1-x)^3}\right)\frac{dx}{dt}$

$= iR + \frac{x}{(1-x)^2}\frac{di}{dt} + \frac{i(1+x)}{(1-x)^3}\frac{dx}{dt}$

d) All derivatives $= 0$

$i^{eq} = \frac{V_0}{R}$

$\frac{\frac{V_0^2}{R^2}(1+x^{eq})}{2(1-x^{eq})^3} = mg$

$\frac{V_0^2}{R^2}(1+x^{eq}) = 2mg(1-x^{eq})^3$

$2mg(1-x^{eq})^3 - \frac{V_0^2}{R^2}(1+x^{eq}) = 0$

Cubic polynomial has a maximum of 3 roots.

$\therefore$ At most 3 EQ points.

Problem 3. (25 points.)

Consider a reluctance rotating motor of a single coil, with the inductance sinusoidal on the rotor position:

$$L(\theta) = 1 + 2\cos(2\theta)$$

The flux linkage is linear in current i.

a) Find the maximum possible torque when $i = 1$ Ampere;

b) Plot the following paths on the $\theta - i$ plane for the cycle: First, the current changes from 0 to 1 Ampere while the angle is fixed at 0°; Second, the angle increases to 90° while i stays at 1 Ampere; Third, the current decreases to 0 with the angle fixed at 90°; Finally, the angle decreases to 0° with the current staying at 0.

c) Find the energy from electrical (EFE) and the energy from mechanical (EFM) over this cycle. Is it a generator or motor? Explain why.

a) $\lambda = L(\theta)\, i$

$= (1+2\cos(2\theta))\, i$

$W_m' = \int_0^{i} \lambda\, d\hat{i}$

$= \dfrac{1+2\cos(2\theta)}{2}\, i^2$

$T^e = \dfrac{\partial W_m'}{\partial \theta}$

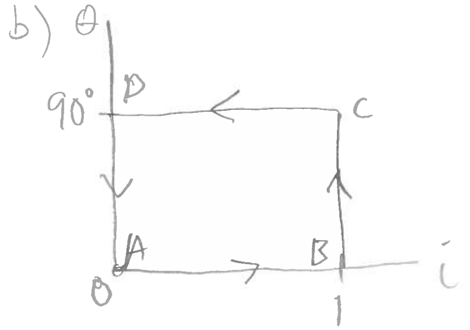
$= -2\sin(2\theta)\, i^2$

$i = 1A$

$T^e = -2\sin 2\theta$

$|T^e_{max}| = |T^e(\theta = 45°)| = 2\ Nm$

b)



c) $EFM_{A\to B} = -\int_0^0 T^e\, d\theta = 0$

$EFM_{B\to C} = -\int_0^{\frac{\pi}{2}} -2\sin(2\theta) \times 1^2\, d\theta$

$= -\cos(2\theta)\Big]_0^{\frac{\pi}{2}}$

$= 2$

$EFM_{C\to D} = -\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} T^e\, d\theta = 0$

$EFM_{D\to A} = -\int_{\frac{\pi}{2}}^{0} -2\sin(2\theta) \times 0^2\, d\theta$

$= 0$

$\therefore EFM_{cycle} = 2\ J$

$EFE_{cycle} + EFM_{cycle} = 0$

$EFE_{cycle} = -2\ J$

$\therefore EFM_{cycle} > 0$

$\therefore$ Generator.

Problem 4. (25 points.)

A dynamic system is modeled as:

$$\dot{x}_1 = -6x_1 + 2x_2$$
$$\dot{x}_2 = x_1^2 - 2x_2 + 5$$

a) Find all equilibrium points.
b) Determine the eigenvalues at each equilibrium point.
c) Determine which equilibrium points are stable and which are unstable.
d) For initial conditions $x_1(0) = 0$ and $x_2(0) = 0$ find the responses for x_1 and x_2 at time equals 0.01 and 0.02 seconds using Euler's method with a time step of 0.01 seconds.

a) $\dot{x} = 0$

$-6x_1 + 2x_2 = 0$ ①

$x_1^2 - 2x_2 + 5 = 0$ ②

①: $x_2 = 3x_1$

②: $x_1^2 - 2(3x_1) + 5 = 0$

$x_1^2 - 6x_1 + 5 = 0$

$(x_1 - 5)(x_1 - 1) = 0$

$x_1 = 5, 1$

$x_2 = 15, 3$

$x^{eq} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

b) $A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$

$= \begin{bmatrix} -6 & 2 \\ 2x_1 & -2 \end{bmatrix}$

For $x^{eq} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$

$A = \begin{bmatrix} -6 & 2 \\ 10 & -2 \end{bmatrix}$

$|\lambda I - A| = \begin{vmatrix} \lambda+6 & -2 \\ -10 & \lambda+2 \end{vmatrix} = 0$

$(\lambda+6)(\lambda+2) - 20 = 0$

$\lambda^2 + 8\lambda - 8 = 0$

$\lambda = \frac{-8 \pm \sqrt{64 + 4\times 8}}{2}$

$= 0.899, -8.899$

Unstable

For $x^{eq} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$A = \begin{bmatrix} -6 & 2 \\ 2 & -2 \end{bmatrix}$

$|\lambda I - A| = \begin{vmatrix} \lambda+6 & -2 \\ -2 & \lambda+2 \end{vmatrix} = 0$

$(\lambda+6)(\lambda+2) - 4 = 0$

$\lambda^2 + 8\lambda + 8 = 0$

$\lambda = \frac{-8 \pm \sqrt{64 - 4\times 8}}{2}$

$= -1.17, -6.83$

Stable

c) $\begin{bmatrix} 5 \\ 15 \end{bmatrix}$ is unstable

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is stable d)

$$X(0.01) = X(0) + f(X(0))\Delta t$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0+0 \\ 0+0+5 \end{bmatrix} 0.01$$

$$= \begin{bmatrix} 0 \\ 0.05 \end{bmatrix}$$

$$X(0.02) = X(0.01) + f(X(0.01))\Delta t$$

$$= \begin{bmatrix} 0 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0+2\times 0.05 \\ 0-2\times 0.05+5 \end{bmatrix} 0.01$$

$$= \begin{bmatrix} 0.001 \\ 0.099 \end{bmatrix}$$